Chapter 18

Cartesian System and Straight Lines

Exercise

1. What is the perimeter of the triangle with vertices $A(-4, 2)$, $B(0, -1)$ and $C(3, 3)$?
   (a) $7+3\sqrt{2}$ (b) $10+5\sqrt{2}$
   (c) $11+6\sqrt{2}$ (d) $5+\sqrt{2}$
2. In what ratio does the point $\left(1, -\frac{7}{2}\right)$ divides the join of the points $(-2, -4)$ and $\left(2, -\frac{10}{3}\right)$?
   (a) 1 : 2 (b) 1 : 3
   (c) 3 : 1 (d) 2 : 1
3. What is the area of the triangle formed by the lines $y = 3x$, $y = 6x$ and $y = 9$.
   (a) $\frac{27}{4}$ sq. units (b) $\frac{27}{2}$ sq. units
   (c) $\frac{19}{4}$ sq. units (d) $\frac{19}{2}$ sq. units
4. The value of k for which the lines $2x - 3y + k = 0$, $3x - 4y - 13 = 0$ and $8x - 11y - 33 = 0$ are concurrent, is
   (a) 20 (b) -7
   (c) 7 (d) -20
5. The area of the figure formed by $a|x| + b|y| + c = 0$ is
   (a) $\frac{c^2}{|ab|}$ (b) $\frac{2c^2}{|ab|}$
   (c) $\frac{c^2}{2|ab|}$ (d) None of these
6. The image of the point $(-1, 3)$ by the line $x - y = 0$ is
   (a) $(3, -1)$ (b) $(1, -3)$
   (c) $(-1, -1)$ (d) $(3, 3)$
7. Length of the median from B on AC, in ΔABC having vertices $A(-1, 3)$, $B(1, -1)$, $C(5, 1)$ is
   (a) $\sqrt{18}$ (b) $\sqrt{10}$
   (c) $2\sqrt{3}$ (d) 4
8. Points (1, 2) and (– 2, 1) are
   (a) on the same side of the line $4x + 2y = 1$
   (b) on the line $4x + 2y = 1$
   (c) on the opposite side of $4x + 2y = 1$
   (d) None of these
9. Points on the line $x + y = 4$ that lie at a unit distance from the line $4x + 3y - 10 = 0$ are
   (a) (3, 1) and (– 7, 11) (b) (– 3, 7) and (2 , 2)
   (c) (– 3, 7) and (– 7, 11) (d) None of these
10. The line $x + y = 4$ divides the line joining (– 1, 1) and (5, 7) in the ratio $\lambda : 1$, then the value of λ is
    (a) 2 (b) $\frac{1}{2}$
    (c) 3 (d) None of these
11. A line passes through (2, 2) and is perpendicular to the line $3x + y = 3$. Its y-intercept is
    (a) $\frac{1}{3}$ (b) $\frac{2}{3}$
    (c) 1 (d) $\frac{4}{3}$
12. The equation of the straight line which passes through the point (1, – 2) and cuts off equal intercepts from the axes will be
    (a) $x + y = 1$ (b) $x - y = 1$
    (c) $x + y + 1 = 0$ (d) $x - y - 2 = 0$
13. A straight line passing through $P(1, 2)$ is such that its intercept between the axes is trisected at P. Its equation is
    (a) $x + 2y = 5$ (b) $x - y + 3 = 0$
    (c) $x + y - 3 = 0$ (d) $x + y + 3 = 0$
14. If the line $y = mx$ meets the lines $x + 2y - 1 = 0$ and $2x - y + 3 = 0$ at the same point, then m is equal to
    (a) 1 (b) -1
    (c) 2 (d) -2

15. Line $x + 2y - 8 = 0$ is the perpendicular bisector of AB. If B is (3, 5), then coordinates of A are
(a) (2, 1) (b) (1, 2)
(c) (2, 2) (d) (1, 1)

16. The foot of the perpendicular from the point (0, 5) on the line $3x - 4y - 5 = 0$ is
(a) (1, 3) (b) (2, 3)
(c) (3, 2) (d) (3, 1)

17. The line segment joining the points (1, 2) and (– 2, 1) is divided by the line $3x + 4y = 7$ in the ratio
(a) 3 : 4 (b) 4 : 3
(c) 9 : 4 (d) 4 : 9

18. If t_1, t_2, t_3 are distinct, the points $(t_1, 2at_1 + at_1^3)$, $(t_2, 2at_2 + at_2^3)$, $(t_3, 2at_3 + at_3^3)$ are collinear if
(a) $t_1t_2t_3 = 1$ (b) $t_1 + t_2 + t_3 = t_1t_2t_3$
(c) $t_1 + t_2 + t_3 = 0$ (d) $t_1 + t_2 + t_3 = -1$

19. The straight lines $x + y - 4 = 0$, $3x + y - 4 = 0$, $x + 3y - 4 = 0$ form a triangle which is
(a) isosceles (b) right angled
(c) equilateral (d) None of these

