CHAPTER 12

Matrices

Exercise

1. If $E(\theta) = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$, then $E(\alpha)\,E(\beta)$ is equal to
   (a) $E(0^\circ)$ (b) $E(\alpha\beta)$
   (c) $E(\alpha + \beta)$ (d) $E(\alpha - \beta)$

2. If $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, J = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$, then B equals
   (a) $I \cos\theta + J \sin\theta$ (b) $I \sin\theta + J \cos\theta$
   (c) $I \cos\theta - J \sin\theta$ (d) $-I \cos\theta + J \sin\theta$

3. If A, B are two square matrices such that $AB = A$ and $BA = B$, then
   (a) A, B are idempotent (b) only A is idempotent
   (c) only B is idempotent (d) None of these

4. If A is a skew-symmetric matrix and n is a positive integer, then A^n is
   (a) a symmetric matrix (b) skew-symmetric matrix
   (c) diagonal matrix (d) None of these

5. If A, B are symmetric matrices of the same order then $AB - BA$ is
   (a) symmetric matrix (b) skew-symmetric matrix
   (c) null matrix (d) unit matrix

6. If $A = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$ = and $n \in N$, then A^n is equal to
   (a) $2^n A$ (b) $2^{n-1} A$
   (c) nA (d) None of these

7. If $A = \begin{bmatrix} a & b \\ b & a \end{bmatrix}$ and $A^2 = \begin{bmatrix} \alpha & \beta \\ \beta & \alpha \end{bmatrix}$, then
   (a) $\alpha = a^2 + b^2, \beta = ab$
   (b) $\alpha = a^2 + b^2, \beta = 2ab$
   (c) $\alpha = a^2 + b^2, \beta = a^2 - b^2$
   (d) $\alpha = 2ab, \beta = a^2 + b^2$

8. If $J_1 = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$ and $J_2 = \begin{bmatrix} 1 & 0 \\ -1 & 0 \end{bmatrix}$ then J_1^2 equals to
   (a) I (b) J_2
   (c) $-I$ (d) $-J_2$

9. If a matrix A is such that $3A^3 + 2A^2 + 5A + I = 0$, then A^{-1} is equal to
   (a) $-(3A^2 + 2A + 5)$ (b) $3A^2 + 2A + 5$
   (c) $3A^2 - 2A - 5$ (d) None of these

10. A and B be 3×3 matrices. Then $AB = 0$ implies
    (a) $A = 0$ and $B = 0$
    (b) $|A| = 0$ and $|B| = 0$
    (c) either $|A| = 0$ or $|B| = 0$
    (d) $A = 0$ or $B = 0$

11. If A and B are two matrices such that $AB = B$ and $BA = A$, then $A^2 + B^2$ is equal to
    (a) $2AB$ (b) $2BA$
    (c) $A + B$ (d) AB

12. If $A = \begin{bmatrix} 4 & x+2 \\ 2x-3 & x+1 \end{bmatrix}$ is symmetric, then x is equal to
    (a) 3 (b) 5
    (c) 2 (d) 4

13. If $A + B = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$ and $A - 2B = \begin{bmatrix} -1 & 1 \\ 0 & -1 \end{bmatrix}$, then A is equal to
    (a) $\begin{bmatrix} 1 & 1 \\ 2 & 1 \end{bmatrix}$ (b) $\begin{bmatrix} \frac{2}{3} & \frac{1}{3} \\ \frac{1}{3} & \frac{2}{3} \end{bmatrix}$
    (c) $\begin{bmatrix} \frac{1}{3} & \frac{1}{3} \\ \frac{2}{3} & \frac{1}{3} \end{bmatrix}$ (d) None of these

14. From the matrix equation $AB = AC$ we can conclude $B = C$ provided
    (a) A is singular (b) A is non-singular
    (c) A is symmetric (d) A is square

15. If $A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, B = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$, then $(A + B)^2$ equals
    (a) $A^2 + B^2$ (b) $A^2 + B^2 + 2AB$
    (c) $A^2 + B^2 + AB - BA$ (d) None of these

