

EXERCISE 17B

For SSC CHSL Exam

1. For what value(s) of
- k
- will the expression

$$p + \frac{1}{9}\sqrt{p} + k^2 \text{ be a perfect square?}$$

SSC CHSL 10/06/2022 (Shift-2)

(a) $k = \pm \frac{1}{8}$ (b) $k = \pm \frac{1}{9}$

(c) $k = \pm \frac{1}{21}$ (d) $k = \pm \frac{1}{18}$

2. If
- $x + 2y = 10$
- and
- $2xy = 9$
- , then one of the value of
- $x - 2y$
- is:

SSC CHSL 10/06/2022 (Shift-2)

(a) 8 (b) 6

(c) 10 (d) 12

3. If
- $x^2 - 9x + 1 = 0$
- , what is the value of
- $x^8 - 6239 + 1 = ?$

SSC CHSL 9/06/2022 (Shift-3)

(a) 1 (b) 0

(c) -1 (d) 2

4. If
- $(ab + bc + ca) = 0$
- , then what is the value of

$$\left(\frac{1}{a^2 - bc} + \frac{1}{b^2 - ca} + \frac{1}{c^2 - ab} \right) ?$$

SSC CHSL 9/06/2022 (Shift-1)

(a) 2 (b) 0

(c) 1 (d) $a + b + c$

5. If
- $x - y = 25$
- and
- $xy = 78$
- , then what is the value of
- $x^2 + y^2$
- ?

SSC CHSL 9/06/2022 (Shift-1)

(a) 625 (b) 781

(c) 103 (d) 756

6. If
- $2z = x + y$
- , then the value of
- $\frac{x}{x-z} + \frac{y}{y-z}$
- .

SSC CHSL 9/06/2022 (Shift-1)

(a) 0 (b) 1

(c) 2 (d) 5

7. If
- $x^3 + y^3 = 416$
- and
- $x + y = 8$
- , then find
- $x^4 + y^4$

SSC CHSL 8/06/2022 (Shift-3)

(a) 3002 (b) 3204

(c) 3004 (d) 3104

8. If
- $px^3 + x^2 + 3x + q$
- is exactly divisible by
- $(x + 2)$
- and
- $(x - 2)$
- , then the values of
- p
- and
- q
- are

SSC CHSL 8/06/2022 (Shift-3)

(a) $p = -\frac{3}{4}$ and $q = 4$ (b) $p = \frac{3}{4}$ and $q = 4$

(c) $p = \frac{3}{4}$ and $q = -4$ (d) $p = -\frac{3}{4}$ and $q = -4$

9. If
- $x - \frac{1}{x} = 11$
- and
- $x > 0$
- , what is the value of

$$\left(x^2 - \frac{1}{x^2} \right) ?$$

SSC CHSL 8/06/2022 (Shift-2)

(a) $-11\sqrt{123}$ (b) $55\sqrt{5}$

(c) $11\sqrt{123}$ (d) $-55\sqrt{5}$

10. What is the value of the following expression?

$$2^2 \left(\frac{x^a}{x^b} \right)^{(a+b)} \times 3^2 \left(\frac{x^b}{x^c} \right)^{(b+c)} \times 6^{-2} \left(\frac{x^c}{x^a} \right)^{(a+c)}$$

SSC CHSL 8/06/2022 (Shift-2)

(a) 1 (b) 0

(c) 4 (d) 9

11. If
- $x = \sqrt{10} + \sqrt{11}$
- ,
- $y = \sqrt{10} - \sqrt{11}$
- , then value of

$$7x^2 - 50xy + 7y^2$$

SSC CHSL 8/06/2022 (Shift-1)

(a) 344 (b) 704

(c) 1360 (d) 386

12. If
- $x + y + z = 0$
- , then find the value of the

$$\frac{x^2}{x^2 - yz} + \frac{y^2}{y^2 - xz} + \frac{z^2}{z^2 - xy}$$

SSC CHSL 07/06/2022 (Shift-3)

(a) 2 (b) 1

(c) 3 (d) 0

13. If
- $P(x) = (x^3 - 8)(x + 1)$
- and
- $Q(x) = (x^3 + 1)(x - 2)$
- the LCM of
- $P(x)$
- and
- $Q(x)$
- is :

SSC CHSL 07/06/2022 (Shift-2)

(a) $(x^2 + 2x + 4)(x^2 + 4x + 1)$

(b) $(x + 1)(x - 2)(x^2 + 2x + 4)(x^2 - x + 1)$

(c) $(x + 1)^2(x - 2)^2(x^2 + 2x + 4)(x^2 + 4x + 1)$

(d) $(x - 2)(x + 1)$

14. If $x + y = 13$, then $(x - 8)^3 + (y - 5)^3$ is :

SSC CHSL 7/06/2022 (Shift-2)

- (a) 2197 (b) 0
(c) 169 (d) 13

15. If $a + b = \sqrt{7}$ and $a - b = \sqrt{5}$, then the value of $8ab(a^2 + b^2) - (a^2 - b^2)$

SSC CHSL 7/06/2022 (Shift-1)

- (a) 19 (b) 23
(c) 27 (d) 21

16. If $p = 38$, then the value of $p(p^2 + 3p + 3)$ is _____.

SSC CHSL 6/06/2022 (Shift-3)

- (a) 39313 (b) 59319
(c) 39318 (d) 59318

17. If $a + b = 8$ and $a - b = 6$ then find the value of ' ab '.

SSC CHSL 6/06/2022 (Shift-2)

- (a) 6 (b) 8
(c) 5 (d) 7

18. If $x^2 + (4 - \sqrt{3})x - 1 = 0$, then what is the value of

$x^2 + \frac{1}{x^2}$? SSC CHSL 12/08/2021 (Shift-3)

- (a) $9 - 8\sqrt{3}$ (b) $21 - 12\sqrt{3}$
(c) $21 - 8\sqrt{3}$ (d) $17 - 8\sqrt{3}$

19. If $x + \frac{1}{x} = \sqrt{3}$, then one of the value of $x^3 - \frac{1}{x^3}$ is:

SSC CHSL 12/08/2021 (Shift-3)

- (a) $4\sqrt{11}$ (b) 32
(c) $4\sqrt{13}$ (d) 36

20. If $(7x - 10y) = 8$ and $xy = 5$, then what is the value of $49x^2 + 100y^2$?

SSC CHSL 12/08/2021 (Shift-2)

- (a) 764 (b) 632
(c) 623 (d) 746

21. The value of $a^3 + b^3 + c^3 - 3abc$, when $a = 125, b = 127$ and $c = 129$, is

SSC CHSL 12/08/2021 (Shift-2)

- (a) 4725 (b) 4572
(c) 4752 (d) 3752

22. If $a = \frac{\sqrt{5} + 2}{\sqrt{5} - 2}$ and $b = \frac{\sqrt{5} - 2}{\sqrt{5} + 2}$, then the value of

$2a^2 + 2b^2 - 5ab$ is equal to:

SSC CHSL 12/08/2021 (Shift-2)

- (a) 693 (b) 649
(c) 635 (d) 639

23. Given that

$3\sqrt{3}x^3 - 8y^3 = (\sqrt{3}x + Ay)(3x^2 + By^2 + Cxy)$, the value of $(A^2 + B^2 - C^2)$ is :

SSC CHSL 12/08/2021 (Shift-1)

