



Implementation of integer order PID controller and fractional order PID controller using genetic algorithm for maglev system

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ABSTRACT

Maglev is the latest technology which has been implemented in most of the foreign countries. It is a method by which an object is suspended in air by means of magnetic force. As it does not have moving parts, the wear and tear of the parts are minimal. But the problem with the magnetic levitation system is its stability. So in order to stabilize the system, several controllers must be used. This paper chooses integer order PID (IOPID) and fractional order PID (FOPID) controllers to stabilize the system. As PID controllers alone cannot meet the required specifications, an optimization technique is used to find out the controller parameters. Due to its high potential for global optimization, genetic algorithm (GA) has been used to tune the controller parameters. On an overall, this paper presents a study on the comparison between IOPID and FOPID controllers with and without genetic algorithm (GA) in magnetic levitation system.

Keywords: Magnetic levitation, IOPID, FOPID, Genetic algorithm, PID

1. INTRODUCTION

All practical maglev systems are inherently open loop unstable one. So it is necessary to implement controllers to stabilize the system. Here in this paper, two PID controllers namely IOPID and FOPID controllers are used for the stabilization. Fractional order controllers are more efficient than integer order controllers [7]. On using FOPID it could satisfy at most 5 robustness criteria as compared to the usual classical PID controller. IOPID controllers only have three controller parameters they are; (proportional gain K_p , integral gain K_i , and derivative gain K_d) to be tuned for three robustness criteria. But FOPID has additional two parameters λ and μ which are orders of integrator operator and derivative operator respectively. On having 5 parameters FOPID can provide more adjustable time and frequency responses of the control system allowing fulfillment

of better as well as robust performance. GA is used for tuning of controller parameters to achieve optimal performance indices. GA plays an important role in the evaluation of better solutions [8]. A laboratory model of the maglev system is chosen in this paper. Initially, a complete non-linear model of the system is developed [2]. Based on the linearized model, a constant gain IOPID and FOPID are considered. Even though PID controllers are used, the system is not able to attain required specifications. So in order to meet required specifications GA based IOPID and FOPID controllers are implemented in the system [4-5-6]. All the simulation results are analyzed.

2. MAGNETIC LEVITATION SYSTEM MODELLING

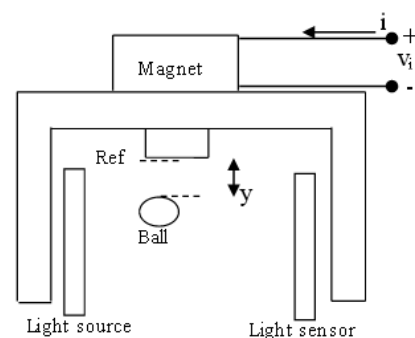


Fig. 1: Schematic of a magnetic levitation system

The layout of a typical magnetic levitation system is illustrated in fig. 1 [2]. This arrangement involves the adjustment of magnetic energy or force in order to balance or counteract the gravitational pull exerted on the object (a small light ferromagnetic ball in this case). The system consists of an electromagnet, a steel ball, a light source and a light detector. The control voltage v_i is applied to the system and on controlling the input voltage the current i flowing through the coil can be controlled. As magnetic flux ϕ is directly

proportional to the current flowing through the coil i , the position of the ball y can be controlled on varying the applied voltage.

Restricted to the vertical direction only, the motion of the ball is monitored by a properly arranged pair of a light emitter and a light detector so that the instantaneous position of the ball can be fed back for the purpose of control computation. This control effort (generated by an electromagnetic circuit) is to ensure that the ball is brought to, and kept at, the desired position. As the ball's position deviates, due to an external disturbance, from the set point, the sensor output changes accordingly so that the right amount of control effort is computed and used to bring the ball back to the set point and keep it there.