16. If $A = \begin{bmatrix} i & 0 \\ 0 & i \end{bmatrix}$ and $B = \begin{bmatrix} 0 & -i \\ -i & 0 \end{bmatrix}$, then $(A + B)(A - B)$ is equal to
    (a) $A^2 - B^2$ (b) $A^2 + B^2$
    (c) $A^2 - B^2 + BA + AB$ (d) None of these

17. If A is 3×4 matrix and B is a matrix such that $A'B$ and BA' are both defined. Then B is of the type
    (a) 3×4 (b) 3×3
    (c) 4×4 (d) 4×3

18. If $A = \begin{bmatrix} 2 & 3 \\ 5 & -2 \end{bmatrix}$ be such that $A^{-1} = kA$, then k equals
    (a) 19 (b) $\frac{1}{19}$
    (c) –19 (d) $-\frac{1}{19}$

19. If $A = \begin{bmatrix} 2 & 4 \\ 4 & 3 \end{bmatrix}, X = \begin{bmatrix} n \\ 1 \end{bmatrix}, B = \begin{bmatrix} 8 \\ 11 \end{bmatrix}$ and $AX = B$, then n is equal to
    (a) 1 (b) 2
    (c) 4 (d) None of these

20. If the matrix AB is zero, then
    (a) $A = O$ and $B = O$
    (b) $A = O$ or $B = O$
    (c) It is not necessary that either $A = O$ or $B = O$.
    (d) All these statements are wrong.

21. If A and B are two matrices such that $A + B$ and AB are both defined, then
    (a) A and B are two matrices not necessarily of same order
    (b) A and B are square matrices of same order
    (c) Number of columns of A = number of rows of B
    (d) None of these

22. The construction of 3×4 matrix A whose element a_{ij} is given by $\frac{(i+j)^2}{2}$ is :
    (a) $\begin{bmatrix} 2 & 9/2 & 8 & 25 \\ 9 & 4 & 5 & 18 \\ 8 & 25 & 18 & 49 \end{bmatrix}$
    (b) $\begin{bmatrix} 2 & 9/2 & 25/2 \\ 9/2 & 5/2 & 5 \\ 25 & 18 & 25 \end{bmatrix}$
    (c) $\begin{bmatrix} 2 & 9/2 & 8 & 25/2 \\ 9/2 & 8 & 25/2 & 18 \\ 8 & 25/2 & 18 & 49/2 \end{bmatrix}$
    (d) None of these

23. If $\begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix} A \begin{bmatrix} -3 & 2 \\ 5 & -3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, then the matrix A is equal to
    (a) $\begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$
    (c) $\begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$ (d) $\begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}$

24. For any 2×2 matrix A, if $A\,(adj.\,A) = \begin{bmatrix} 10 & 0 \\ 0 & 10 \end{bmatrix}$, then $|A|$ equals
    (a) 0 (b) 10
    (c) 20 (d) 100

25. Which of the following matrices is not invertible ?
    (a) $\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ (b) $\begin{bmatrix} -1 & -1 \\ -1 & 2 \end{bmatrix}$
    (c) $\begin{bmatrix} 2 & 3 \\ 4 & 6 \end{bmatrix}$ (d) $\begin{bmatrix} 2 & -2 \\ 1 & 1 \end{bmatrix}$

26. If $A = \begin{bmatrix} 1 & 0 \\ \frac{1}{2} & 1 \end{bmatrix}$ then A^{50} is
    (a) $\begin{bmatrix} 1 & 0 \\ 0 & 50 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & 0 \\ 50 & 1 \end{bmatrix}$
    (c) $\begin{bmatrix} 1 & 25 \\ 0 & 1 \end{bmatrix}$ (d) $\begin{bmatrix} 1 & 0 \\ 25 & 1 \end{bmatrix}$

27. If A is square matrix such that $A^2 = A$, then $|A|$ equals
    (a) 0 or 1
    (b) – 2 or 2
    (c) – 3 or 3
    (d) None of these

28. If 1, ω, ω^2 are the cube roots of unity, for what value of m is the matrix $\begin{bmatrix} 1 & \omega & m \\ \omega & m & 1 \\ m & 1 & \omega \end{bmatrix}$ singular?
    (a) 0 (b) 1
    (c) ω (d) ω^2

29. Consider the following in respect of the matrix $A=\begin{bmatrix}-1 & 1\\ 1 & -1\end{bmatrix}$ **[NDA-I 2016]**

1. $A^2=-A$ 2. $A^3=4A$

Which of the above is/are correct?