- (a) 4 (b) 8
(c) 12 (d) 0

24. If $3x - 2y + 3 = 0$, then what will be the value of $27x^3 + 54xy + 30 - 8y^3$?

SSC CHSL 12/08/2021 (Shift-1)

- (a) -27 (b) 57
(c) -57 (d) 3

25. If $\sqrt{x} + \frac{1}{\sqrt{x}} = 2\sqrt{3}$, then what will be the value of

$x^4 + \frac{1}{x^4}$? SSC CHSL 12/08/2021 (Shift-1)

- (a) 10402 (b) 9606
(c) 9602 (d) 10406

26. If $3a - b = 1$ and $ab = 4$, then one of the value of $(9a^2 - b^2)$ is:

SSC CHSL 11/08/2021 (Shift-3)

- (a) 7 (b) 8
(c) 5 (d) 6

27. If $a + 5b = 25$ and $ab = 20$, then one of the value of $(a - 5b)$ is:

SSC CHSL 11/08/2021 (Shift-3)

- (a) 15 (b) 14
(c) 16 (d) 13

28. If $x = 555, y = 556$ and $z = 557$, then find the value of the $x^3 + y^3 + z^3 - 3xyz$.

SSC CHSL 11/08/2021 (Shift-3)

- (a) 5006 (b) 5002
(c) 5008 (d) 5004

29. If $a - b = 7$ and $a^2 + b^2 = 169$ where $a, b > 0$, then the value of $3a + b$ is

SSC CHSL 11/08/2021 (Shift-2)

- (a) 44 (b) 41
(c) 38 (d) 46

30. If $a - \frac{24}{a} = 5$, where $a > 0$, then the value of the

$a^2 + \frac{64}{a^2}$ is : SSC CHSL 11/08/2021 (Shift-2)

- (a) 45 (b) 56
(c) 60 (d) 65

31. x, y are two positive numbers such that $x > y$. If $x^4 + y^4 = 706$ and $xy = 15$, then the value of the $2x + 3y$ is:

SSC CHSL 11/08/2021 (Shift-2)

- (a) 19 (b) 20
(c) 18 (d) 15

32. $3x + 5y = 14$ and $xy = 6$, then what is the value of $9x^2 + 25y^2$? **SSC CHSL 11/08/2021 (Shift-1)**
- (a) 16 (b) 14
(c) 20 (d) 182
33. If $a^2 + b^2 + c^2 + 48 = 8(a + b + c)$, then the value of the $\sqrt[3]{a^3 - b^3 + c^3}$? **SSC CHSL 11/08/2021 (Shift-1)**
- (a) 6 (b) 4
(c) 3 (d) 2
34. If $x^4 + x^{-4} = 47$, $x > 0$, then the value of the $(2x - 3)^2$ is: **SSC CHSL 11/08/2021 (Shift-1)**
- (a) 9 (b) 3
(c) 5 (d) 7
35. If $a + b + c = 11$ and $ab + bc + ca = 115$ then what is the value of $a^3 + b^3 + c^3 - 3abc$? **SSC CHSL 10/08/2021 (Shift-3)**
- (a) 368 (b) 638
(c) 386 (d) 836
36. If $(x - 1.5)^3 + (x - 4)^3 + (x - 3.5)^3 = 3(x - 1.5)(x - 4)(x - 3.5)$, then what is the value of the x ? **SSC CHSL 10/08/2021 (Shift-3)**
- (a) 1 (b) 3
(c) 6 (d) 9
37. If $a + b + c = 5$, $a^2 + b^2 + c^2 = 27$ and $a^3 + b^3 + c^3 = 125$, then the value of the $\frac{abc}{5}$ is: **SSC CHSL 10/08/2021 (Shift-3)**
- (a) -1 (b) 5
(c) -5 (d) 1
38. If $1 + 4x^2 + 16x^4 = 512$ and $1 - 2x + 4x^2 = 64$, then the value of the $1 + 2x + 4x^2$ is: **SSC CHSL 10/08/2021 (Shift-2)**
- (a) 6 (b) 8
(c) 12 (d) 10
39. If $x + \frac{1}{3x} = 5$ then the value of the $27x^3 + \frac{1}{x^3}$ will be **SSC CHSL 10/08/2021 (Shift-2)**
- (a) 3240 (b) 3024
(c) 3042 (d) 3420
40. If $x^6 - 6\sqrt{6}y^6 = (x^2 + Ay^2)(x^4 + Bx^2y^2 + Cy^4)$, then what will be the value of $(A^2 - B^2 + C^2)$? **SSC CHSL 10/08/2021 (Shift-1)**
- (a) 27 (b) 42
(c) 36 (d) 18

SOLUTIONS

1. (d) $p + \frac{1}{q}\sqrt{p} + k^2 = A = \sqrt{p}$, $B = k$
 $A^2 + B^2 + 2AB = (A + B)^2$
 $2AB = \frac{1}{9}\sqrt{p}$
 $k = B = \frac{1}{9 \times 2} = \pm \frac{1}{18}$
2. (a) $x + 2y = 10$, $2xy = 9$
 $(x - 2y)^2 = (x + 2y)^2 - 8xy$
 $(x - 2y)^2 = 100 - 36 = 64$
 $x - 2y = 8$
3. (b) $x^2 - 9x + 1 = 0 \Rightarrow x + \frac{1}{x} = 9$
 $x^2 + \frac{1}{x^2} = 9^2 - 1 = 79$
 $x^4 + \frac{1}{x^4} = 79^2 - 1 = 6239$
 $x^8 + 1 = 6239x^4$
 $x^8 - 6239x^4 + 1 = 0$
4. (b) $ab + bc + ca = 0$
 $\frac{1}{a^2 - bc} + \frac{1}{b^2 - ac} + \frac{1}{c^2 - ab}$
 $= \frac{1}{a^2 + ab + ca} + \frac{1}{b^2 + bc + ab} + \frac{1}{c^2 + bc + ca}$
 $= \frac{ab + bc + ca}{abc(a + b + c)} = 0$
5. (b) $x - y = 25$
 $xy = 78$
 $x^2 + y^2 = (x - y)^2 + 2xy$
 $= 25^2 + 2 \times 78$
 $= 781$
6. (c) $2z = x + y$
 $\frac{x}{x - z} + \frac{y}{y - z} = \frac{x(y - z) + y(x - z)}{(x - z)(y - z)}$
 $= \frac{2xy - z(x + y)}{xy - z(x + y) + z^2}$
 $= \frac{2xy - 2z^2}{xy - 2z^2 + z^2}$
 $= \frac{2(xy - z^2)}{xy - z^2}$
 $= 2$

7. (d) $x^3 + y^3 = 416, x + y = 8$
 $(x + y)^3 = x^3 + y^3 + 3xy(x + y)$
 $8^3 = 416 + 3xy(8)$
 $xy = 4$
 $(x + y)^2 = x^2 + y^2 + 2xy$
 $x^2 + y^2 = 64 - 8$
 $= 56$
 $(x^2 + y^2)^2 = x^4 + y^4 + 2x^2y^2$
 $x^4 + y^4 = 56^2 - 2 \times 16$
 $= 3136 - 32$
 $= 3104$

8. (d) $px^3 + x^2 + 3x + q = 0$

$x + 2 = 0$ $x = -2$	$x - 2 = 0$ $x = 2$
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$p(-2)^3 + (-2)^2 + 3(-2) + q = 0$
 $-8p + q = 2$... (i)

$x = 2$
 $p(2)^3 + (2)^2 + 3(2) + q = 0$
 $8p + q = -10$... (ii)