20. The coordinates B and C are (1, – 2), (2, 3). A lies on the line $2x + y - 2 = 0$. The area of the triangle ABC is 8 square units. Then, the vertex A is
(a) (1, 4) (b) (– 1, 4)
(c) (– 1, – 4) (d) (1, – 4)

21. The orthocentre and centroid of a triangle are (– 3, 5) and (3, 3) respectively then the circumentre is
(a) (0, 4) (b) (6, – 2)
(c) (0, 8) (d) (6, 2)

22. The equation $kx^2 + 4xy + 5y^2 = 0$ represents two lines inclined at angle π, if k is
(a) $\frac{5}{4}$ (b) $\frac{4}{5}$
(c) $-\frac{4}{5}$ (d) None of these

23. The angle between the pair of lines represented by $2x^2 - 5xy + 3y^2 = 0$ is
(a) 60º (b) $\tan^{-1}\left(\frac{1}{5}\right)$
(c) $\tan^{-1}\left(\frac{7}{6}\right)$ (d) 30º

24. The equation $2x^2 - 3xy - py^2 + x + qy - 1 = 0$ represents two mutually perpendicular lines is
(a) $p = 3, q = 2$
(b) $p = 2, q = 3$
(c) $p = -2, q = 3$
(d) $p = 2, q = -\frac{9}{2}$

25. The equation $8x^2 + 8xy + 2y^2 + 26x + 13y + 15 = 0$ represents a pair of straight lines. The distance between them is
(a) $\frac{7}{\sqrt{5}}$ (b) $\frac{7}{2\sqrt{5}}$
(c) $\sqrt{\frac{7}{5}}$ (d) None of these

26. If $\lambda x^2 - 10xy + 12y^2 + 5x - 16y - 3 = 0$ represents a pair of straight lines, the value of λ is
(a) 4 (b) 3
(c) 2 (d) 1

27. The image of the point (1, 3) in the line $x + y - 6 = 0$ is
(a) (3, 5) (b) (5, 3)
(c) (1, – 3) (d) (0, – 4)

28. If the lines $x + ay + a = 0$, $bx + y + b = 0$ and $cx + cy + 1 = 0$ (a, b, c being distinct $\neq 1$) are concurrent, then the value of $\frac{a}{a-1} + \frac{b}{b-1} + \frac{c}{c-1}$ is
(a) – 1 (b) 0
(c) 1 (d) None of these

29. Line segment joining the points (1, 2) and (k, 1) is divided by the lines $3x + 4y - 7 = 0$ in the ratio 4 : 9 then k is equal to
(a) – 2 (b) 2
(c) – 3 (d) 3

30. The coordinates of foot of the perpendiucular drawn from the point (2, 4) on the line $x + y = 1$ are
(a) $\left(\frac{1}{2}, \frac{3}{2}\right)$ (b) $\left(-\frac{1}{2}, \frac{3}{2}\right)$
(c) $\left(\frac{3}{2}, -\frac{1}{2}\right)$ (d) $\left(-\frac{1}{2}, \frac{3}{2}\right)$

31. Let PS be the median of the triangle with vertices $P(2, 2)$, $Q(6, -1)$ and $R(7, 3)$. The equation of the line passing through (1, – 1) and parallel to PS is
(a) $2x - 9y - 7 = 0$ (b) $2x - 9y - 11 = 0$
(c) $2x + 9y - 11 = 0$ (d) $2x + 9y + 7 = 0$

32. What angle does the line segment joining (5, 2) and (6, – 15) subtend at (0, 0) ?
(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{4}$
(c) $\frac{\pi}{2}$ (d) $\frac{3\pi}{4}$

33. If the coordinates of the points A, B and C be (– 1, 5), (0, 0) and (2, 2) respectively and D be the middle point of BC, then the equation of the perpendicular drawn from B to the line AD is
(a) $x + 2y = 0$ (b) $2x + y = 0$
(c) $x - 2y = 0$ (d) $2x - y = 0$

34. The lines $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ are perpendicular to each other, if
(a) $a_1b_2 - a_2b_1 = 0$ (b) $a_1a_2 + b_1b_2 = 0$
(c) $a_1^2b_2 + b_1^2a_2 = 0$ (d) $a_1b_1 + a_2b_2 = 0$

35. Angle between the lines $\frac{x}{a}+\frac{y}{b}=1$ and $\frac{x}{a}-\frac{y}{b}=1$ is
(a) $2\tan^{-1}\frac{b}{a}$ (b) $\tan^{-1}\frac{2ab}{a^2+b^2}$
(c) $\tan^{-1}\frac{a^2-b^2}{a^2+b^2}$ (d) None of these

36. If the middle points of the sides BC, CA and AB of ΔABC be (1, 3), (5, 7) and (– 5, 7) respectively, then the equation of the side AB is
(a) $x-y-2=0$ (b) $x-y+12=0$
(c) $x+y-12=0$ (d) None of these

Directions (Q. Nos. 37-39): Consider a parallelogram whose vertices are $A(1, 2)$, $B(4, y)$, $C(x, 6)$ and $D(3, 5)$ taken in order.