(a) Only 1
(b) Only 2
(c) Both 1 and 2
(d) Neither 1 nor 2

30. If A is a square matrix of order 3 and det A = 5, then what is det $\{2(A)^{-1}\}$ equal to? **[NDA-II 2016]**

(a) $\frac{1}{10}$ (b) $\frac{2}{5}$
(c) $\frac{8}{5}$ (d) $\frac{1}{40}$

31. What is $[x\ y\ z]\begin{bmatrix}a & h & g\\ h & b & f\\ g & f & c\end{bmatrix}$ equal to? **[NDA-II 2016]**

(a) $[ax+hy+gz \quad h+b+f \quad g+f+c]$
(b) $\begin{bmatrix}a & h & g\\ hx & by & fz\\ g & f & c\end{bmatrix}$
(c) $\begin{bmatrix}ax+hy+gz\\ hx+by+fz\\ gx+fy+cz\end{bmatrix}$
(d) $[ax+hy+gz \quad hx+by+fz \quad gx+fy+cz]$

32. If $m=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$ and $n=\begin{bmatrix}0 & 1\\ -1 & 0\end{bmatrix}$, then what is the value of the determinant of $m\cos\theta - n\sin\theta$? **[NDA-II 2016]**

(a) – 1 (b) 0
(c) 1 (d) 2

33. If $f(x)=\begin{bmatrix}\cos x & -\sin x & 0\\ \sin x & \cos x & 0\\ 0 & 0 & 1\end{bmatrix}$, then which of the following are correct ? **[NDA-II 2016]**

I. $f(\theta)\times f(\phi)=f(\theta+\phi)$.
II. The value of the determinant of the matrix $f(\theta)\times f(\phi)$ is 1.
III. The determinant of $f(x)$ is an even function.

Select the correct answer using the code given below.

(a) I and II
(b) II and III
(c) I and III
(d) I, II and III

34. If $A=\begin{bmatrix}1 & -1\\ 2 & 3\end{bmatrix}$ and $B=\begin{bmatrix}2 & 3\\ -1 & -2\end{bmatrix}$, then which of the following is/are correct?

I. $AB(A^{-1}B^{-1})$ is a unit matrix. **[NDA-II 2016]**
II. $(AB)^{-1}=A^{-1}B^{-1}$.

Select the correct answer using the code given below.

(a) I only
(b) II only
(c) Both I and II
(d) Neither I nor II

35. If $A=\begin{bmatrix}\alpha & 2\\ 2 & \alpha\end{bmatrix}$ and det $(A^3)=125$, then α is equal to **[NDA-I 2017]**

(a) ± 1 (b) ± 2
(c) ± 3 (d) ± 5

36. If B is a non-singular matrix and A is a square matrix, then the value of det $(B^{-1}AB)$ is equal to **[NDA-I 2017]**

(a) det (B) (b) det (A)
(c) det (B^{-1}) (d) det (A^{-1})

37. If $A=\begin{bmatrix}\cos\alpha & \sin\alpha\\ -\sin\alpha & \cos\alpha\end{bmatrix}$, then what is AA^T equal to (where A^T is transpose of A)? **[NDA-I 2017]**

(a) Null matrix (b) Identity matrix
(c) A (d) $-A$

38. $A=\begin{bmatrix}x+y & y\\ x & x-y\end{bmatrix}, B=\begin{bmatrix}3\\ -2\end{bmatrix}$ and $C=\begin{bmatrix}4\\ -2\end{bmatrix}$.