From (i) + (ii)
 $q = -4$
 $p = -\frac{3}{4}$

9. (b) $x - \frac{1}{x} = 11 \Rightarrow x + \frac{1}{x}$
 $= \sqrt{11^2 - 4} = \sqrt{125}$
 $= 5\sqrt{5}$
 $x^2 - \frac{1}{x^2} = \left(x - \frac{1}{x}\right) \times \left(x + \frac{1}{x}\right)$
 $= 11 \times 5\sqrt{5} = 55\sqrt{5}$

10. (a) $2^2 \left(\frac{x^a}{x^b}\right)^{(a+b)} \times 3^2 \left(\frac{x^b}{x^c}\right)^{(b+c)} \times 6^{-2} \left(\frac{x^c}{x^a}\right)^{(a+c)}$
 $= \frac{4 \times 9}{36} \times x^{(a-b)(a+b)} \times x^{(b-c)(b+c)} \times x^{(c-a)(c+a)}$
 $x^{a^2 - b^2 + b^2 - c^2 + c^2 - a^2} = x^0 = 1$

11. (a) $x = \sqrt{10} + \sqrt{11}$
 $y = \sqrt{10} - \sqrt{11}$
 $x^2 = 21 + 2\sqrt{110}$
 $y^2 = 21 - 2\sqrt{110}$

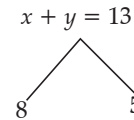
$xy = -1$
 $= 7x^2 - 50xy + 7y^2$
 $= 7(21 + 2\sqrt{110}) - 50(-1) + 7(21 - 2\sqrt{110})$
 $= 147 + 147 + 14\sqrt{110} - 14\sqrt{110} + 50$
 $= 344$

12. (a) $x + y + z = 0 \Rightarrow x = -y - z, y = -x - z, z = -x - y$

$\frac{x^2}{x^2 - yz} + \frac{y^2}{y^2 - xz} + \frac{z^2}{z^2 - xy}$
 $= \frac{x^2}{-xy - xz - yz} + \frac{y^2}{-xy - yz - xz}$
 $+ \frac{z^2}{-zy - zx - xy}$
 $= \frac{x^2 + y^2 + z^2}{-(xy + xz + yz)}$
 $= \frac{-2(xy + yz + xz)}{-(xy + xz + yz)} = 2$

13. (b) $P(x) = (x^3 - 8)(x + 1)$
 $= (x^3 - 2^3)(x + 1)$
 $P(x) = (x - 2)(x^2 + 2x + 4)(x + 1)$
 $Q(x) = (x^3 + 1^3)(x - 2)$
 $Q(x) = (x + 1)(x^2 - x + 1)(x - 2)$
 $\text{LCM} = (x + 1)(x - 2)(x^2 + 2x + 4)(x^2 - x + 1)$

14. (b) Let $x = 8, y = 5$



$(x - 8)^3 + (y - 5)^3 = (8 - 8)^3 + (5 - 5)^3$
 $= 0$

15. (a) $a + b = \sqrt{7}$,
 $a - b = \sqrt{5}$
 $a = \frac{\sqrt{7} + \sqrt{5}}{2}$,
 $b = \frac{\sqrt{7} - \sqrt{5}}{2}$

$ab = \frac{1}{2}$
 $a^2 + b^2 = (a + b)^2 - 2ab$
 $= 7 - 1$
 $= 6$

$8ab(a^2 + b^2) - (a - b)^2 = 8 \times \frac{1}{2} \times 6 - (\sqrt{5})^2$
 $= 24 - 5 = 19$

16. (d) $P = 38$
 $P(p^2 + 3p + 3) = p^3 + 3p^2 + 3p$
 $(p^3 + 3p^2 + 3p + 1) - 1 = (p + 1)^3 - 1$
 $= (38 + 1)^3 - 1$
 $59319 - 1 = 59318$
17. (d) $a + b = 8$
 $a - b = 6$
 $a = \frac{8+6}{2}$
 $= 7$
 $b = 1$
 $ab = 7 \times 1$
 $= 7$
18. (c) $x^2 + (4 - \sqrt{3})x - 1 = 0$
 $x - \frac{1}{x} = \sqrt{3} - 4$
 $x^2 + \frac{1}{x^2} = (\sqrt{3} - 4)^2 + 2$
 $= 3 + 16 - 8\sqrt{3} + 2$
 $= 21 - 8\sqrt{3}$
19. (d) $x + \frac{1}{x} = \sqrt{13}$
 $\Rightarrow x - \frac{1}{x} = \sqrt{(\sqrt{13})^2 - 4}$
 $= 3$
 $x^3 - \frac{1}{x^3} = 3^3 + 3 \times 3$
 $= 36$
20. (a) $7x - 10y = 8$
 $xy = 5$
 $\frac{49x^2 + 100y^2}{(7x)^2 + (10y)^2}$
 $= \frac{(7x - 10y)^2 + 2 \times 7x \times 10y}{8^2 + 2 \times 7 \times 10 \times 5}$
 $= \frac{64 + 700}{8^2 + 700}$
 $= \frac{764}{708}$
21. (b) $a = 125$
 $b = 127$
 $c = 129$
 $a^3 + b^3 + c^3 - 3abc$
 $= \frac{1}{2}(a + b + c)[(a - b)^2$
 $+ (b - c)^2 + (c - a)^2]$
 $= \frac{1}{2}(125 + 127 + 129)(4 + 4 + 16)$
 $= 381 \times 12 = 4572$

22. (d) $a = \frac{\sqrt{5} + 2}{\sqrt{5} - 2}$
 $= 9 + 4\sqrt{5}$
 $b = \frac{\sqrt{5} - 2}{\sqrt{5} + 2}$
 $= 9 - 4\sqrt{5}$
 $a^2 = 81 + 80 + 72\sqrt{5}$
 $= 161 + 72\sqrt{5}$
 $b^2 = 161 - 72\sqrt{5}$
 $ab = (9 + 4\sqrt{5})(9 - 4\sqrt{5})$
 $= 81 - 80$
 $= 1$
 $2a^2 + 2b^2 - 5ab$
 $= 2(161 + 72\sqrt{5}) + 2(161 - 72\sqrt{5}) - 5 \times 1$
 $= 322 + 322 - 5$
 $= 639$
23. (b) $3\sqrt{3}x^3 - 8y^3 = (\sqrt{3}x + Ay)(3x^2 + By^2 + Cxy)$
 $(\sqrt{3}x)^3 - (2y)^3 = (\sqrt{3}x + Ay)(3x^2 + By^2 + Cxy)$
 $A = -2$
 $B = 4$
 $C = 2\sqrt{3}$
 $A^2 + B^2 - C^2 = 4 + 16 - 12$
 $= 8$
24. (d) $3x - 2y + 3 = 0$
 $3x - 2y = -3$
 $(3x - 2y)^3 = (-3)^3$
 $27x^3 + 18xy(3x - 2y) - 8y^3 = -27$
 $27x^3 + 54xy - 8y^3 + 27 = 0$
 $27x^3 + 54xy - 8y^3 + 30 = 3$
25. (c) $\sqrt{x} + \frac{1}{\sqrt{x}} = 2\sqrt{3}$
 $x + \frac{1}{x} = (2\sqrt{3})^2 - 2 = 10$
 $x^2 + \frac{1}{x^2} = 10^2 - 2 = 98,$
 $x^4 + \frac{1}{x^4} = 98^2 - 2 = 9602$
26. (a) $3a - b = 1, ab = 4$
 $(3a + b)^2 = (3a - b)^2 + 12ab$
 $= 1 + 12 \times 4 = 49$

