CHAPTER 10

Binomial Theorem

Exercise

1. Coefficient of x^{-4} in $\left(\frac{3}{2}-\frac{3}{x^2}\right)^{10}$ is

(a) $\frac{405}{226}$ (b) $\frac{504}{289}$

(c) $\frac{450}{263}$ (d) None of these

2. The number of terms in the expansion of $(1+5\sqrt{2}x)^9+(1-5\sqrt{2}x)^9$ is

(a) 5 (b) 7

(c) 9 (d) 10

3. The value of $\sum_{k=0}^{n}(-1)^k\ {}^nC_k$ is

(a) – 1 (b) 2^k

(c) 2^n (d) 0

4. If the 4th term in the expansion of $\left(ax+\frac{1}{x}\right)^n$ is 5/2, then the values of a and n are

(a) $\frac{1}{2},6$ (b) 1, 3

(c) $\frac{1}{2},3$ (d) cannot be found

5. If the coefficients of second, third and fourth terms in the expansion of $(1 + x)^{2n}$ are in AP, then

(a) $2n^2 + 9n + 7 = 0$ (b) $2n^2 - 9n + 7 = 0$

(c) $2n^2 - 9n - 7 = 0$ (d) None of these

6. If the coefficients of $(2r + 4)$th and $(r - 2)$th terms in the expansion of $(1 + x)^{18}$ are equal, then the value of r is

(a) 5 (b) 6

(c) 7 (d) 9

7. The middle term in the expansion of $\left(x+\frac{1}{x}\right)^{10}$ is

(a) ${}^{10}C_1.\frac{1}{x}$ (b) ${}^{10}C_5$

(c) ${}^{10}C_6$ (d) ${}^{10}C_7\,x$

8. The sum of the coefficients of the polynomial $(1 + x - 3x^2)^{21430}$ is

(a) – 1 (b) 1

(c) 0 (d) None of these

9. If the coefficient of $(r + 1)$th term in the expansion of $(1 + x)^{2n}$ be equal to that of $(r + 3)$th term, then

(a) $n - r + 1 = 0$ (b) $n - r - 1 = 0$

(c) $n + r + 1 = 0$ (d) None of these

10. If n is even, then the greatest coefficient in the expansion of $(x + a)^n$ is

(a) ${}^nC_{\frac{n}{2}+1}$ (b) ${}^nC_{\frac{n}{2}-1}$

(c) ${}^nC_{\frac{n}{2}}$ (d) None of these

11. If n is even and rth term has the greatest coefficient in the binomial expansion of $(1 + x)^n$, then

(a) $r=\frac{n}{2}$ (b) $r=\frac{n}{2}+1$

(c) $r=\frac{n}{2}-1$ (d) None of these

12. If C_0, C_1, C_2,....,C_n denote the coefficients in the expansion of $(1 + x)^n$, then the value of $\sum_{r=1}^{n} rC_r$ is

(a) $n\ .\ 2^{n-1}$ (b) $(n + 1)2^n$

(c) $(n + 1)\ .\ 2^{n-1}$ (d) $(n + 2)\ .\ 2^{n-1}$

13. If C_0, C_1, C_2,....,C_n denote the binomial coefficients in the expansion of $(1 + x)^n$, then the value of $\sum_{r=0}^{n}(r+1)C_r$ is

(a) $n\,2^n$ (b) $(n+1)2^{n-1}$
(c) $(n+2)2^{n-1}$ (d) $(n-2)2^{n-1}$

14. The expression $[x+(x^3-1)^{1/2}]^5+[x-(x^3-1)^{1/2}]^5$ is a polynomial of degree
(a) 5 (b) 6
(c) 7 (d) 8

15. The coefficient of x^3 in $\left(\sqrt{x^5}+\frac{3}{\sqrt{x^3}}\right)^6$ is
(a) 0 (b) 120
(c) 420 (d) 540

16. The greatest coefficient in the expansion of $(1+x)^{10}$ is
(a) $\frac{10!}{5!\,6!}$ (b) $\frac{10!}{(5!)^2}$
(c) $\frac{10!}{5!\,7!}$ (d) None of these

17. The coefficient of x^{53} in the expansion $\sum_{m=0}^{100} {}^{100}C_m (x-3)^{100-m}.2^m$ is
(a) ${}^{100}C_{47}$ (b) ${}^{100}C_{53}$
(c) $-{}^{100}C_{53}$ (d) $-{}^{100}C_{100}$