If $AB=C$, then what is A^2 equal to? **[NDA-I 2017]**

(a) $\begin{bmatrix}4 & 8\\ -4 & -16\end{bmatrix}$ (b) $\begin{bmatrix}4 & -4\\ 8 & -16\end{bmatrix}$
(c) $\begin{bmatrix}-4 & -8\\ 4 & 12\end{bmatrix}$ (d) $\begin{bmatrix}-4 & -8\\ 8 & 12\end{bmatrix}$

39. Consider the set A of all matrices of order 3 × 3 with entries 0 or 1 only. Let B be the subset of A consisting of all matrices whose determinant is 1. Let C be the subset of A consisting of all matrices whose determinant is – 1. Then, which one of the following is correct? **[NDA-I 2017]**

(a) C is empty
(b) B has as many elements as C
(c) $A=B\cup C$
(d) B has thrice as many elements as C

40. If $A=\begin{bmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{bmatrix}$, then what is A^3 equal to? **[NDA-I 2017]**

(a) $\begin{bmatrix}\cos 3\theta & \sin 3\theta\\ -\sin 3\theta & \cos 3\theta\end{bmatrix}$ (b) $\begin{bmatrix}\cos^3\theta & \sin^3\theta\\ -\sin^3\theta & \cos^3\theta\end{bmatrix}$
(c) $\begin{bmatrix}\cos 3\theta & -\sin 3\theta\\ \sin 3\theta & \cos 3\theta\end{bmatrix}$ (d) $\begin{bmatrix}\cos^3\theta & -\sin^3\theta\\ \sin^3\theta & \cos^3\theta\end{bmatrix}$

ANSWERS

1.	(c)	**2.**	(a)	**3.**	(a)	**4.**	(d)	**5.**	(b)	**6.**	(b)	**7.**	(b)	**8.**	(c)	**9.**	(d)	**10.**	(c)
11.	(c)	**12.**	(b)	**13.**	(c)	**14.**	(b)	**15.**	(a)	**16.**	(a)	**17.**	(a)	**18.**	(d)	**19.**	(b)	**20.**	(c)
21.	(b)	**22.**	(c)	**23.**	(a)	**24.**	(b)	**25.**	(c)	**26.**	(d)	**27.**	(a)	**28.**	(d)	**29.**	(b)	**30.**	(c)
31.	(d)	**32.**	(c)	**33.**	(d)	**34.**	(d)	**35.**	(c)	**36.**	(b)	**37.**	(b)	**38.**	(c)	**39.**	(b)	**40.**	(a)
41.	(b)	**42.**	(a)	**43.**	(c)	**44.**	(b)	**45.**	(a)	**46.**	(c)	**47.**	(a)	**48.**	(b)	**49.**	(b)	**50.**	(a)
51.	(a)	**52.**	(a)	**53.**	(a)	**54.**	(b)	**55.**	(c)	**56.**	(b)	**57.**	(b)	**58.**	(a)	**59.**	(a)	**60.**	(b)
61.	(a)	**62.**	(b)	**63.**	(b)	**64.**	(d)	**65.**	(b)	**66.**	(a)	**67.**	(c)	**68.**	(c)	**69.**	(a)	**70.**	(a)
71.	(c)	**72.**	(c)	**73.**	(d)	**74.**	(c)	**75.**	(c)	**76.**	(a)	**77.**	(a)	**78.**	(d)	**79.**	(a)	**80.**	(c)

Explanations

1. (c) $E(\alpha)E(\beta) = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}\begin{bmatrix} \cos\beta & \sin\beta \\ -\sin\beta & \cos\beta \end{bmatrix}$

$= \begin{bmatrix} \cos(\alpha+\beta) & \sin(\alpha+\beta) \\ -\sin(\alpha+\beta) & \cos(\alpha+\beta) \end{bmatrix}$

$= E(\alpha + \beta)$

2. (a) $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, J = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$

$I\cos\theta = \begin{bmatrix} \cos\theta & 0 \\ 0 & \cos\theta \end{bmatrix},$

$J\cos\theta = \begin{bmatrix} 0 & \sin\theta \\ -\sin\theta & 0 \end{bmatrix}$

$I\cos\theta + J\sin\theta = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} = B$

3. (a) Given $AB = A$ and $BA = B$

Now, $A^2 = A \times A = (AB) \times A = A(BA) = AB$

$\Rightarrow A^2 = A \Rightarrow A$ is idempotent.