$$3a + b = 7$$

$$9a^2 - b^2 = (3a + b)(3a - b) - 7 \times 1 = 7$$

27. (a) $a + 5b = 25,$

$$ab = 20$$

$$(a - 5b)^2 = (a + 5b)^2 - 20ab$$

$$= 625 - 400 = 225$$

$$a - 5b = \sqrt{225} = 15$$

28. (d) $x = 555$

$$y = 556$$

$$z = 557$$

$$x^3 + y^3 + z^3 - 3xyz$$

$$= \frac{1}{2}(x + y + z)[(x - y)^2 + (y - z)^2 + (z - x)^2]$$

$$= \frac{1}{2} \times 1668(1 + 1 + 4)$$

$$= 5004$$

29. (b) $a - b = 7$

$$\begin{array}{l} \downarrow \quad \downarrow \\ 12 - 5 = 7 \end{array}$$

$$a^2 + b^2 = 169$$

$$\begin{array}{l} \downarrow \quad \downarrow \\ 12^2 + 5^2 = 169 \end{array}$$

$$a = 12$$

$$b = 5$$

$$3a + b = 41$$

30. (d) $\frac{a - 24}{a} = 5$

$$a^2 - 5a - 24 = 0$$

$$a = 8$$

$$a^2 + \frac{64}{a^2} = 64 + \frac{64}{64} = 64 + 1$$

$$= 65$$

31. (a) $x^4 + y^4 = 706$

$$x = 5$$

$$y = 3$$

$$2x + 3y = 10 + 9$$

$$= 19$$

32. (a) $3x + 5y = 14$

$$xy = 6$$

$$9x^2 + 25y^2 = (3x)^2 + (5y)^2$$

$$= (3x + 5y)^2 - 30xy$$

$$= 14^2 - 30 \times 6$$

$$= 196 - 180$$

$$= 16$$

33. (b) $a^2 + b^2 + c^2 + 48 = 8(a + b + c)$

$$a^2 - 8a + 16 + b^2 + 8b - 16 + c^2 - 8c + 16$$

$$= 0$$

$$= (a - 4)^2 + (b - 4)^2 + (c - 4)^2 = 0$$

$$a = 4$$

$$b = 4$$

$$c = 4$$

$$\sqrt[3]{a^3 - b^3 + c^3} = \sqrt[3]{4^3 - 4^3 + 4^3}$$

$$= 4$$

34. (c) $x^4 + \frac{1}{x^4} = 47 \Rightarrow x^2 + \frac{1}{x^2}$

$$= \sqrt{47 + 2}$$

$$= 7$$

$$x + \frac{1}{x} = \sqrt{7 + 2}$$

$$= 3$$

$$x^2 - 3x = -1$$

$$(2x - 3)^2 = 4x^2 + 9 - 12x$$

$$= 4(x^2 - 3x) + 9$$

$$= 4(-1) + 9$$

$$= 5$$

35. (d) $a + b + c = 11,$

$$ab + bc + ca = 15$$

$$a^3 + b^3 + c^3 - 3abc =$$

$$(a + b + c)[(a + b + c)^2 - 3(ab + bc + ca)]$$

$$= 11[11^2 - 3(15)]$$

$$= 11 \times 76$$

$$= 836$$

36. (b) $(x + 1.5)^3 + (x - 4)^3 + (x - 3.5)^3$

$$= 3(x - 1.5)(x - 4)(x - 3.5)$$

$$a^3 + b^3 + c^3 = 3abc$$

$$\text{then } (a + b + c) = 0$$

$$x - 1.5 + x - 4 + x - 3.5 = 0$$

$$3x = 9$$

$$x = 3$$

37. (a) $a + b + c = 5.$

$$a^2 + b^2 + c^2 = 27$$

$$a^3 + b^3 + c^3 = 125$$

$$ab + bc + ca = \frac{(a + b + c)^2 - (a^2 + b^2 + c^2)}{2}$$

$$= \frac{25 - 27}{2} = -1$$

$$a^3 + b^3 + c^3 - 3abc$$

$$= (a + b + c)[a^2 + b^2 + c^2 - (ab + bc + ca)]$$

$$- 3abc = 5(27 + 1) - 125$$

$$- 3abc = 140 - 125$$

$$abc = -5$$

$$\frac{abc}{5} = \frac{-5}{5} = -1$$

38. (b) $1 + 4x^2 + 16x^4 = 512$

$$1 - 2x + 4x^2 = 64$$

$$1 + 2x + 4x^2 = ?$$

$$1 + 4x^2 + 16x^4 = (1 - 2x + 4x^2)(1 + 2x + 4x^2)$$

$$1 + 2x + 4x^2 = \frac{1 + 4x^2 + 16x^4}{1 - 2x + 4x^2}$$

$$= \frac{512}{64} = 8$$

39. (a) $x + \frac{1}{3x} = 5 \Rightarrow 3x + \frac{1}{x} = 15$

$$27x^3 + \frac{1}{x^3} = 15^3 - 15 \times 3 \times 3$$

$$= 3375 - 135 = 3240$$

40. (c) $x^6 - 6\sqrt{6}y^6 = (x^2 + Ay^2)(x^4 + Bx^2y^2 + Cy^4)$

$$(x^2)^3 - (\sqrt{6}y^2)^3 = (x^2 - Ay^2)(x^4 + Bx^2y^2 + Cy^4)$$

$$A = -\sqrt{6}$$

$$B = \sqrt{6}$$

$$C = 6$$

$$A^2 - B^2 + C^2 = (-\sqrt{6})^2 - (\sqrt{6})^2 + 6^2$$

$$= 6 - 6 + 36$$

$$= 36$$

EXERCISE 17C

For SSC CGL and CPO Exams

1. If $a + b + c = 11$ and $ab + bc + ca = 28$, then find the value of the $a^3 + b^3 + c^3 - 3abc$

SSC CGL 19/04/2022 (Shift-3)

- (a) 1639 (b) 407
(c) 2255 (d) 1093

2. If $(x + y)^3 - (x - y)^3 - 3y(2x^2 - 3y^2) = ky^3$ then find the value of k .

SSC CGL 19/04/2022 (Shift-3)

- (a) 10.5 (b) 8
(c) 1 (d) 10

3. If $x^2 - 5x - 1 = 0$, what is the value of

$$\frac{x^6 - x^4 + x^2 - 1}{x^3} ?$$

SSC CGL 19/04/2022 (Shift-2)

- (a) 145 (b) 140
(c) 130 (d) 135

4. If $a^2 + b^2 + 49c^2 + 18 = 2(b + 28c - a)$, then the value of $(2a - b + 7c)$ is :

SSC CGL 19/04/2022 (Shift-2)

- (a) 5 (b) -3
(c) -4 (d) 1

5. If $x + y + z = 7$, $xy + yz + zx = 8$, then what is the value of $x^3 + y^3 + z^3 - 3xyz$?

SSC CGL 19/04/2022 (Shift-1)

- (a) 200 (b) 150
(c) 125 (d) 175

6. If $\sqrt{x} - \frac{1}{\sqrt{x}} = \sqrt{3}$, then what is the value of

$$x^4 + \frac{1}{x^4} ?$$

SSC CGL 18/04/2022 (Shift-3)