18. The term independent of x in $\left[\sqrt{\frac{x}{3}}+\sqrt{\frac{3}{2x^2}}\right]^{10}$ is
(a) term does not exist (b) ${}^{10}C_1$
(c) $\frac{5}{12}$ (d) 1

19. If the rth term in the expansion of $\left(\frac{x}{3}-\frac{2}{x^2}\right)^{10}$ contains x^4 then r is equal to
(a) 2 (b) 3
(c) 4 (d) 5

20. The 14th term from the end in the expansion of $(\sqrt{x}-\sqrt{y})^{17}$ is
(a) ${}^{17}C_5 x^6(-\sqrt{y})^6$ (b) ${}^{17}C_6(\sqrt{x})^{11}y^3$
(c) ${}^{17}C_4 x^{13/2}y^2$ (d) None of these

21. The first four terms in the expansion of $(1-x)^{3/2}$ are
(a) $1-\frac{3}{2}x+\frac{3}{8}x^2+\frac{1}{16}x^2$ (b) $1-\frac{3}{2}x+\frac{3}{8}x^2-\frac{1}{16}x^3$
(c) $1-\frac{3}{2}x+\frac{3}{8}x^2+\frac{1}{16}x^3$ (d) None of the above

22. The number of terms in the expansion of $(x+y+z)^{10}$ is
(a) 11 (b) 33
(c) 66 (d) 1000

23. In the expansion of $\left(x^2-\frac{1}{3x}\right)^9$, the term without x is equal to
(a) $\frac{28}{81}$ (b) $\frac{-28}{243}$
(c) $\frac{28}{243}$ (d) None of these

24. The number of terms in the expansion of $(1+5x+10x^2+10x^3+5x^4+x^5)^{20}$ is
(a) 100 (b) 101
(c) 120 (d) None of these

25. If $(1-x+x^2)=a_0+a_1x+a_2x^2+\ldots a_{2n}x^{2n}$ then $a_0+a_2+a_4+\ldots.\,a_{2n}$ equals :
(a) $\frac{3^n+1}{2}$ (b) $\frac{3^n-1}{2}$
(c) $\frac{1-3^n}{2}$ (d) $3^n+\frac{1}{2}$

26. In the expansion of $(1+x^2)^{40}\left(x^2+2+\frac{1}{x^2}\right)^{-5}$ the coefficient of x^{20} is
(a) ${}^{30}C_{10}$ (b) 1
(c) ${}^{30}C_{10}$ (d) None of these

27. The coefficient of x^5 in the expansion of $(1+x^2)^4(1+x)^4$ is
(a) 30 (b) 60
(c) 40 (d) none of these

28. If $(1+x)^n=\sum_{r=0}^{n} C_r x^r$, then $\left(1+\frac{C_1}{C_0}\right)\left(1+\frac{C_2}{C_1}\right)\ldots\ldots\left(1+\frac{C_n}{C_{n-1}}\right)$ is equal to
(a) $\frac{n^{n-1}}{(n-1)!}$ (b) $\frac{(n+1)^{n-1}}{(n-1)!}$
(c) $\frac{(n+1)^n}{n!}$ (d) $\frac{(n+1)^{n+1}}{n!}$

29. The coefficient of the term independent of x in the expansion of $\left(ax+\frac{b}{x}\right)^{14}$ is
(a) $14!\,a^7b^7$ (b) $\frac{14!}{7!}a^7b^7$
(c) $\frac{14!}{(7!)^2}a^7b^7$ (d) $\frac{14!}{(7!)^3}a^7b^7$

30. $aC_0+(a+b)\,C_1+(a+2b)\,C_2+\ldots\ldots(a+nb)C_n$ is equal to
(a) $(2a+nb)2^n$ (b) $(2a+nb)2^{n-1}$
(c) $(na+2b)2^n$ (d) $(na+2b)2^{n-1}$

ANSWERS

1.	(d)	**2.**	(a)	**3.**	(d)	**4.**	(a)	**5.**	(b)	**6.**	(b)	**7.**	(b)	**8.**	(b)	**9.**	(b)	**10.**	(c)
11.	(b)	**12.**	(a)	**13.**	(c)	**14.**	(c)	**15.**	(d)	**16.**	(b)	**17.**	(c)	**18.**	(a)	**19.**	(b)	**20.**	(c)
21.	(c)	**22.**	(c)	**23.**	(c)	**24.**	(b)	**25.**	(a)	**26.**	(c)	**27.**	(c)	**28.**	(c)	**29.**	(c)	**30.**	(b)