Similarly, $B^2 = B \times B = (BA)B$

$= B(AB) = BA$

$\Rightarrow B^2 = B \Rightarrow B$ is idempotent.

4. (d) Given, A is skew symmetric. Then,

$A^T = -A \Rightarrow (A^T)^n = (-A)^n$

$\Rightarrow (A^n)^T = (-1)^n A^n$

$\Rightarrow (A^n)^T \begin{cases} A^n, & \text{if } n \text{ is even} \\ -A^n, & \text{if } n \text{ is odd} \end{cases}$

5. (b) $(AB - BA)^T = (AB)^T - (BA)^T$

$= B^T A^T - A^T B^T$

$= BA - AB$ $\{\because A^T = A$ and $B^T = B)$

$(AB - BA)^T = -(AB - BA)\}$

$\Rightarrow AB - BA$ is skew symmetric matrix.

6. (b) $A = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

$A^2 = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix} = 2A$

$A^3 = A(2A) = 2(AA) = 2A^1 = 2(2A)$

$A^3 = 2^2 A$

Hence, $A^n = 2^{n-1} A$

7. (b) $A = \begin{bmatrix} a & b \\ b & a \end{bmatrix}$

$A^2 = \begin{bmatrix} a & b \\ b & a \end{bmatrix}\begin{bmatrix} a & b \\ b & a \end{bmatrix} = \begin{bmatrix} a^2+b^2 & 2ab \\ 2ab & a^2+b^2 \end{bmatrix}$

But given, $A^2 = \begin{bmatrix} \alpha & \beta \\ \beta & \alpha \end{bmatrix}$

So, $\alpha = a^2 + b^2$ and $\beta = 2ab$

8. (c) $J_1^2 = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$

$= \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} = -\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = -1$

9. (d) $3A^3 + 2A^2 + 5A + I = 0$

$\Rightarrow I = -(3A^3 + 2A^2 + 5A)$

$\Rightarrow I = -(3A^3 + 2A^2 + 5A)\, A^{-1}$

$\Rightarrow A^{-1} = -(3A + 2A + 5I)$

10. (c) Given that $AB = 0$

$\Rightarrow |AB| = 0 \Rightarrow |A| = |B| = 0$

$\Rightarrow |A| = 0$ or $|B| = 0$

11. (c) Given $AB = B$ and $BA = A$

Therefore, $A^2 + B^2 = A \cdot A + B \cdot B$

$= A(BA) + B(AB) = (AB)A + (BA)B$

$= BA + AB = A + B$

12. (b) $A = \begin{bmatrix} 4 & x+2 \\ 2x-3 & x+1 \end{bmatrix}$ is symmetric

So, $a_{12} = a_{21}$

$\Rightarrow x + 2 = 2x - 3 \Rightarrow x = 5$

13. (c) $A + B = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$ ...(i)

$A - 2B = \begin{bmatrix} -1 & 1 \\ 0 & -1 \end{bmatrix}$ ...(ii)

Subtract eq. (ii) – 2(i)

$A = \begin{bmatrix} \frac{1}{3} & \frac{1}{3} \\ \frac{2}{3} & \frac{1}{3} \end{bmatrix}$

14. (b) $AB = AC$

$\Rightarrow A^{-1}(AB) = A^{-1}(AC)$

$\Rightarrow (A^{-1}A)B = (A^{-1}A)C \Rightarrow IB = IC$

$\Rightarrow B = C$

$\Rightarrow A^{-1}$ exist, *i.e.*, $|A| \neq 0$

$\Rightarrow A$ is non-singular matrix.