- (a) 531 (b) 721
(c) 623 (d) 527

7. If $2\sqrt{2}x^3 - 3\sqrt{3}y^3$

$$= (\sqrt{2}x - \sqrt{3}y)(Ax^2 - Bxy + Cy^2)$$

then the value of the $\sqrt{A^2 + B^2 + C^2}$ is :

SSC CGL 18/04/2022 (Shift-2)

- (a) $\sqrt{19}$ (b) $\sqrt{11}$
(c) $\sqrt{17}$ (d) $\sqrt{21}$

8. If $a^2 + b^2 + c^2 + 49c^2 + 18 = 2(b - 28c - a)$ then the value of the $(a - b - 7c)$ is ?

SSC CGL 18/04/2022 (Shift-2)

- (a) 4 (b) 3
(c) 2 (d) 1

9. If $a + b - c = 5$ and $ab - bc - ca = 10$, then find the value of the $a^2 + b^2 + c^2$.

SSC CGL 18/04/2022 (Shift-1)

- (a) 40 (b) 5
(c) 45 (d) 15

10. If $\left(x^2 + \frac{1}{x^2}\right) = 23$, $x > 0$, what is the value of

$$\left(x^3 + \frac{1}{x^3}\right) = ?$$

SSC CGL 18/04/2022 (Shift-1)

- (a) 140 (b) 110
(c) -110 (d) -140

11. If $x + y + z = 2$, $xy + yz + zx = -11$, and $xyz = -12$, then what is the value of the $x^3 + y^3 + z^3$?

SSC CGL 13/04/2022 (Shift-3)

- (a) 36 (b) 38
(c) 40 (d) 42

12. If $x + y + z = 18$, $xyz = 81$ and $xy + yz + zx = 90$, then find the value of the $\sqrt{x^3 + y^3 + z^3 + 3xyz}$

SSC CGL 13/04/2022 (Shift-2)

- (a) 6 (b) 12
(c) 9 (d) 10

13. If $27x^3 - 64y^3 = (Ax + By)(Cx^2 - Dy^2 + 12xy)$ then the value of $4A + B + 3C + 2D$ is:

SSC CGL Tier II 03/02/2022

- (a) 5 (b) 3
(c) -3 (d) -4

14. If $x = 32.5$, $y = 34.6$ and $z = 30.9$ then the value of $x^3 + y^3 + z^3 - 3xyz$ is $0.98k$, where k is equal to
SSC CGL Tier II (03/02/2022)
 (a) 1,033 (b) 933
 (c) 1,026 (d) 921
15. If $x^3 - 3x + 1 = 0$, then the value of $\frac{\left(x^4 + \frac{1}{x^2}\right)}{\left(x^2 + 5x + 1\right)}$ is :
SSC CGL Tier II (03/02/2022)
 (a) $\frac{9}{4}$ (b) $\frac{27}{8}$
 (c) $\frac{5}{2}$ (d) 2
16. If $x^2 - \sqrt{7}x + 1 = 0$, then what is the value of the $x^5 + \frac{1}{x^5}$?
SSC CGL 29/01/2022 (Shift-1)
 (a) $25\sqrt{7}$ (b) $21\sqrt{7}$
 (c) $27\sqrt{7}$ (d) $19\sqrt{7}$
17. If $x + \frac{1}{x} = 7$, then $x^2 + \frac{1}{x^2}$ is equal to :
SSC CGL 28/08/2021 (Shift-1)
 (a) 47 (b) 49
 (c) 61 (d) 51
18. If $2x + 3y + 1 = 0$, then what is the value of $(8x^3 + 8 + 27y^3 - 18xy)$?
SSC CGL 24/08/2021 (Shift-1)
 (a) 7 (b) -9
 (c) -7 (d) 9
19. If $a^4 + b^4 + a^2b^2 = 273$ and $a^2 + b^2 - ab = 21$, then one of the value of $\left(\frac{1}{a} + \frac{1}{b}\right)$ is :
SSC CGL 28/08/2021 (Shift-1)
 (a) $-\frac{9}{4}$ (b) $-\frac{3}{4}$
 (c) $\frac{9}{8}$ (d) $\frac{3}{2}$
20. If $(2x + y)^3 - (x - 2y)^3 = (x + 3y)[Ax^2 + By^2 + Cxy]$, then what is the value of the $(A + 2B + C)$?
SSC CGL 23/08/2021 (Shift-3)
 (a) 13 (b) 7
 (c) 14 (d) 10
21. If $9(a^2 + b^2) + c^2 + 20 = 12(a + 2b)$, then the value of $\sqrt{6a + 9b + 2c}$ is :
SSC CGL 23/08/2021 (Shift-3)
- (a) 3 (b) 4
 (c) 2 (d) 6
22. If $x + y + z = 2$, $x^3 + y^3 + z^3 - 3xyz = 74$, then the value of $(x^2 + y^2 + z^2)$ is equal to:
SSC CGL 23/08/2021 (Shift-3)
 (a) 22 (b) 29
 (c) 26 (d) 24
23. If $x^4 + x^2y^2 + y^4 = 21$ and $x^2 + xy + y^2 = 3$, then what is the value of $(-xy)$?
SSC CGL 23/08/2021 (Shift-2)
 (a) 2 (b) 1
 (c) -1 (d) -2
24. If $x + \frac{1}{x} = \frac{17}{4}$, $x > 1$, then what is the value of $x - \frac{1}{x}$?
SSC CGL 23/08/2021 (Shift-1)
 (a) $\frac{9}{4}$ (b) $\frac{3}{2}$
 (c) $\frac{8}{3}$ (d) $\frac{15}{4}$
25. If $2x^2 + 7x + 5 = 0$. then what is the value of $x^3 + \frac{125}{8x^3}$
SSC CGL 23/08/2021 (Shift-1)
 (a) $12\frac{5}{8}$ (b) $16\frac{5}{8}$
 (c) $10\frac{5}{8}$ (d) $18\frac{5}{8}$
26. If $8 + 2px^2 - 36x - 27x^3 = (2 - 3x)^3$, then what is the value of p ?
SSC CGL 23/08/2021 (Shift-1)
 (a) 27 (b) 54
 (c) 9 (d) -27
27. If $x - y = 4$ and $x^3 - y^3 = 316$, $y > 0$ then the value of $x^4 - y^4$ is:
SSC CGL 20/08/2021 (Shift-1)
 (a) 2482 (b) 2320
 (c) 2500 (d) 2401
28. What is the coefficient of the x in the expansion of the $(3x - 4)^3$?
SSC CGL 20/08/2021 (Shift-3)
 (a) -144 (b) -108
 (c) 108 (d) 144
29. If $x + y + z = 3$, $xy + yz + zx = -12$ and $xyz = -16$ then the value of $\sqrt{x^3 + y^3 + z^3 + 13}$
SSC CGL 20/08/2021 (Shift-3)