Explanations

1. (d) General term in the expansion of $\left(\frac{3}{2}-\frac{3}{x^2}\right)^{10}$ is

$$T_{r+1} = {}^{10}C_r\left(\frac{3}{2}\right)^{10-r}\left(\frac{-3}{x^2}\right)^r$$

$$= (-1)^r\,{}^{10}C_r\frac{(3)^{10}}{2^{10-r}}.x^{-2r}$$

For the coefficient of x^{-4}, put $-2r = -4$

$\Rightarrow r = 2$

So, coefficient of $x^{-4} = (-1)^2\,{}^{10}C_2\frac{(3)^{10}}{2^8}$

$= {}^{10}C_2\frac{3^{10}}{2^8}$

2. (a) $(1+5\sqrt{2}x)^9$ has 10 terms out of which 5 terms cancel out with the 5 terms of $(1-5\sqrt{2}x)^9$

$\therefore$ Total number of terms on simplification = 5

3. (d) $\because (1+x)^n = {}^nC_0 + {}^nC_1 + {}^nC_2x^2 + {}^nC_3x^3 + \ldots {}^nC_nx^n$

Put $x = -1$

$0 = {}^nC_0 - {}^nC_1 + {}^nC_2 + {}^nC_3 + \ldots.\ {}^nC_n$

So, $\sum_{k=0}^{n}(-1)^k\ {}^nC_k = 0$

4. (a) In the expansion of $\left(ax+\frac{1}{x}\right)^n$, $T_4 = \frac{5}{2}$

$\Rightarrow {}^nC_3\ (ax)^{n-3}\left(\frac{1}{x}\right)^3 = \frac{5}{2}$

${}^nC_3\ a^{n-3}x^{n-6} = \frac{5}{2}$

On comparing, $n - 6 = 0 \Rightarrow n = 6$

and ${}^nC_3\,a^{n-3} = \frac{5}{2}$

$\frac{n!}{3!(n-3!)}a^{n-3} = \frac{5}{2}$ $\{\because n = 6\}$

$a^{6-3} = \frac{1}{8} \Rightarrow a^2 = \frac{1}{8} \Rightarrow a = \frac{1}{2}$

5. (b) Coefficients of T_2, T_3, T_4 are in A.P.

$\Rightarrow 2T_3 = T_2 + T_4 \Rightarrow 2.\ {}^{2n}C_2 = {}^{2n}C_1 + {}^{2n}C_3$

$\Rightarrow 2.\frac{(2n)!}{2!(2n-2)!} = \frac{(2n)!}{1!(2n-1)!} + \frac{(2n)!}{3!(2n-3)!}$

$\Rightarrow \frac{1}{2n-2} = \frac{1}{(2n-1)(2n-2)} + \frac{1}{6}$

$\Rightarrow 2n^2 - 9n + 7 = 0$

6. (b) Coefficient of $(2r + 4)$th and $(r - 2)$th terms are equal in the expansion of $(1 + x)^{18}$.

${}^{18}C_{2r+3} = {}^{18}C_{r-3}$ $\{\because {}^nC_x = {}^nC_y \Rightarrow x + y = n\}$

$\Rightarrow (2r + 3) + (r - 3) = 18 \Rightarrow r = 6$

7. (b) Here, $n = 10$

So, middle term = $\frac{10}{2}+1$ = 6th term

$T_{5+1} = {}^{10}C_5(x)^{10-5}\left(\frac{1}{x}\right)^5 = {}^{10}C_5$

8. (b) For the sum of coefficients, put $x = 1$.

So, sum of coefficients = $(1 + 1 - 3)^{21430} = 1$

9. (b) Coefficient of $(r + 1)$th term

= Coefficient of $(r + 3)$th term

$= {}^{2n}C_r = {}^{2n}C_{r+2}$

$\Rightarrow r + r + 2 = 2n \Rightarrow n - r - 1 = 0$

10. (c) If n is even, then number of terms $(n + 1)$, i.e., odd.