15. (a) $AB = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 0 & -i \\ -i & 0 \end{bmatrix} = \begin{bmatrix} i & 0 \\ 0 & i \end{bmatrix}$

$BA = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} -i & 0 \\ 0 & i \end{bmatrix}$

$AB = -BA$

So, $A^2 + AB + BA + B^2 = A^2 + B^2$

16. (a) $AB = \begin{bmatrix} i & 0 \\ 0 & i \end{bmatrix}\begin{bmatrix} 0 & -i \\ -i & 0 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

$BA = \begin{bmatrix} 0 & -i \\ -i & 0 \end{bmatrix}\begin{bmatrix} i & 0 \\ 0 & i \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

$(A + B)(A - B) = A^2 - AB + BA - B^2 = A^2 - B^2$

17. (a) A is of 3×4 order

$\Rightarrow A'$ is of 4×3 order

$\Rightarrow A'B = (4 \times 3)(3 \times n)$

$\Rightarrow BA' = (3 \times n)(4 \times 3)$

Thus, $n = 4$ satisfies the problem.

Hence, $B = 3 \times 4$ matrix.

18. (d) $A^{-1} = kA$

$\left(-\frac{1}{19}\right)\begin{bmatrix} -2 & -3 \\ -5 & 2 \end{bmatrix} = k\begin{bmatrix} 2 & 3 \\ 5 & -2 \end{bmatrix} \Rightarrow k = \frac{1}{19}$

19. (b) $A = \begin{bmatrix} 2 & 4 \\ 4 & 3 \end{bmatrix}, X = \begin{bmatrix} n \\ 1 \end{bmatrix}, B = \begin{bmatrix} 8 \\ 11 \end{bmatrix}$

$AX = B$

$\begin{bmatrix} 2n+4 \\ 4n+3 \end{bmatrix} = \begin{bmatrix} 8 \\ 11 \end{bmatrix}$

$\Rightarrow 2n + 4 = 8 \Rightarrow n = 2$

20. (c) If $AB = 0$, then, it is not necessary that either $A = 0$ or $B = 0$

21. (b) If $A + B$ and AB both are defined for two matrices A and B, then it is obvious that A and B are the square matrices of same order.

22. (c) $a_{ij} = \frac{(i+j)^2}{2}$

So, $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \end{bmatrix}_{3\times4}$

$= \begin{bmatrix} 2 & \frac{9}{2} & 8 & \frac{25}{2} \\ \frac{9}{2} & 8 & \frac{25}{2} & 18 \\ 8 & \frac{25}{2} & 18 & \frac{49}{2} \end{bmatrix}$

23. (a) $\begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix} A \begin{bmatrix} -3 & 2 \\ 5 & -3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$A = \begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix}^{-1}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} -3 & 2 \\ 5 & -3 \end{bmatrix}^{-1}$

$= \begin{bmatrix} 2 & -1 \\ -3 & 2 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 3 & 2 \\ 5 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$

24. (b) $A(\text{adj } A) = |A|\, I$

$\Rightarrow \begin{bmatrix} 10 & 0 \\ 0 & 10 \end{bmatrix} = |A|\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \Rightarrow \begin{bmatrix} 10 & 0 \\ 0 & 10 \end{bmatrix} = \begin{bmatrix} |A| & 0 \\ 0 & |A| \end{bmatrix}$

$\Rightarrow |A| = 10$

25. (c) $\because \begin{vmatrix} 2 & 3 \\ 4 & 6 \end{vmatrix} = 0$

So, $\begin{bmatrix} 2 & 3 \\ 4 & 6 \end{bmatrix}$ is not invertible.

26. (d) $A = \begin{bmatrix} 1 & 0 \\ \frac{1}{2} & 1 \end{bmatrix}$

$A^2 = \begin{bmatrix} 1 & 0 \\ \frac{1}{2} & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ \frac{1}{2} & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ \frac{2}{2} & 1 \end{bmatrix}$

$A^3 = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 1/2 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 3/2 & 1 \end{bmatrix}$

So, $A^{50} = \begin{bmatrix} 1 & 0 \\ 50/2 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 25 & 1 \end{bmatrix}$

27. (a) $\because A^2 = A$

$\Rightarrow |A^2| = |A|$ or $|A^2| - |A| = 0$

$\Rightarrow |A|\{|A| - 1\} = 0 \Rightarrow |A| = 0$ or 1

28. (d) $\because$ Given matrix is singular.