- (a) 11 (b) 9
(c) 10 (d) 8
30. If $\sqrt{x} - \frac{1}{\sqrt{x}} = \sqrt{7}$, then the value of $x^2 + \frac{1}{x^2}$ is:
SSC CGL 20/08/2021 (Shift-2)
- (a) 60 (b) 75
(c) 81 (d) 79
31. If $(56\sqrt{7}x^3 - 2\sqrt{2}y^3) \div (2\sqrt{7}x - \sqrt{2}y)$
 $= Ax^2 + B y^2 - Cxy$,
then find the value of $A + B - \sqrt{14}C$.
SSC CGL 20/08/2021 (Shift-2)
- (a) 19 (b) 10
(c) 58 (d) 38
32. If $\frac{x}{y} + \frac{y}{x} = 2$, ($x, y \neq 0$), then the value of $(x - y)$
is:
SSC CGL 20/08/2021 (Shift-2)
- (a) -2 (b) 1
(c) 2 (d) 0
33. If $(2a + \frac{3}{a} - 1) = 11$, what is the value of
 $(4a^2 + \frac{9}{a^2})$?
SSC CGL 20/08/2021 (Shift-1)
- (a) 110 (b) 148
(c) 132 (d) 121
34. If $a^2 + b^2 + c^2 + 216 = 12(a + b - 2c)$, then
 $\sqrt{ab - bc - ca}$ is
SSC CGL 20/08/2021 (Shift-1)
- (a) $6\sqrt{5}$ (b) $4\sqrt{5}$
(c) $3\sqrt{5}$ (d) $8\sqrt{5}$
35. If $a + b + c = 0$, then what is the value of
 $\frac{(b+c)^2}{bc} + \frac{(c+a)^2}{ca} + \frac{(a+b)^2}{ab}$
SSC CGL 18/08/2021 (Shift-3)
- (a) 1 (b) -3
(c) -1 (d) 3
36. If $x + y + z = 13$, $x^2 + y^2 + z^2 = 133$ and $x^3 + y^3 + z^3 = 847$, then the value of the $\sqrt[3]{xyz}$ is:
SSC CPO 24/11/2020 (Shift-3)
- (a) 8 (b) -9
(c) -6 (d) 7
37. If $a^3 + b^3 = 217$ and $a + b = 7$ then the value of ab is:
SSC CPO 24/11/2020 (Shift-3)

- (a) 6 (b) -1
(c) -6 (d) 7
38. If $a^2 + b^2 + c^2 + 84 = 4(a - 2b + 4c)$, then
 $\sqrt{ab - bc + ca}$ is equal to:
SSC CPO 24/11/2020 (Shift-3)
- (a) $5\sqrt{10}$ (b) $4\sqrt{10}$
(c) $2\sqrt{10}$ (d) $\sqrt{10}$
39. If $ab + bc + cd = 8$ and $a + b + c = 12$ then $a^2 + b^2 + c^2$ is equal to:
SSC CPO 14/03/2019 (Shift-1)
- (a) 160 (b) 144
(c) 134 (d) 128
40. The value of $\frac{(0.013)^3 + (0.007)(0.000049)}{(0.007)^2 + 0.013(0.013 - 0.007)}$ is:
SSC CPO 13/12/2019 (Shift-3)
- (a) 0.07 (b) 0.02
(c) 0.06 (d) 0.04

SOLUTIONS

1. (b) $a + b + c = 11$
 $ab + bc + ca = 28$
 $a^3 + b^3 + c^3 - 3abc = 11(11^2 - 3 \times 28)$
 $= 11(121 - 84)$
 $= 407$
2. (c) $(x + y)^3 - (x - y)^3 - 3y(2x^2 - 3y^2)$
 $= ky^3$
 $x^3 + y^3 + 3x^2y + 3xy^2 - x^3 + y^3 + 3x^2y - 3xy^2 - 6yx^2$
 $+ 9y^3 = ky^3$
 $11y^3 = ky^3$
3. (d) $x^2 - 5x - 1 = 0$
 $\Rightarrow x - \frac{1}{x} = 5$
 $x^3 - \frac{1}{x^3} = 5^3 + 3 \times 5$
 $= 140$
 $\frac{x^6 - x^4 + x^2 - 1}{x^3} = x^3 - x + \frac{1}{x} - \frac{1}{x^3}$
 $= \left(x^3 - \frac{1}{x^3}\right) - \left(x - \frac{1}{x}\right)$
 $= 140 - 5$
 $= 135$
4. (d) $a^2 + b^2 + 49c^2 + 18 = 2(b + 28c - a)$
 $a^2 + b^2 + (7c)^2 + 18 - 2b - 56c + 2a = 0$
 $a^2 + 2a + 1 + b^2 - 2b + 1 + (7c)^2 - 56c + 16 = 0$
 $(a + 1)^2 + (b - 1)^2 + (7c - 4)^2 = 0$
 $a + 1 = 0$
 $b - 1 = 0$

$$7c - 4 = 0$$

$$a = -1 \quad b = 1$$

$$c = \frac{4}{7}$$

$$2a - b + 7c = -2 - 1 + 4 = 1$$

$$\begin{aligned} 5. \text{ (d)} \quad x + y + z &= 7, xy + yz + zx = 8 \\ x^3 + y^3 + z^3 - 3xyz &= (x + y + z)[(x + y + z)^2 - 3(xy + yz + zx)] \\ &= 7[7^2 - 3(8)] = 7 \times 25 = 175 \end{aligned}$$

$$\begin{aligned} 6. \text{ (d)} \quad \sqrt{x} - \frac{1}{\sqrt{x}} &= \sqrt{3} \\ \Rightarrow x + \frac{1}{x} &= \sqrt{3}^2 + 2 = 5 \end{aligned}$$

$$x^2 + \frac{1}{x^2} = 5^2 - 2 = 23$$

$$x^4 + \frac{1}{x^4} = 23^2 - 2 = 527$$

$$\begin{aligned} 7. \text{ (a)} \quad 2\sqrt{2}x^3 - 3\sqrt{3}y^3 &= (\sqrt{2}x - \sqrt{3}y)(Ax^2 - Bxy + cy^2) \\ &= (\sqrt{2}x)^3 - (\sqrt{3}y)^3 \end{aligned}$$

$$\begin{aligned} &= (\sqrt{2}x - \sqrt{3}y)(Ax^2 - Bxy + Cy^2) \\ A &= 2, \\ B &= -\sqrt{2} \times \sqrt{3} = -\sqrt{6} \\ C &= 3 \end{aligned}$$

$$\sqrt{A^2 + B^2 + C^2} = \sqrt{4 + 6 + 9} = \sqrt{19}$$

$$\begin{aligned} 8. \text{ (c)} \quad a^2 + b^2 + 4ac^2 + 18 &= 2(b - 28c - a) \\ a^2 + b^2 + (7c)^2 + 18 - 2b + 56c - 2a &= 0 \\ a^2 + 2a + 1 + b^2 - 2b + 1 + (7c)^2 + 56c + 16 &= 0 \\ (a + 1)^2 + (b - 1)^2 + (7c + 4)^2 &= 0 \\ a &= -1 \\ b &= 1 \\ c &= -\frac{4}{7} \end{aligned}$$

$$a - b - 7c = -1 - 1 + 4 = 2$$

$$\begin{aligned} 9. \text{ (b)} \quad a + b + c &= 5, ab - bc - ca = 10 \\ a^2 + b^2 + c^2 &= (a + b + c)^2 - 2(ab - bc - ca) \\ &= 5^2 - 2 \times 10 = 5 \end{aligned}$$

$$\begin{aligned} 10. \text{ (b)} \quad x^2 + \frac{1}{x^2} &= 23 \\ \Rightarrow x + \frac{1}{x} &= \sqrt{23 + 2} = 5 \end{aligned}$$

$$x^3 + \frac{1}{x^3} = 5^3 - 3 \times 5 = 110$$

$$\begin{aligned} 11. \text{ (b)} \quad x + y + z &= 2 \\ xy + yz + zx &= -11 \\ xyz &= -12 \\ x^2 + y^2 + z^2 &= (x + y + z)[(x + y + z)^2 - 3(xy + yz + zx)] + 3xyz \\ &= 2[2^2 - 3(-11)] + 3(-12) \\ &= 2 \times 37 - 36 \\ &= 38 \end{aligned}$$