So, greatest coefficient = Middle term coefficient

i.e., $\left(\frac{n}{2}+1\right)$ th term coefficient

So, greatest coefficient = ${}^nC_{n/2}$

11. (b) Same as above,

Greatest term = $\left(\frac{n}{2}+1\right)^{\text{th}}$ term

So, $T_{(r-1)+1} = {}^nC_{\frac{n}{2}}x^{\frac{n}{2}}$

$\Rightarrow r - 1 = \frac{n}{2} \Rightarrow r = \frac{n}{2}+1$

12. (a) $\sum_{r=1}^{n} r\,{}^nC_r$

$\because \frac{{}^nC_r}{{}^{n-1}C_{r-1}} = \frac{n}{r}$

So, $\sum_{r=1}^{n} r\,{}^{n}C_r = \sum_{r=1}^{n} n.{}^{n-1}C_{r-1}$

$= n \sum_{r-1=0}^{n} {}^{n-1}C_{r-1} = n.2^{n-1}$

13. (c) $\sum_{r=0}^{n} (r+1)\ {}^{n}C_r$

$= \sum_{r=0}^{n} r\,{}^{n}C_r + \sum_{r=0}^{n} {}^{n}C_r$

$= n \cdot 2^{n-1} + 2^n = 2^{n-1}(n+2)$

14. (c) Let $\sqrt{x^3 - 1} = y$

Now, given expression = $(x + y)^5 + (x - y)^5$

$= 2[x^5 + {}^5C_2 x^3 y^2 + {}^5C_4\, xy^4]$

$= 2[x^5 + 10\,x^3\,(x^3 - 1) + 5x\,(x^3 - 1)^2]$

$\therefore$ Degree of the polynomial = Highest power of $x = 7$

15. (d) $T_{r+1} = {}^6C_r\ (\sqrt{x^5})^{6-r}\left(\frac{3}{\sqrt{x^3}}\right)^r$

$= {}^6C_r\,(3)^r\ (x)^{\frac{30-8r}{2}}$

For coefficient of x^3, put $\frac{30-8r}{2} = 3$

$\Rightarrow r = 3$

Hence, coefficient of $x^3 = {}^6C_3\ 3^3 = 540$

16. (b) Greatest coefficient in $(1 + x)^{10}$

= Middle term coefficient

$= \left(\frac{10}{2}+1\right)$th = 6th term coefficient

$= {}^{10}C_5 = \frac{10!}{(5!)^2}$

17. (c) $\sum_{m=0}^{100} {}^{100}C_m\,(x-3)^{100-m}\,.\,2^m = (x - 3 + 2)^{100}$

$= (x - 1)^{100} = (1 - x)^{100}$

Coefficient of $x^{53} = -\,{}^{100}C_{53}$

18. (a) $T_{r+1} = {}^{10}C_r \left(\sqrt{\frac{x}{3}}\right)^{10-r}\left(\sqrt{\frac{3}{2x^2}}\right)^{2r}$

$= {}^{10}C_r = \frac{(\sqrt{3})^{2r-10}}{(\sqrt{2})^r}(x)^{\frac{10-3r}{2}}$

For term independent of x, put $\frac{10-3r}{2} = 0$

$\Rightarrow r = \frac{10}{3}$

It is not possible.

So, there exists no term independent of x.

19. (b) $T_{(r-1)+1} = {}^{10}C_{r-1}\left(\frac{x}{3}\right)^{10-r+1}\left(-\frac{2}{x^2}\right)^{r-1}$

$= {}^{10}C_{r-1}\,\frac{(-2)^{r-1}}{(3)^{11-r}}.x^{13-3r}$

For term containing x^4,

Put $13 - 3r = 4 \Rightarrow r = 3$

20. (c) Here, $n = 17$

So, number of terms = 17 + 1 = 18

14th term from last = (18 – 14 + 1)th term from starting = 5th term from starting

So, $T_{4+1} = {}^{17}C_4\,(\sqrt{x})^{13}(-\sqrt{y})^4 = {}^{17}C_4\,x^{13/2}\,y^2$

21. (c) $(1+x)^n = 1 + nx + \frac{n(n-1)x^2}{2!} + \frac{n(n-1)(n-2)x^3}{3!} + \ldots$

So, $(1-x)^{3/2} = 1 - \frac{3}{2}x + \frac{\frac{3}{2}\left(\frac{3}{2}-1\right)x^2}{2!} - \frac{\left(\frac{3}{2}\right)\left(\frac{3}{2}-1\right)\left(\frac{3}{2}-2\right)x^3}{31}$