37. What is the value of $AC^2 - BD^2$? **[NDA-I 2016]**
(a) 25 (b) 30
(c) 36 (d) 40

38. What is the point of intersection of the diagonals? **[NDA-I 2016]**
(a) $\left(\frac{7}{2},4\right)$ (b) (3, 4)
(c) $\left(\frac{7}{2},5\right)$ (d) (3, 5)

39. A straight line intersects x and y axes at P and Q, respectively. If (3, 5) is the middle point of PQ, then what is the area of the ΔPOQ? **[NDA-I 2016]**
(a) 12 sq. units (b) 15 sq. units
(c) 20 sq. units (d) 30 sq. units

Directions (Q. Nos. 40-41): Consider the two lines $x+y+1=0$ and $3x+2y+1=0$

40. What is the equation of the line passing through the point of intersection of the given lines and parallel to X-axis? **[NDA-I 2016]**
(a) $y+1=0$ (b) $y-1=0$
(c) $y-2=0$ (d) $y+2=0$

41. What is the equation of the line passing through the point of intersection of the given lines and parallel to Y-axis? **[NDA-I 2016]**
(a) $x+1=0$ (b) $x-1=0$
(c) $x-2=0$ (d) $x+2=0$

42. $(a, 2b)$ is the mid-point of the line segment joining the points (10, – 6) and $(k, 4)$. If $a-2b=7$, then what is the value of k? **[NDA-I 2016]**
(a) 2 (b) 3
(c) 4 (d) 5

43. What is the acute angle between the lines represented by the equations $y-\sqrt{3}x-5=0$ and $\sqrt{3}y-x+6=0$? **[NDA-I 2016]**
(a) 30º (b) 45º
(c) 60º (d) 75º

44. An equilateral triangle has one vertex at (0, 0) and another at $(3, \sqrt{3})$. What are the coordinates of the third vertex? **[NDA-II 2016]**
(a) $(0, 2\sqrt{3})$ only
(b) $(3, -\sqrt{3})$ only
(c) $(0, 2\sqrt{3})$ or $(3, -\sqrt{3})$
(d) Neither $(0, 2\sqrt{3})$ nor $(3, -\sqrt{3})$

45. What is the equation of the right bisector of the line segment joining (1, 1) and (2, 3)? **[NDA-II 2016]**
(a) $2x+4y-11=0$ (b) $2x-4y-5=0$
(c) $2x-4y-11=0$ (d) $x-y+1=0$

46. If the point (a, a) lies between the lines $|x+y|=2$, then which one of the following is correct? **[NDA-II 2016]**
(a) $|a|<2$ (b) $|a|<\sqrt{2}$
(c) $|a|<1$ (d) $|a|<\frac{1}{\sqrt{2}}$

47. What is the equation of the straight line which passes through the point of intersection of the straight lines $x+2y=5$ and $3x+7y=17$ and is perpendicular to the straight line $3x+4y=10$? **[NDA-II 2016]**
(a) $4x+3y+2=0$ (b) $4x-y+2=0$
(c) $4x-3y-2=0$ (d) $4x-3y+2=0$

48. If (a , b) is at unit distance from the line $8x+6y+1=0$, then which of the following conditions are correct? **[NDA-II 2016]**
I. $3a-4b-4=0$ II. $8a+6b+11=0$
III. $8a+6b-9=0$
Select the correct answer using the code given below.
(a) I and II (b) II and III
(c) I and III (d) I, II and III

49. A straight line cuts off an intercept of 2 units on the positive direction of X-axis and passes through the point (– 3, 5). What is the foot of the perpendicular drawn from the point (3, 3) on this line? **[NDA-II 2016]**
(a) (1, 3) (b) (2, 0)
(c) (0, 2) (d) (1, 1)

50. If a vertex of a triangle is (1, 1) and the mid points of two sides of the triangle through this vertex are (–1, 2) and (3, 2), then the centroid of the triangle is **[NDA-I 2017]**
(a) $\left(-\frac{1}{3},\frac{7}{3}\right)$ (b) $\left(-1,\frac{7}{3}\right)$
(c) $\left(\frac{1}{3},\frac{7}{3}\right)$ (d) $\left(1,\frac{7}{3}\right)$

51. The incentre of the triangle with vertices $A(1, \sqrt{3})$, $B(0, 0)$ and $C(2, 0)$ is **[NDA-I 2017]**
(a) $\left(1,\frac{\sqrt{3}}{2}\right)$ (b) $\left(\frac{2}{3},\frac{1}{\sqrt{3}}\right)$

(c) $\left(\frac{2}{3}, \frac{\sqrt{3}}{2}\right)$ (d) $\left(1, \frac{1}{\sqrt{3}}\right)$