So, $\begin{vmatrix} 1 & \omega & m \\ \omega & m & 1 \\ m & 1 & \omega \end{vmatrix} = 0$

$\Rightarrow \begin{vmatrix} 1+\omega+m & \omega & m \\ 1+\omega+m & m & 1 \\ 1+\omega+m & 1 & \omega \end{vmatrix} = 0 \quad \{C_1 + C_2 + C_3\}$

or $(1+\omega+m)\begin{vmatrix} 1 & \omega & m \\ 1 & m & 1 \\ 1 & 1 & \omega \end{vmatrix} = 0 \Rightarrow 1+\omega+m = 0$

or $m = -(1+\omega)$

$m = -(-\omega^2)$ $\{\because 1 + \omega + \omega^2 = 0\}$

$m = \omega^2$.

29. (b) Given $A = \begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix}$

$A^2 = A \cdot A = \begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix} = \begin{bmatrix} 2 & -2 \\ -2 & 2 \end{bmatrix}$

$= -2\begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix} = -2A$

Now, $A^3 = A^2 \times A = (-2A) \times A = -2(A^2)$

$= -2(-2A) = 4A$

Hence, only statement 2 is correct.

30. (c) Given, order of matrix A is 3×3.

and $|A| = 5$

Now det $\{2(A^{-1})\} = |2A^{-1}| = 2^3 |A^{-1}|$

$= 8|A|^{-1} = 8 \cdot \frac{1}{|A|} = \frac{8}{5}$

31. (d) $[x\ y\ z]\begin{bmatrix} a & h & g \\ h & b & f \\ g & f & c \end{bmatrix}$

$= [ax + hy + gz \quad hx + by + fz \quad gx + fy + cz]$

32. (c) $m \cos\theta - n \sin\theta = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\cos\theta + \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}\sin\theta$

$= \begin{bmatrix} \cos\theta & 0 \\ 0 & \cos\theta \end{bmatrix} - \begin{bmatrix} 0 & \sin\theta \\ -\sin\theta & 0 \end{bmatrix}$

$= \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$

determinant of $m \cos\theta - n \sin\theta$

$= \cos^2\theta - (-\sin^2\theta)$

$= 1$

33. (d) $f(\theta) \times f(\phi) = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} \cos\phi & -\sin\phi & 0 \\ \sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$\begin{bmatrix} \cos\theta\cos\phi - \sin\theta\sin\phi & -\cos\theta\sin\phi - \sin\theta\cos\phi & 0 \\ \sin\theta\cos\phi + \cos\theta\sin\phi & -\sin\theta\sin\phi + \cos\theta\cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$\begin{bmatrix} \cos(\theta+\phi) & -\sin(\theta+\phi) & 0 \\ \sin(\theta+\phi) & \cos(\theta+\phi) & 0 \\ 0 & 0 & 1 \end{bmatrix} = f(\theta+\phi)$

Determinant of $f(\theta) \times f(\phi)$

$= \cos^2(\theta + \phi) + \sin^2(\theta + \phi) = 1$

$|f(x)| = \begin{vmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{vmatrix} = \cos^2 x + \sin^2 x = 1$

Here, $|f(-x)| = |f(x)|$

So, $f(x)$ is an even function.

Hence, all the three statements are correct.

34. (d) Given $A = \begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 3 \\ -1 & -2 \end{bmatrix}$

$\Rightarrow$ adj $A = \begin{bmatrix} 3 & 1 \\ -2 & 1 \end{bmatrix}$ and adj $B = \begin{bmatrix} -2 & -3 \\ 1 & 2 \end{bmatrix}$

Also, $|A| = 3 + 2 = 5$ and $|B| = -4 + 3 = -1$

So, $A^{-1} = \frac{\text{adj} A}{|A|} = \begin{bmatrix} 3/5 & 1/5 \\ -2/5 & 1/5 \end{bmatrix}$

$B^{-1} = \frac{\text{adj} B}{|B|} = \begin{bmatrix} 2 & 3 \\ -1 & -2 \end{bmatrix}$

$AB(A^{-1}B^{-1}) = \begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}\begin{bmatrix} 2 & 3 \\ -1 & -2 \end{bmatrix}$

$\begin{bmatrix} 3/5 & 1/5 \\ -2/5 & 1/5 \end{bmatrix}\begin{bmatrix} 2 & 3 \\ -1 & -2 \end{bmatrix}$

$= \begin{bmatrix} 3 & 5 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 1 & 7/5 \\ -1 & -7/5 \end{bmatrix} = \begin{bmatrix} -2 & 10 \\ 1 & 7/5 \end{bmatrix} \neq I$

So, statement 1 is not correct.