$$\begin{aligned} 12. \text{ (a)} \quad x + y + z &= 18, \\ xyz &= 81 \\ xy + yz + zx &= 90 \\ x^2 + y^2 + z^2 &= 18^2 - 2 \times 90 = 144 \\ x^3 + y^3 + z^3 &= (x + y + z)[x^2 + y^2 + z^2 - (xy + yz + zx)] + 3xyz \\ &= 18[144 - 90] + 3 \times 81 = 1215 \end{aligned}$$

$$\begin{aligned} \sqrt[4]{x^3 + y^3 + z^3 + xyz} &= \sqrt[4]{1215 + 81} \\ &= \sqrt[4]{1296} = 6 \end{aligned}$$

$$\begin{aligned} 13. \text{ (b)} \quad 27x^3 - 64y^3 &= (Ax + By)(Cx^2 - Dy^2 + 12xy) \\ (3x)^3 - (4y)^3 &= (Bx + 4y)(9x^2 + 16y^2 + 12xy) \\ A &= 3 \\ B &= -4 \\ C &= 9 \\ D &= -16 \end{aligned}$$

$$4A + B + 3C + 2D = 12 - 4 + 27 - 32 = 3$$

$$\begin{aligned} 14. \text{ (a)} \quad x &= 32.5, y = 34.6, z = 30.9 \\ x^3 + y^3 + z^3 - 3xyz &= 0.98k \\ \frac{1}{2}(x + y + z)[(x - y)^2 + (y - z)^2 + (z - x)^2] &= 0.98k \\ \frac{1}{2} \times 98[4.41 + 13.69 + 2.56] &= 0.98k \\ k &= \frac{98 \times 20.66}{2 \times 0.98} = 1033 \end{aligned}$$

$$15. \text{ (a)} \quad x^2 - 3x + 1 = 0$$

$$x + \frac{1}{x} = 3$$

$$x^3 + \frac{1}{x^3} = 3^3 - 3 \times 3 = 18$$

$$\frac{\left(x^4 + \frac{1}{x^2}\right) \times \frac{1}{x}}{\left(x^2 + 5x + 1\right) \times \frac{1}{x}} = \frac{x^3 + \frac{1}{x^3}}{x + \frac{1}{x} + 5} = \frac{18}{8} = \frac{9}{4}$$

$$\begin{aligned}
 16. \text{ (a)} \quad x^2 - \sqrt{7}x + 1 &= 0 \Rightarrow x + \frac{1}{x} = \sqrt{7} \\
 x^2 + \frac{1}{x^2} &= (\sqrt{7})^2 - 2 = 5 \\
 x^3 + \frac{1}{x^3} &= (\sqrt{7})^3 - 3 \times \sqrt{7} = 4\sqrt{7} \\
 x^5 + \frac{1}{x^5} &= \left(x^2 + \frac{1}{x^2}\right) \left[\left(x^3 + \frac{1}{x^3}\right) + \left(x + \frac{1}{x}\right)\right] \\
 &= 5(4\sqrt{7} + \sqrt{7}) = 25\sqrt{7}
 \end{aligned}$$

$$\begin{aligned}
 17. \text{ (a)} \quad x + \frac{1}{x} &= 7 \\
 x^2 + \frac{1}{x^2} &= 7^2 - 2 = 47
 \end{aligned}$$

$$\begin{aligned}
 18. \text{ (a)} \quad 2x + 3y + 1 &= 0 \\
 \Rightarrow (2x + 3y)^3 &= -1^3 \\
 8x^3 + 8 + 27y^3 - 18xy &= (8x^3 + 27y^3 - 18xy) + 8 \\
 &= (2x + 3y)^3 + 8 \\
 &= -1 + 8 = 7
 \end{aligned}$$

$$\begin{aligned}
 19. \text{ (b)} \quad a^4 + b^4 + a^2b^2 &= 273, a^2 + b^2 - ab = 21 \\
 (a^2 + b^2 + ab)(a^2 + b^2 - ab) &= a^4 + b^4 + a^2b^2
 \end{aligned}$$

$$\begin{aligned}
 a^2 + b^2 + ab &= \frac{273}{21} \\
 &= 13 \\
 a^2 + b^2 &= \frac{21 + 13}{2} = 17,
 \end{aligned}$$

$$\begin{aligned}
 ab &= -4, \\
 a + b &= 3 \\
 \frac{1}{a} + \frac{1}{b} &= \frac{a + b}{ab} = \frac{3}{-4}
 \end{aligned}$$

$$\begin{aligned}
 20. \text{ (d)} \quad (2x + y)^3 - (x - 2y)^3 &= (x + 3y)[Ax^2 + By^2 + Cxy] \\
 &= (2x + y)^3 - (x - 2y)^3 \\
 &= (2x + y - x + 2y)[(2x + y)^2 \\
 &\quad + (x - 2y)^2 + (2x + y)(x - 2y)] \\
 &= (x + 3y)[7x^2 + 3y^2 - 3xy]
 \end{aligned}$$

$$A = 7,$$

$$B = 3,$$

$$C = -3$$

$$A + 2B + C = 7 + 6 - 3$$

$$= 10$$

$$\begin{aligned}
 21. \text{ (b)} \quad 9(a^2 + b^2) + c^2 + 20 &= 12(a + 2b) \\
 9a^2 - 12a + 4 + 9b^2 - 24b + 16 + c^2 &= 0 \\
 (3a - 2)^2 + (3b - 4)^2 + c^2 &= 0 \\
 3a - 2 &= 0 \\
 3b - 4 &= 0 \\
 c &= 0
 \end{aligned}$$

$$a = \frac{2}{3}$$

$$b = \frac{4}{3}$$

$$\sqrt{6a + 9b + 2c} = \sqrt{4 + 12 + 0} = 4$$

$$\begin{aligned}
 22. \text{ (c)} \quad x + y + z &= 2, x^3 + y^3 + z^3 - 3xyz = 74 \\
 x^3 + y^3 + z^3 - 3xyz &= \frac{(x + y + z)}{2} [3(x^2 + y^2 + z^2) - (x + y + z)]
 \end{aligned}$$

$$x^2 + y^2 + z^2 = \frac{74 \times 2}{2 \times 3} + \frac{2^2}{3} = 26$$

$$\begin{aligned}
 23. \text{ (a)} \quad x^4 + x^2y^2 + y^4 &= 21, x^2 + xy + y^2 \\
 &= 3
 \end{aligned}$$

$$x^4 + x^2y^2 + y^4 = (x^2 - xy + y^2)(x^2 + xy + y^2)$$

$$x^2 - xy + y^2 = \frac{21}{3} = 7$$

$$2xy = -4$$

$$xy = -2$$

$$-xy = 2$$

$$24. \text{ (d)} \quad x + \frac{1}{x} = \frac{17}{4}$$

$$x - \frac{1}{x} = \sqrt{\left(\frac{17}{4}\right)^2 - 4}$$

$$= \sqrt{\frac{289}{4} - 16}$$

$$x - \frac{1}{x} = \frac{15}{4}$$

$$25. \text{ (b)} \quad 2x^2 - 7x + 5 = 0$$

$$x + \frac{5}{2x} = \frac{7}{2}$$

$$x^3 + \frac{125}{8x^3} = \left(\frac{7}{2}\right)^3 - 3 \times \frac{7}{2} \times \frac{5}{2}$$

$$= \frac{343}{8} - \frac{105}{4}$$

$$= \frac{133}{8}$$

$$= 16\frac{5}{8}$$

$$\begin{aligned}
 26. \text{ (a)} \quad (2 - 3x)^3 &= 8 + 2px^2 - 36x - 27x^3 \\
 8 - 27x^3 - 36x + 54x^2 &= 8 + 2px^2 - 36x - 27x^3
 \end{aligned}$$