52. If the three consecutive vertices of a parallelogram are (– 2, – 1), (1, 0) and (4, 3), then what are the coordinates of the fourth vertex? **[NDA-I 2017]**
(a) (1, 2) (b) (1, 0)
(c) (0, 0) (d) (1, – 1)

53. What is the ratio in which the point $C\left(-\frac{2}{7}, -\frac{20}{7}\right)$ divides the line joining the points A(– 2, – 2) and B(2, – 4)? **[NDA-I 2017]**
(a) 1 : 3 (b) 3 : 4
(c) 1 : 2 (d) 2 : 3

54. What is the equation of the straight line parallel to $2x + 3y + 1 = 0$ and passes through the point (– 1, 2)? **[NDA-I 2017]**
(a) $2x + 3y - 4 = 0$ (b) $2x + 3y - 5 = 0$
(c) $x + y - 1 = 0$ (d) $3x - 2y + 7 = 0$

55. What is the acute angle between the pair of straight lines $\sqrt{2}x + \sqrt{3}y = 1$ and $\sqrt{3}x + \sqrt{2}y = 2$? **[NDA-I 2017]**
(a) $\tan^{-1}\left(\frac{1}{2\sqrt{6}}\right)$ (b) $\tan^{-1}\left(\frac{1}{\sqrt{2}}\right)$
(c) $\tan^{-1}(3)$ (d) $\tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$

56. If the centroid of a triangle formed by $(7, x)$, $(y, -6)$ and (9, 10) is (6, 3), then the values of x and y are respectively. **[NDA-I 2017]**
(a) 5, 2 (b) 2, 5
(c) 1, 0 (d) 0, 0

57. The points (a, b), (0, 0), $(-a, -b)$ and (ab, b^2) are **[NDA-II 2017]**
(a) the vertices of a parallelogram
(b) the vertices of a rectangle
(c) the vertices of a square
(d) collinear

58. The angle between the lines $x + y - 3 = 0$ and $x - y + 3 = 0$ is α and the acute angle between the lines $x - \sqrt{3}y + 2\sqrt{3} = 0$ and $\sqrt{3}x - y + 1 = 0$ is β. Which one of the following is correct? **[NDA-II 2017]**
(a) $\alpha = \beta$ (b) $\alpha > \beta$
(c) $\alpha < \beta$ (d) $\alpha = 2\beta$

59. The distance of the point (1, 3) from the line $2x + 3y = 6$, measured parallel to the line $4x + y = 4$, is **[NDA-II 2017]**
(a) $\frac{5}{\sqrt{13}}$ units (b) $\frac{3}{\sqrt{17}}$ units
(c) $\sqrt{17}$ units (d) $\frac{\sqrt{17}}{2}$ units

60. The equation of straight line which cuts off an intercept of 5 units on negative direction of Y-axis and makes and angle 120° with positive direction of X-axis is **[NDA-II 2017]**
(a) $y + \sqrt{3}x + 5 = 0$ (b) $y - \sqrt{3}x + 5 = 0$
(c) $y + \sqrt{3}x - 5 = 0$ (d) $y - \sqrt{3}x - 5 = 0$

ANSWERS

1.	(b)	**2.**	(c)	**3.**	(a)	**4.**	(b)	**5.**	(b)	**6.**	(a)	**7.**	(b)	**8.**	(c)	**9.**	(a)	**10.**	(b)
11.	(d)	**12.**	(c)	**13.**	(c)	**14.**	(b)	**15.**	(d)	**16.**	(d)	**17.**	(d)	**18.**	(c)	**19.**	(a)	**20.**	(b)
21.	(d)	**22.**	(b)	**23.**	(b)	**24.**	(b)	**25.**	(b)	**26.**	(c)	**27.**	(a)	**28.**	(c)	**29.**	(a)	**30.**	(d)
31.	(d)	**32.**	(c)	**33.**	(c)	**34.**	(b)	**35.**	(a)	**36.**	(b)	**37.**	(c)	**38.**	(a)	**39.**	(d)	**40.**	(d)
41.	(b)	**42.**	(a)	**43.**	(a)	**44.**	(c)	**45.**	(a)	**46.**	(a)	**47.**	(d)	**48.**	(b)	**49.**	(d)	**50.**	(d)
51.	(d)	**52.**	(a)	**53.**	(b)	**54.**	(a)	**55.**	(a)	**56.**	(a)	**57.**	(d)	**58.**	(b)	**59.**	(d)	**60.**	(a)

Explanations

1. (b) Vertices of triangle are A (– 4,2), B (0, –1) and C (3, 3)

Then, $AB = \sqrt{(0+4)^2 + (-1-2)^2} = 5$

$BC = \sqrt{(3-0)^2 + (3+1)^2} = 5$

$CA = \sqrt{(-4-3)^2 + (2-3)^2} = 5\sqrt{2}$

Now, perimeter of ΔABC

$= AB + BC + CA$

$= 10 + 5\sqrt{2}$

2. (c) Let the points $\left(1, \frac{-7}{2}\right)$ divides the join of the points $(-2, -4)$ and $\left(2, \frac{-10}{3}\right)$ in $m : 1$.