The electromechanical model of the maglev system illustrated in Fig.1 can be classified into two systems:

1. A mechanical system and
2. An electrical system

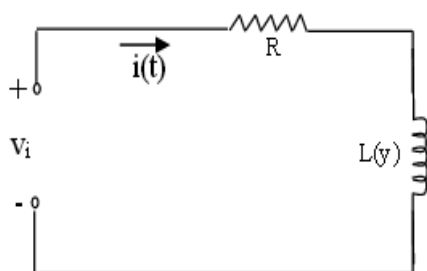


Fig.2: Electrical circuit subsystem of the maglev system

Fig.2 is the representation of the electric circuit subsystem of the magnetic levitation system. It is a series combination of a linear resistor, with resistance R , and a non-linear inductor, with inductance $L(y)$. Where R and L are the resistance and inductance of the electromagnetic coil.

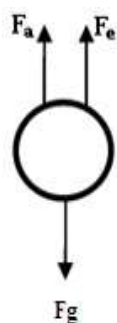


Fig. 3: A free-body diagram showing forces acting on the ball

The free body diagram showing forces acting on the ball is illustrated in Fig.3. When the steel ball is placed under the electromagnet, it encounters three forces- the electromagnetic force F_e , accelerating force due to the mass of the ball F_a and the gravitational force F_g . In order to suspend the ball, the sum of forces F_a and F_e should be maintained equal to the gravitational force F_g .

2.1. Non-linear model the system

However, the magnetic levitation has unstable nonlinear dynamics which should be taken in the count. Most of the contributions require measurements of position, velocity and electric current and thus state observers should be synthesized to estimate the unavailable signals of the nonlinear dynamical system. To determine the complete model of the system, two important dynamic equations, one representing the variations of

the magnetic flux with time (based on Fig . 2) and the other the Newtonian equation of motion of the ball based on forces acting on its as shown in Fig.3 are required.

From Fig.2, when a voltage is applied to the coil results in Fig.3 are required.

From Fig.2, when a voltage is applied to the coil results in a current i governed by the differential equation:

$$\frac{d\phi(t,y)}{dt} + Ri(t) = v_i \tag{1}$$

Where $\phi(t,y)$ is the magnetic flux in Webers, $i(t)$ is the current in amperes, R is the resistance in ohms, v_i is the source voltage in volts, and t is time in second.

Since the magnetic flux around a coil is directly proportional to the current flow in the coil inductance being a factor of proportionality, thus,

$$\phi(t, y) = L(y)i(t) \tag{2}$$

Differentiating (2) with respect to time and substituting the result into (1) yield

$$L(y) \frac{di(t)}{dt} + \left[R + \frac{dL(y)}{dy} \frac{dy}{dt} \right] i(t) = v_i \tag{3}$$

Where $y(t)$ is the distance between the electromagnet and the ball, and $L(y)$ is the total inductance of the circuit in Henry.

Also, from Fig. 3,

$$F_a + F_e = F_g \tag{4}$$

Where F_a is the accelerating force, due to the mass of the ball, F_e is the magnetic force, and F_g is the gravitational force.

Since,

$$F_a = m \frac{d^2y}{dt^2} \text{ and } F_g = mg$$

Therefore, (4) can be rewritten as

$$m \frac{dv}{dt} = mg - F_e \tag{5}$$

In (5), m is the mass of the ball in kg, v is the velocity of the ball in m/s , and g is the acceleration due to gravity in m/s^2 .

Equation (3) and (5), which constitute the mathematical representation of the system, can be developed further by redefining $L(y)$ and F_e and finding appropriate expressions for them, respectively, as shown by the following derivations.

$$L(y) = L_c + L_b \tag{6}$$

L_c Which is fixed, is the inductance due to electromagnet coil; L_b is the inductance due to ball. Because L_b is inversely proportional to the distance between the electromagnet and the ball, it implies that if L_0 is the inductance that corresponds to a set point position y_0 , then the inductance L_b , that corresponds to an instantaneous position, y , is expressed as

$$L_b = \frac{L_0 y_0}{y} \tag{7}$$

Therefore, putting (7) in (6) gives

$$L(y) = L_c + \frac{L_0 y_0}{y} \tag{8}$$

Further, the magnetic force, F_e , is defined as the rate of change of work done with distance as the ball is moved from one position to the other by the force, and is given as:

$$F_e = - \frac{dW}{dy} \tag{9}$$

Where W is the energy stored in the magnetic field is:

$$W = \frac{1}{2} L(y) i^2$$

Hence, (9) gives

$$F_e = \frac{1}{2} L_0 y_0 \frac{i^2}{y^2} \tag{10}$$

Which further can be reduced to:

$$F_e = K \frac{i^2}{y^2} \quad (11)$$

Where K is called magnetic force constant = $\frac{1}{2}L_0y_0$

Let the state variables and inputs be defined as;

$$x_1 = y; x_2 = dy/dt; x_3 = i; u = v_i$$

Now substituting (8) into (3) and (11) into (5), we get the equivalent nonlinear model of the system,

$$\frac{dx_1}{dt} = x_2 \quad (12)$$

$$\frac{dx_2}{dt} = g - \frac{K}{m} \left[\frac{x_3}{x_1} \right]^2 \quad (13)$$

$$\frac{dx_3}{dt} = \frac{1}{L(y)} u - \left[\frac{R}{L(y)} - \frac{2K}{L(y)} \frac{x_2}{x_1^2} \right] x_3 \quad (14)$$

2.2 A linearized model of the system

As can be seen in the model just developed, the maglev system is nonlinear. To improving the system performance for small range operation the above nonlinear model is linearized about a nominal operating point $x_0(t)$, which corresponds to a nominal input, u_0 , using a Taylor series[3].

By using the Taylor series expansion we get the linearized model as [2],

$$\frac{d\Delta x_1}{dt} = \Delta x_2 \quad (15)$$

$$\frac{d\Delta x_2}{dt} = \frac{2K}{m} \frac{x_{03}^2}{x_{01}^3} \Delta x_1 - \frac{2K}{m} \frac{x_{03}}{x_{01}^2} \Delta x_3 \quad (16)$$

$$\frac{d\Delta x_3}{dt} = \frac{4Kx_{02}x_{03}}{Lx_{01}^3} \Delta x_1 + \frac{2Kx_{03}}{Lx_{01}^2} \Delta x_2 - \left[\frac{R}{L} - \frac{2Kx_{02}}{Lx_{01}^2} \right] \Delta x_3 + \frac{1}{L} \Delta u \quad (17)$$

2.3 State space model of the system

From these equations the state space model of the system is as follows;

$$\begin{bmatrix} \Delta x_1 \\ \Delta x_2 \\ \Delta x_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ \frac{2K}{m} \frac{x_{03}^2}{x_{01}^3} & 0 & -\frac{2K}{m} \frac{x_{03}}{x_{01}^2} \\ \frac{4Kx_{02}x_{03}}{Lx_{01}^3} & \frac{2Kx_{03}}{Lx_{01}^2} & -\left[\frac{R}{L} - \frac{2Kx_{02}}{Lx_{01}^2} \right] \end{bmatrix} \begin{bmatrix} \Delta x_1 \\ \Delta x_2 \\ \Delta x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \frac{1}{L} \end{bmatrix} \Delta u \quad (18)$$

$$\Delta y = [1 \ 0 \ 0] \begin{bmatrix} \Delta x_1 \\ \Delta x_2 \\ \Delta x_3 \end{bmatrix} \quad (19)$$

Now, the nominal operating point of the system can be deduced by considering the behavior of the system at an equilibrium point.

Thus the equation becomes,

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ \frac{2KI^2}{my_0^2} & 0 & -\frac{2KI}{my_0^2} \\ 0 & 0 & -\left[\frac{R}{L} \right] \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \frac{1}{L} \end{bmatrix} \Delta u \quad (20)$$

$$y = [1 \ 0 \ 0] \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \quad (21)$$

Results, where $I=x_{03}$; $y_0 = x_{01}$. Note that the incremental symbol, Δ has been dropped. While this makes the model appear more compact, however it does not change the meaning and interpretation of the model. Also in the same equation, L has been assumed to be equivalent to L_c since $L_c \gg L_0$ and under a properly tuned compensator, $y=y_0$. On converting the state space model into transfer function following equation is formed:

$$\frac{Y(s)}{U(s)} = \frac{\frac{1-2KI}{Lmy_0^2}}{s^3 + \frac{R}{L_c}s^2 - \frac{2KI^2}{my_0^3}s - \frac{2KI^2}{my_0^3} \frac{R}{L_c}} \quad (22)$$