Now, $(AB)(A^{-1}B^{-1}) \neq I$

$\Rightarrow (AB)^{-1} \neq A^{-1}B^{-1}$

So, statement 2 is also not correct.

35. (c) $A = \begin{bmatrix} \alpha & 2 \\ 2 & \alpha \end{bmatrix}$

$\Rightarrow |A| = \begin{bmatrix} \alpha & 2 \\ 2 & \alpha \end{bmatrix} = \alpha^2 - 4$

Given, $|A^3| = 125 \Rightarrow |A|^3 = 125 \Rightarrow |A| = 5$

$\Rightarrow \alpha^2 - 4 = 5$

$\Rightarrow \alpha^2 = 9 \Rightarrow \alpha = \pm 3$

36. (b) $|B^{-1}AB| = |B^{-1}||A||B| = |B|^{-1}|A||B| = |A|$

$\{\because$ Given $|B| \neq 0\}$

37. (b) $AA^T = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}\begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$

$$= \begin{bmatrix} \cos^2 \alpha + \sin^2 \alpha & 0 \\ 0 & \sin^2 \alpha + \cos^2 \alpha \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$$

38. (c) Given, $AB = C$

$$\Rightarrow \begin{bmatrix} x+y & y \\ x & x-y \end{bmatrix} \begin{bmatrix} 3 \\ -2 \end{bmatrix} = \begin{bmatrix} 4 \\ -2 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 3x+y \\ x+2y \end{bmatrix} = \begin{bmatrix} 4 \\ -2 \end{bmatrix}$$

$\Rightarrow$ $3x + y = 4$ and $x + 2y = -2$

On solving, $x = 2$ and $y = -2$

Then, $A = \begin{bmatrix} 0 & -2 \\ 2 & 4 \end{bmatrix}$

$$\Rightarrow A^2 = \begin{bmatrix} 0 & -2 \\ 2 & 4 \end{bmatrix} \begin{bmatrix} 0 & -2 \\ 2 & 4 \end{bmatrix} = \begin{bmatrix} -4 & -8 \\ 8 & 12 \end{bmatrix}$$

39. (b) Given, A is the set of all matrices of order 3×3 with entries 0 and 1 only. It will behave like a universal set here.

While B and C are the subsets of A.

Given, determinant of matrices of set $B = 1$ and determinant of matrices of set $C = -1$

This is possible only when any two rows or any two columns of the matrices in set C are interchanged.

Hence, B has as many matrices as in set C.

40. (a) $A^2 = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$

$$= \begin{bmatrix} \cos^2\theta - \sin^2\theta & \cos\theta\sin\theta + \sin\theta\cos\theta \\ -\sin\theta\cos\theta - \cos\theta\sin\theta & -\sin^2\theta + \cos^2\theta \end{bmatrix}$$

$$= \begin{bmatrix} \cos 2\theta & \sin 2\theta \\ -\sin 2\theta & \cos 2\theta \end{bmatrix}$$

Now, $A^3 = \begin{bmatrix} \cos 2\theta & \sin 2\theta \\ -\sin 2\theta & \cos 2\theta \end{bmatrix} \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$

$$= \begin{bmatrix} \cos 2\theta\cos\theta - \sin 2\theta\sin\theta & \cos 2\theta\sin\theta + \sin 2\theta\cos\theta \\ -\sin 2\theta\cos\theta - \cos 2\theta\sin\theta & -\sin 2\theta\sin\theta + \cos 2\theta\cos\theta \end{bmatrix}$$

$$= \begin{bmatrix} \cos 3\theta & \sin 3\theta \\ -\sin 3\theta & \cos 3\theta \end{bmatrix}$$