Then, $1 = \frac{m \times 2 + 1 \times -2}{m+1}$

$\Rightarrow m + 1 = 2m - 2 \Rightarrow m = 3$

Hence, required ratio $m : 1 = 3 : 1$

3. (a) Given lines are $y = 3x$, $y = 6x$ and $y = 9$

Intersection points of these lines are (0, 0), (3, 9) and $\left(\frac{3}{2}, 9\right)$.

So, area of triangle formed by these lines

$$= \frac{1}{2}\begin{vmatrix} 0 & 0 & 1 \\ 3 & 9 & 1 \\ 3/2 & 9 & 1 \end{vmatrix} = \frac{1}{2}\left\{27 - \frac{27}{2}\right\} = \frac{27}{4} \text{ sq. units.}$$

4. (b) Given, lines $2x - 3y + k = 0$, $3x - 4y - 13 = 0$ and $8x - 11y - 33 = 0$ are concurrent.

So, $\begin{vmatrix} 2 & -3 & k \\ 3 & -4 & -13 \\ 8 & -11 & -33 \end{vmatrix} = 0$

$\Rightarrow 2(132 - 143) + 3(-99 + 104) + k(-33 + 32) = 0$

$\Rightarrow -22 + 15 - k = 0 \Rightarrow k = -7$

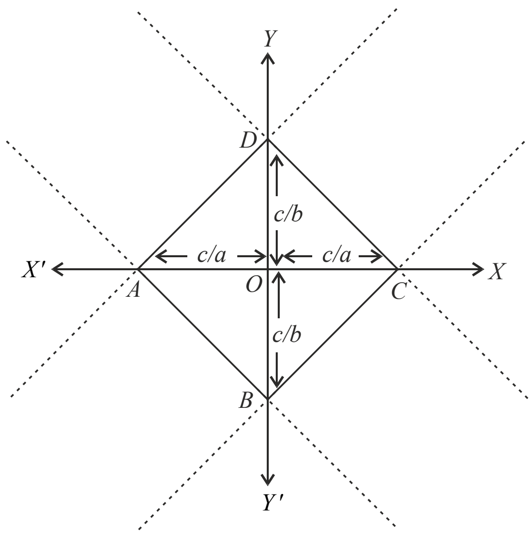
5. (b) Given line $a|x| + b|y| + c = 0$ represents four lines which are as follows :

$$ax + by + c = 0 \Rightarrow \frac{x}{(-c/a)} + \frac{y}{(-c/b)} = 1 \quad \text{...(i)}$$

$$ax + by - c = 0 \Rightarrow \frac{x}{(c/a)} + \frac{y}{(c/b)} = 1 \quad \text{...(ii)}$$

$$ax - by + c = 0 \Rightarrow \frac{x}{(-c/a)} + \frac{y}{(c/b)} = 1 \quad \text{...(iii)}$$

$$ax - by - c = 0 \Rightarrow \frac{x}{(c/a)} + \frac{y}{(-c/b)} = 1 \quad \text{...(iv)}$$



These above four lines form a quadrilateral as shown in the figure. So, area of quadrilateral $ABCD$

$= 4 \times$ Area of ΔOCD

$$= 4 \times \left\{\frac{1}{2} \times \left|\frac{c}{a}\right| \times \left|\frac{c}{b}\right|\right\} = \frac{2c^2}{|ab|}$$

6. (a) Image (h, k) of the point $(-1, 3)$ on the line $x - y = 0$ is given by

$$\frac{h - x_1}{a} = \frac{k - y_1}{b} = -\frac{2(ax_1 + by_1 + c)}{a^2 + b^2}$$

Here, $x_1 = -1$, $y_1 = 3$, $a = 1$, $b = -1$, $c = 0$

So, $\frac{h+1}{1} = \frac{k-3}{-1} = -2\left(\frac{1 \times -1 - 1 \times 3 + 0}{1+1}\right)$

$\Rightarrow h + 1 = 3 - k = 4 \Rightarrow h = 3$ and $k = -1$

Hence, image of $(-1, 3)$ by the lines $x - y = 0$ is $(3, -1)$.

7. (b) Mid point of $A(-1, 3)$ and $C(5,1)$ is

$M\left(\frac{-1+5}{2}, \frac{3+1}{2}\right)$ *i.e.*, $M(2,2)$

Length of $BM = \sqrt{(2-1)^2 + (2+1)^2} = \sqrt{1+9} = \sqrt{10}$

8. (c) Let, $L = 4x + 2y - 1$

The, equation of line $4x + 2y - 1 = 0$

For point (1, 2), $L = 4 + 4 - 1 = 7 > 0$

and for point $(-2, 1)$, $L = -8 + 2 - 1 = -7 < 0$

Hence, the two points (1, 2) and $(-2, 1)$ are on the opposite sides of the line $L = 0$

9. (a) Let any point on the line $x + y = 4$ is $(a, 4 - a)$.