$= 1 - \frac{3x}{2} + \frac{3x^2}{8} + \frac{x^3}{16}$

22. (c) Number of terms = $\frac{(n+1)(n+2)}{2}$ {Put $n = 10$}

$= \frac{11 \times 12}{2} = 66$

23. (c) $T_{r+1} = {}^9C_r\,(x^2)^{9-r}\left(-\frac{1}{3x}\right)^r = \left(-\frac{1}{3}\right)^r\ {}^9C_r\,x^{18-3r}$

For term without x, put $18 - 3r = 0$

$\Rightarrow r = 6$

So, term without $x = \left(-\frac{1}{3}\right)^6\ {}^9C_6 = \frac{28}{243}$

24. (b) $(1 + 5x + 10x^2 + 10x^3 + 5x^4 + x^5)^{20}$

$= [(1 + x)^5]^{20}$

$= (1 + x)^{100}$

So, number of terms = 101

25. (a) Given $(1 - x + x^2)^n = a_0 + a_1x + a_2x^2 + \ldots + a_{2n}x^{2n}$

Putting $x = 1$ and -1 and adding, we get

$1 + 3^n = 2\,(a_0 + a_2 + \ldots + a_{2n})$

$\Rightarrow a_0 + a_2 + \ldots a_{2n} = \frac{3^n + 1}{2}$

26. (c) $(1+x^2)^{40}\left(x^2 + 2 + \frac{1}{x^2}\right)^{-5}$

$$= (1+x^2)^{40}\left\{\left(x+\frac{1}{x}\right)^2\right\}^{-5}$$

$$= (1+x^2)^{40}\frac{(1+x^2)^{-10}}{x^{-10}} = x^{10}(1+x^2)^{30}$$

For the coefficient of x^{20}, we have to get the coefficient of x^{10} in the expansion of $(1 + x^2)^{30}$ which is ${}^{30}C_5$.

$\because\ {}^nC_r = {}^nC_{n-r}$

So, coefficient of x^{20} is ${}^{30}C_{25}$.

27. (c) $(1 + x^2)^4\,(1 + x)^4$

$= ({}^4C_0 + {}^4C_1x^2 + {}^4C_2x^4 + {}^4C_3x^6 + {}^4C_4x^8)$
$\quad + ({}^4C_0 + {}^4C_1x + {}^4C_2x^2 + {}^4C_3x^3 + {}^4C_4x^4)$

So, coefficient of x^5

$= {}^4C_1 \,.\, {}^4C_3 + {}^4C_2 \,.\, {}^4C_1 = 40$

28. (c) $\dfrac{{}^nC_r}{{}^nC_{r-1}} = \dfrac{n-r+1}{r}$

Putting $r = 1, 2, 3, \ldots n$

$$\frac{C_1}{C_0} = \frac{n}{1}, \frac{C_2}{C_1} = \frac{n-1}{2}, \frac{C_3}{C_2} = \frac{n-2}{3}, \ldots.$$

$$\left(1+\frac{C_1}{C_0}\right)\left(1+\frac{C_2}{C_1}\right)\left(1+\frac{C_3}{C_2}\right)\ldots\left(1+\frac{C_n}{C_{n-1}}\right)$$

$$= \left(1+\frac{n}{1}\right)\left(1+\frac{n-1}{2}\right)\left(1+\frac{n-2}{3}\right)\ldots n \text{ factors}$$

$$= \left(\frac{n+1}{1}\right)\left(\frac{n+1}{2}\right)\left(\frac{n+1}{3}\right)\ldots n \text{ factors}$$

$$= \frac{(n+1)^n}{n!}$$

29. (c) $T_{r+1} = {}^{14}C_r\,(ax)^{14-r}\left(\dfrac{b}{x}\right)^r$

$= {}^{14}C_r a^{14-r} \cdot b^r x^{14-2r}$

For term independent of x, put $r = 7$

So, coefficient of the term independent of x

$= {}^{14}C_7\, a^7 b^7 = \dfrac{14!}{(7!)^2}(ab)^7$

30. (b) $aC_0 + (a + b)\, C_1 + (a + 2b)\, C_2 + \ldots + (a + nb)\, C_n$

$= a\{C_0 + C_1 + C_2 + \ldots C_n\}$
$\quad + b\{C_1 + 2C_2 + 3C_3 + \ldots nC_n\}$

$= a.2^n + b \,.\, n \,.\, 2^{n-1} = (2a + nb)2^{n-1}$