3.MAGNETIC LEVITATION DESIGN

For system design following numerical values are used;

Table 1: Numerical values of parameters

Parameter	Value	Unit
R	31.1	Ω
L_c	0.109	H
g	9.81	m/s^2
K	0.000659	Nm^2/A^2
m	0.01058	kg
I	0.125	A
y_0	0.01	m

If numerical values are inserted into the equation (22) the transfer function can modify as;

$$\frac{Y(s)}{U(s)} = \frac{-1419.6}{s^3 + 283.50s^2 - 1946.5s - 551830} \quad (23)$$

From equation (23), it is obvious that system is unstable as one pole is located at the right half of s-plane.

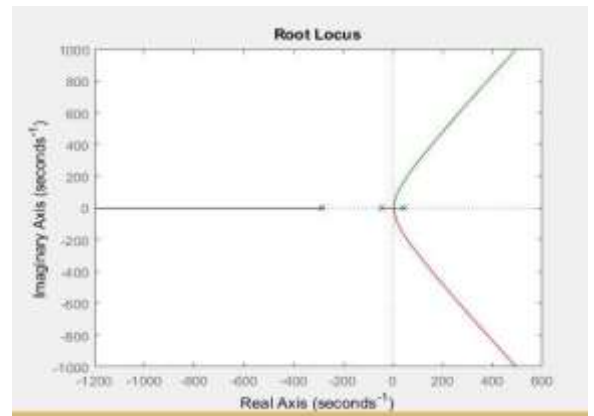


Fig. 4: Root locus of magnetic levitation system

To verify whether the simple gain adjustment will stabilize the system, a constant gain compensator is used. i.e., the transfer function becomes:

$$TF = \frac{221457}{s^3 + 283.50s^2 - 1946.5s - 551830} \quad (25)$$

Our required specifications are as follows

Table 2: Required Specifications

Controller parameter	VALUE
Peak overshoot	$\leq 5\%$
Rise time	≤ 0.15 sec
Settling time	≤ 0.5 sec

4. SIMULATION AND ANALYSIS

This section covers implementation of Integer order and Fractional order PID controller for the maglev system. Mathematical equations of IOPID and FOPID controllers are given as:

$$C(s) = K_p + \frac{K_i}{s} + K_d s \quad (26)$$

$$C(s) = K_p + \frac{K_i}{s^\lambda} + K_d s^\mu \quad (27)$$

For the simulation of IOPID controller, the parameters obtained from PID tuner is used. While the response of FOPID controller is obtained with the help of FOMCON toolbox. The simulation process is done through MATLAB and the simulation model for IOPID controller is shown in Fig. 5.

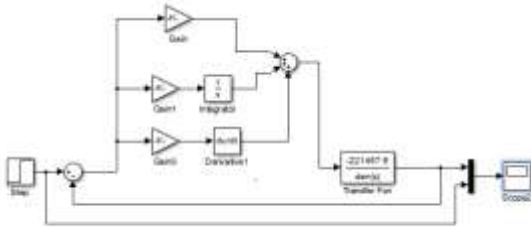


Fig 5: A Simulation model for IOPID controller

The responses for both the controllers were obtained and analyzed. The following table shows the specifications obtained from the responses. Fig. 6 and Fig. 7 show system responses for IOPID and FOPID controller respectively.

Table 3: Controller Parameters

	K_p	K_i	λ	K_d	μ	Over shoot [%]	Settling Time [sec]	Rise Time [sec]
IO PID	25 .4	6 7	-	0. 46	-	15.5	1	0.05
FO PID	29 .4	3 2	0 0 0 2	55	1. 85	5	0.11	0.002

From the specifications mentioned in above table it is clear that on using FOPID controller, rise time and settling time has improved when compared to IOPID controller. But FOPID controller has more overshoot than IOPID controller. Also, both the controllers cannot meet the required specifications. So in order to meet required specifications, genetic algorithm optimization technique is used.