Distance of point $(a, 4 - a)$ from line $4x + 3y - 10 = 0$ is 1.

$$\Rightarrow \left|\frac{4a + 3(4-a) - 10}{\sqrt{4^2 + 3^2}}\right| = 1$$

$\Rightarrow a + 2 = 5$

$\Rightarrow a + 2 = 5$ or $a + 2 = -5$

$\Rightarrow a = 3$ or $a = -7$

Hence, points are (3, 1) and $(-7, 11)$.

10. (b) Let point $P(x, y)$ divides the line joining $(-1, 1)$ and (5, 7) in ratio of $\lambda : 1$.

Then, $x = \frac{5\lambda - 1}{\lambda + 1}$ and $y = \frac{7\lambda + 1}{\lambda + 1}$

Now, this point $P = \left(\frac{5\lambda - 1}{\lambda + 1}, \frac{7\lambda + 1}{\lambda + 1}\right)$ lies on line $x + y = 4$

$\Rightarrow (5\lambda - 1) + (7\lambda + 1) = 4(\lambda + 1)$

$\Rightarrow \lambda = \frac{1}{2}$

11. (d) Equation of a line perpendicular to the line $3x + y - 3 = 0$ is given by $x - 3y + k = 0$

Now, this line passes through (2,2)

$\Rightarrow 2 - 3(2) + k = 0 \Rightarrow k = 4$

Hence, required line is $x - 3y + 4 = 0$

It can be written as $\frac{x}{(-4)}+\frac{y}{(4/3)}=1$

So, y-intercept = 4 / 3

12. (c) Let the line cuts off the equal intercepts of length a on both axes.

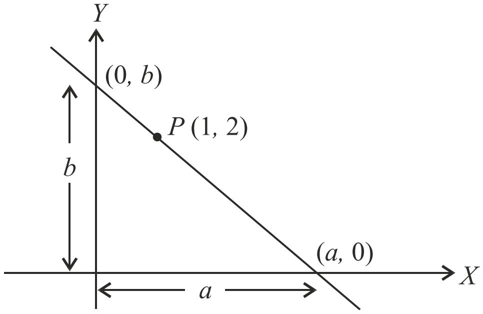
Then, equation of line is $\frac{x}{a}+\frac{y}{b}=1$

This line passes through (1, – 2)

$\Rightarrow a = x + y = 1 - 2 = -1$

Hence, the required line is $x + y + 1 = 0$

13. (c) Let the equation of the line is $\frac{x}{a}+\frac{y}{b}=1$



Point P(1, 2) divides this line in the ratio 1 : 2.

So, coordinates of P are

$$\left(\frac{1\times a\times 2\times 0}{1+2},\frac{1\times 0\times 2\times b}{3}\right) i.e.,\left(\frac{a}{3},\frac{2b}{3}\right)$$

Therefore, $\frac{a}{3}=1$ and $\frac{2b}{3}=2$

$a = 3$ and $b = 3$

So, equation of line is $\frac{x}{3}+\frac{y}{3}=1$

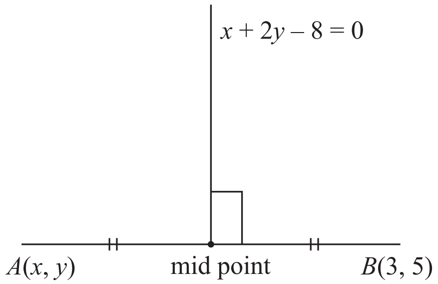
$i.e., x + y - 3 = 0$

14. (b) $\because$ Lines $mx - y = 0$, $x + 2y - 1 = 0$ and $2x - y + 3 = 0$ meet at same point, Therefore, these lines are concurrent.

$$\text{So, }\begin{vmatrix} m & -1 & 0 \\ 1 & 2 & -1 \\ 2 & -1 & 3 \end{vmatrix} = 0$$

$\Rightarrow m(6 - 1) + 1(3 + 2) = 0 \Rightarrow m = -1$

15. (d) Line perpendicular of $x + 2y - 8 = 0$ is given by $2x - y + k = 0$. It passes through (3, 5).



$\Rightarrow k = -1$

So, two perpendicular lines are

$x + 2y - 8 = 0$ and $2x - y - 1 = 0$

Their intersection point is (2, 3).

Now point (2, 3) is the mid point of $A(x, y)$ and $B(3, 5)$

So, $2=\frac{x+3}{2}$ and $3=\frac{y+5}{2}$

$\Rightarrow x = 1$ and $y = 1$

Hence, coordinates of A are (1, 1).

16. (d) Let foot of the perpendicular from the point (0, 5) on the line $3x - 4y - 5 = 0$ is (h, k).

