OBJECTIVE MATHEMATICS

Volume 2

Descriptive Test Series

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CHAPTER-14 : MATRICES

UNIT TEST-1

1. Let S be the set of values of λ, for which the system of equations.

$6\lambda x - 3y + 3z = 4\lambda^2$,

$2x + 6\lambda y + 4z = 1$,

$3x + 2y + 3\lambda z = \lambda$ has no solution. Then $12\sum_{\lambda \in S} |\lambda|$ is equal to ____________.

2. Let $S = \left\{ \begin{pmatrix} -1 & a \\ 0 & b \end{pmatrix}; a, b \in \{1, 2, 3, ...100\} \right\}$ and let $T_n = \{A \in S : A^{n(n+1)} = I\}$ Then the number of elements in $\bigcap_{n=1}^{100} T_n$ is ____________.

3. The number of matrices of order 3×3, whose entries are either 0 or 1 and the sum of all the entries is a prime number, is ____________.

Hints and Solutions

1. (24)

For no solution

$$\begin{vmatrix} 6\lambda & -3 & 3 \\ 2 & 6\lambda & 4 \\ 3 & 2 & 3\lambda \end{vmatrix} = 0$$

$$\Rightarrow 9\lambda^3 - 7\lambda - 2 = 0$$

$$\Rightarrow (\lambda - 1)(3\lambda + 1)(3\lambda + 2) = 0$$

$$\Rightarrow 12\sum_{\lambda \in S} |\lambda| = 12 \times \left(1 + \frac{1}{3} + \frac{2}{3}\right) = 24$$

2. (100)

$$S = \left\{ \begin{pmatrix} -1 & a \\ 0 & b \end{pmatrix} : a, b \in \{1, 2, 3, ..., 100\} \right\}$$

$\because A = \begin{pmatrix} -1 & a \\ 0 & b \end{pmatrix}$ then even powers of A as $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$, if $b = 1$ and $a \in \{1, ..., 100\}$

Here, $n(n + 1)$ is always even.

$\therefore T_1, T_2, T_3, \ldots, T_n$ are all I for $b = 1$ and each value of a.

$$\therefore \bigcap_{n=1}^{100} T_n = 100$$

3. (282)

In a 3×3 order matrix there are 9 entries.

These nine entries are zero or one.

The sum of positive prime entries are 2, 3, 5 or 7.

Total possible matrices

$$= \frac{9!}{2! \cdot 7!} + = \frac{9!}{3! \cdot 6!} + \frac{9!}{5! \cdot 4!} + = \frac{9!}{7! \cdot 2!}$$

$$= 36 + 84 + 126 + 36 = 282$$