Figure 6: Closed loop response of IOPID controller

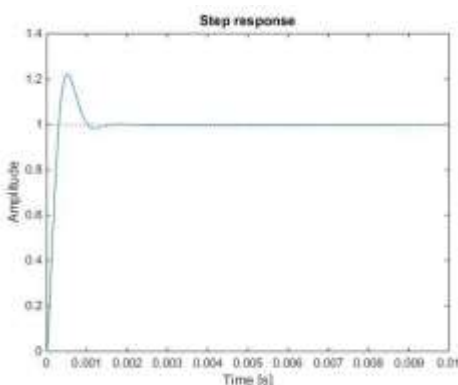


Figure 7: Closed loop response of FOPID controller

In the control system difference between reference and output signal clamping as an error which is actually input signal of the controller. The controller generates an output signal according to error signal value. The main purpose is defined that on minimizing the error signal reference signal is tracked by the closed loop output signal. Fitness function can be evaluated

through error value. If fitness function is minimized the error will be reduced. To minimize the fitness function genetic algorithm based controller is used [1].

Here genetic algorithm optimization is done through GA toolbox. As a fitness function, Integral Time weighed Absolute Error (ITAE) is used. IOPID controller parameters K_p, K_i, K_d are limited lower and upper as $[0 \ 0 \ 0] - [100 \ 100 \ 100]$. FOPID controller parameters K_p, K_i, K_d, λ and μ are bounded as $[0 \ 0 \ 0 \ 0 \ 0] - [100 \ 100 \ 100 \ 2 \ 2]$.

The optimization process is done for both FOPID and IOPID controllers in respect to given above specifications. Following table clarifies results of optimization process and Fig.7 and Fig.8 show the system responses with GA.

Table 4: Controller Parameters

	K_p	K_i	λ	K_d	μ	Overs hoot [%]	Settling Time [sec]	Rise Time [sec]
IO PID	20 .4 8	86	-	1. 21	-	22	0.62	0.15
FO PID	0. 00 3	1. 63	0. 50	1. 46	0. 53	25	.001	0.00 03

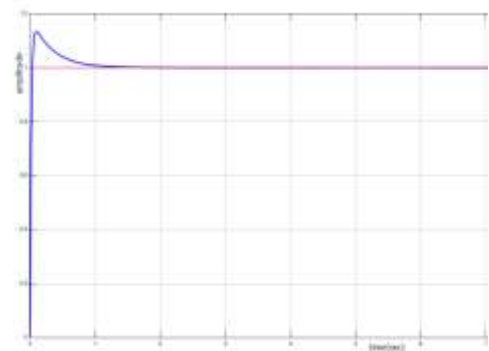


Figure 8: Closed loop response of IOPID controller using GA

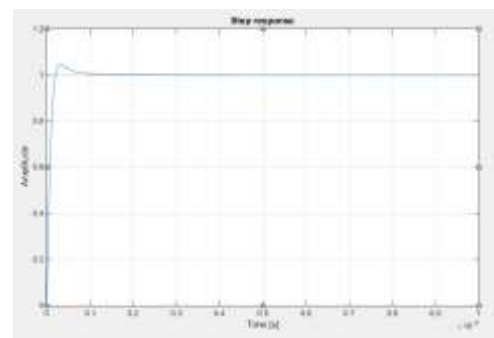


Figure 9: Closed loop response of FOPID controller using GA

On analyzing the responses it is clear that FOPID controller with GA can provide better performance characteristics than IOPID controller. And the characteristics of its response can meet the required specifications of the system.

5. RESULT

Stabilization of magnetic levitation system on satisfying the specifications has been the focus of this paper. From the

responses obtained from the simulation without using any optimization technique, the system cannot meet the required specifications. So genetic algorithm based controllers were implemented. On using GA, optimized values of controller parameters were obtained which improves the responses of controllers. On an overall analysis, we can conclude that FOPID controller is far better than IOPID controller and FOPID controller with GA is able to meet the required specifications as it has more parameters than integer order controller. As an extension, an adaptive controller can be designed and applied to the maglev system for performance enhancement.

6. REFERENCES

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