Then, $\frac{h-0}{3}=\frac{k-5}{-4}=\frac{(3\times 0-4\times 5-5)}{3^2+4^2}$

$\Rightarrow \frac{h}{3}=\frac{k-5}{-4}=1$

$\Rightarrow h = 3$ and $k = 1$

Hence, foot of perpendicular = (3,1)

17. (d) Let the line $3x + 4y - 7 = 0$ divides the join of points (1, 2) and (– 2, 1) in ratio $m : 1$ at point P. Then, coordinates of P are

$$= P\left(\frac{m\times -2+1\times 1}{m+1},\frac{m\times 1+1\times 2}{m+1}\right)$$

$$= P\left(\frac{1-2m}{m+1},\frac{2+m}{m+1}\right)$$

This point lies on the given line

So, $3\left(\frac{1-2m}{m+1}\right)+4\left(\frac{2+m}{m+1}\right)-7=0$

$\Rightarrow 3 - 6m + 8 + 4m - 7m - 7 = 0$

$\Rightarrow m = \frac{4}{9}$

Hence, required ratio = $m : 1 = 4 : 9$

18. (c) Given, $(t_1, 2at_1 + at_1^3)$, $(t_2, 2at_2 + at_1^3)$ and $(t_3, 2at_3 + at_3^3)$ are collinear

$$\text{So, }\begin{vmatrix} t_1 & 2at_1+at_1^3 & 1 \\ t_2 & 2at_2+at_2^3 & 1 \\ t_3 & 2at_3+at_2^3 & 1 \end{vmatrix} = 0$$

$$\Rightarrow 2a\begin{vmatrix} t_1 & t_1 & 1 \\ t_2 & t_2 & 1 \\ t_3 & t_3 & 1 \end{vmatrix} + a\begin{vmatrix} t_1 & t_1^3 & 1 \\ t_2 & t_2^3 & 1 \\ t_3 & t_3^3 & 1 \end{vmatrix} = 0$$

$\Rightarrow (t_1 - t_2)(t_2 - t_3)(t_3 - t_1)(t_1 + t_2 + t_3) = 0$

$\Rightarrow t_1 + t_2 + t_3 = 0$ $\{\because t_1, t_2, t_3$ are distinct$\}$

19. (a) Given lines $x + y - 4 = 0$, $3x + y - 4 = 0$ and $x + 3y - 4 = 0$ form a triangle ΔABC

So, vertices of this triangle = Interception points of these lines

$\Rightarrow$ Vertices of $\Delta ABC = A(0, 4), B(4, 0), C(1,1)$

Here, $AC = BC$

i.e., two sides of ΔABC are equal.
Hence, ΔABC is an isosceles triangle.

20. (b) Let the coordinates of A are (a, b).
This point $A(a, b)$ lies on line $2x + y - 2 = 0$
$\Rightarrow 2a + b - 2 = 0$ ...(i)
Area of $\Delta ABC = 8$ sq. units.
$$\Rightarrow \frac{1}{2}\begin{vmatrix} a & b & 1 \\ 1 & -2 & 1 \\ 2 & 3 & 1 \end{vmatrix} = 8$$
$\Rightarrow 5a - b + 9 = 0$ ...(ii)
On solving eqs. (i) and (ii), we get
$a = -1$ and $b = 4$
Hence, vertex A is $(-1, 4)$.

21. (d) Given, orthocentre and centroid of a triangle are $(-3, 5)$ and $(3, 3)$ respectively.
Let circumcentre is (x, y).
$\because$ Centroid divides the line joining the circumcentre and orthocentre in the ratio 1 : 2.
So, $3 = \dfrac{1x - 3 + 2 \times x}{1 + 2}$ and $3 = \dfrac{1 \times 5 \times 2 \times y}{1 + 2}$
$\Rightarrow x = 6$ and $y = 2$
Hence, circumcentre = (6, 2)

22. (b) Angle between the two lines represented by $kx^2 + 4xy + 5y^2 = 0$ is π
So, $\tan\theta = \dfrac{2\sqrt{h^2 - ab}}{a + b}$
$\Rightarrow \tan\pi = \dfrac{2\sqrt{4 - 5k}}{5 + k}$
$\Rightarrow k = \dfrac{4}{5}$

23. (b) Angle between the pair of lines represented by $2x^2 - 5xy + 3y^2 = 0$ is
$$\tan\theta = \frac{2\sqrt{\frac{25}{4} - (2)(3)}}{2 + 3} \qquad \left\{ \begin{array}{l} a = 2 \\ b = 3 \\ h = -\frac{5}{2} \end{array} \right.$$
$$= \frac{2 \times \frac{1}{2}}{5} \Rightarrow \theta = \tan^{-1}\left(\frac{1}{5}\right)$$

24. (b) Equation $2x^2 - 3xy - py^2 + x + qy - 1 = 0$ represents two mutually perpendicular lines. So, $a + b = 0$
$\Rightarrow 2 - p = 0 \Rightarrow p = 2$
Given equation represents the line
So, $\Delta = abc + 2fgh - af^2 - bg^2 - ch^2 = 0$
$$\Rightarrow 4 - \frac{3q}{4} - \frac{q^2}{2} + \frac{1}{2} + \frac{9}{4} = 0$$
$\Rightarrow 2q^2 + 3q - 27 = 0$
$\Rightarrow q = 3$ or $q = \dfrac{9}{2}$
Hence, $p = 2$ and $q = 3$