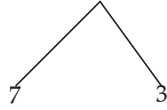
$$2px^2 = 54x^2$$

$$p = \frac{54}{2}$$

$$= 27$$

$$\begin{aligned}
 27. \text{ (b)} \quad & x - y = 4 \\
 & x^3 - y^3 = 316 \\
 & x^3 - y^3 = (x - y)(x^2 + xy + y^2) \\
 & 316 = 4[(x - y)^2 + 3xy] \\
 & 3xy + 16 = 79
 \end{aligned}$$

$$xy = 21$$



$$x = 7$$

$$y = 3$$

$$\begin{aligned}
 x^4 - y^4 &= (x^2 - y^2)(x^2 + y^2) \\
 &= (49 - 9)(49 + 9) \\
 &= 2320
 \end{aligned}$$

$$28. \text{ (d)} \quad (3x - 4)^3 = 27x^3 - 64 - 108x^2 + 144x$$

$$\text{Coefficient of } x = 144$$

$$29. \text{ (c)} \quad x + y + z = 3$$

$$xy + yz + zx = -12$$

$$xyz = -16$$

$$\begin{aligned}
 x^3 + y^3 + z^3 &= (x + y + z)[(x + y + z)^2 - \\
 &\quad 3(xy + yz + zx)] + 3xyz \\
 &= 3[3^2 - 3(-12)] + 3(-16)
 \end{aligned}$$

$$3 \times 45 - 48 = 87$$

$$\begin{aligned}
 \sqrt{x^3 + y^3 + z^3 + 13} &= \sqrt{87 + 13} \\
 &= \sqrt{100} \\
 &= 10
 \end{aligned}$$

$$30. \text{ (d)} \quad \sqrt{x} - \frac{1}{\sqrt{x}} = \sqrt{7}$$

$$\begin{aligned}
 x + \frac{1}{x} &= (\sqrt{7})^2 + 2 \\
 &= 9
 \end{aligned}$$

$$x^2 + \frac{1}{x^2} = 9^2 - 2 = 79$$

$$31. \text{ (c)} \quad \frac{56\sqrt{7}x^3 - 2\sqrt{2}y^3}{2\sqrt{7}x - \sqrt{2}y} = Ax^2 + By^2 - Cxy$$

$$\frac{(2\sqrt{7}x)^3 - (\sqrt{2}y)^3}{2\sqrt{7}x - \sqrt{2}y} = Ax^2 + By^2 - Cxy$$

$$28x^2 + 2\sqrt{14}xy + 2y^2 = Ax^2 + By^2 - Cxy$$

$$A = 28, B = 2, C = -2\sqrt{14}$$

$$A + B - \sqrt{14}C = 28 + 2 - \sqrt{14}(-2\sqrt{14})$$

$$= 30 + 28$$

$$= 58$$

$$32. \text{ (d)} \quad \frac{x}{y} + \frac{y}{x} = 2$$

$$x^2 + y^2 = 2xy$$

$$x^2 + y^2 - 2xy = 0$$

$$(x - y)^2 = 0$$

$$x - y = 0$$

$$33. \text{ (c)} \quad 2a + \frac{3}{a} - 1 = 11$$

$$2a + \frac{3}{a} = 12$$

$$\begin{aligned}
 4a^2 + \frac{9}{a^2} &= 12^2 - 2 \times 2 \times 3 \\
 &= 144 - 12 \\
 &= 132
 \end{aligned}$$

$$34. \text{ (a)} \quad a^2 + b^2 + c^2 + 216 = 12(a + b - 2c)$$

$$a^2 - 12a + 36 + b^2 - 12b + 36 + c^2 + 24c + 144 = 0$$

$$(a - 6)^2 + (b - 6)^2 + (c + 12)^2 = 0$$

$$a = 6$$

$$b = 6$$

$$c = -12$$

$$\sqrt{ab - bc - ca} = \sqrt{36 + 72 + 72}$$

$$= \sqrt{36 \times 5}$$

$$= 6\sqrt{5}$$

$$35. \text{ (d)} \quad a + b + c = 0 \Rightarrow a^3 + b^3 + c^3 = 3abc$$

$$a + b = -c,$$

$$b + c = -a,$$

$$a + c = -b$$

$$\frac{(b + c)^2}{bc} + \frac{(c + a)^2}{ca} + \frac{(a + b)^2}{ab}$$

$$= \frac{(-a)^2}{bc} + \frac{(-b)^2}{ca} + \frac{(-c)^2}{ab}$$

$$\frac{a^2 + b^3 + c^3}{abc} = \frac{3abc}{abc} = 3$$

$$36. \text{ (c)} \quad x + y + z = 13, x^2 + y^2 + z^2 = 133$$

$$x^3 + y^3 + z^3 = 847, xy + yz + zx$$

$$= \frac{13^2 - 133}{2} = 18$$

$$x^3 + y^3 + z^3 - 3xyz$$

$$= (x + y + z)[x^2 + y^2 + z^2 - (xy + yz + zx)]$$

$$- 3xyz = 13(133 - 18) - 847$$

$$xyz = \frac{+648}{-3}$$

$$= -216$$

$$\sqrt[3]{xyz} = \sqrt[3]{216}$$

$$= -6$$

$$\begin{aligned}
 37. \text{ (a)} \quad a^3 + b^3 &= 217 \\
 a + b &= 7 \\
 (a + b)^3 &= a^3 + b^3 + 3ab(a + b) \\
 3ab \times 7 &= 7^3 - 217 \\
 21ab &= 126 \\
 ab &= 6
 \end{aligned}$$

$$\begin{aligned}
 38. \text{ (c)} \quad a^2 + b^2 + c^2 + 84 &= 4(a - 2b + 4c) \\
 a^2 - 4a + 4 + b^2 + 8b + 16 + c^2 - 16c + 64 &= 0 \\
 (a - 2)^2 + (b + 4)^2 + (c - 8)^2 &= 0 \\
 a &= 2 \\
 b &= -4 \\
 c &= 8 \\
 \sqrt{ab - bc + ca} &= \sqrt{-8 + 32 + 16}
 \end{aligned}$$

$$= \sqrt{40}$$

$$= 2\sqrt{10}$$

$$\begin{aligned}
 39. \text{ (d)} \quad a + b + c &= 12, ab + bc + ca = 8 \\
 a^2 + b^2 + c^2 &= (a + b + c)^2 - 2(ab + bc + ca) \\
 &= 144 - 16 = 128
 \end{aligned}$$

$$\begin{aligned}
 40. \text{ (b)} \quad &\frac{(0.013)^2 + (0.007)(0.000049)}{(0.007)^2 + 0.013(0.013 - 0.007)} \\
 a^3 + b^3 &= (a + b)(a^2 + b^2 - ab) \\
 \frac{a^3 + b^3}{a^2 + b^2 - ab} &= a + b = 0.013 + 0.007 \\
 &= 0.02
 \end{aligned}$$