25. (b) Given equation of pair of straight lines is $8x^2 + 8xy + 2y^2 + 26x + 13y + 15 = 0$
$\Rightarrow (2x + y + 5)(4x + 2y + 3) = 0$
So, two parallel lines are $2x + y + 5 = 0$ and
$4x + 2y + 3 = 0$ or $2x + y + \dfrac{3}{2} = 0$.
Distance between these two parallel lines
$$= \left|\frac{5 - 3/2}{\sqrt{4 + 1}}\right| = \frac{7}{2\sqrt{5}}$$

26. (c) $\because \lambda x^2 - 10xy + 12y^2 + 5x - 16y - 3 = 0$ represents a pair of straight lines.
So, $abc + 2fgh - af^2 - bg^2 - ch^2 = 0$
$$\Rightarrow (\lambda)(12)(-3) + 2(-8)\left(\frac{5}{2}\right)(-5) - (\lambda)(-8)^2$$
$$-(12)\left(\frac{5}{2}\right)^2 - (-3)(-5)^2 = 0$$
$\Rightarrow \lambda = 2$

27. (a) Image (h, k) of the point (1, 3) in the lin $x + y - 6 = 0$ is given by
$$\frac{h - 1}{1} = \frac{k - 3}{1} = -\frac{2(1 \times 1 + 1 \times 3 - 6)}{1 + 1}$$
$\Rightarrow h - 1 = k - 3 = 2$
$\Rightarrow h = 3$ and $k = 5$
Hence, image = (3, 5)

28. (c) $\because$ Lines $x + ay + a = 0$, $bx + y + b = 0$ and $cx + cy + 1 = 0$ are concurrent.
$$\Rightarrow \begin{vmatrix} 1 & a & a \\ b & 1 & b \\ c & c & 1 \end{vmatrix} = 0 \Rightarrow \begin{vmatrix} \frac{1}{a} & 1 & 1 \\ 1 & \frac{1}{b} & 1 \\ 1 & 1 & \frac{1}{c} \end{vmatrix} = 0$$
$\{$Apply $R_2 \to R_2 - R_1, R_3 \to R_3 - R_1\}$
$$\Rightarrow \begin{vmatrix} \frac{1}{a} & 1 & 1 \\ 1 - \frac{1}{a} & \frac{1}{b} - 1 & 0 \\ 1 - \frac{1}{a} & 0 & \frac{1}{c} - 1 \end{vmatrix} = 0$$
$$\Rightarrow \frac{1}{a}\left(\frac{1}{b} - 1\right)\left(\frac{1}{c} - 1\right) + \left(\frac{1}{a} - 1\right)\left(\frac{1}{b} - 1\right)$$
$$+ \left(\frac{1}{c} - 1\right)\left(1 - \frac{1}{a}\right) = 0$$
$\Rightarrow (1 - b)(1 - c) + c(1 - a)(1 - b) + b(1 - c)(1 - a) = 0$

$$\Rightarrow \frac{1}{1-a}+\frac{c}{1-c}+\frac{b}{1-b}=0$$

$$\Rightarrow 1-\frac{a}{a-1}+\frac{b}{1-b}+\frac{c}{1-c}=0$$

$$\Rightarrow \frac{a}{a-1}+\frac{b}{b-1}+\frac{c}{c-1}=1$$

29. (a) Let point P divides the line segment joining the points (1, 2) and (k, 1) in ratio 4 : 9.

Then, coordinates of P are $\left(\frac{4k+9}{13},\frac{4+13}{13}\right)$

i.e., $P\left(\frac{4k+9}{13},\frac{22}{13}\right)$

This point P lies on the line $3x + 4y - 7 = 0$

So, $3\left(\frac{4k+9}{13}\right)+4\left(\frac{22}{13}\right)-7=0$

$\Rightarrow 12k + 27 + 88 - 91 = 0$

$\Rightarrow k = -2$

30. (d) Let (h, k) be the foot of the perpendicular drawn from the points (2, 4) on the line $x + y - 1 = 0$

Then, $\frac{h-2}{1}=\frac{k-4}{1}=-\frac{(2\times1+4\times1-1)}{1+1}$

$\Rightarrow h-2=k-4=\frac{-5}{2}$

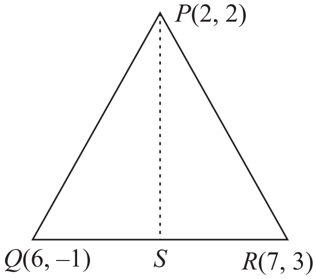
$\Rightarrow h=-\frac{1}{2}$ and $k=\frac{3}{2}$

Hence, foot of the perpendicular are $\left(-\frac{1}{2},\frac{3}{2}\right)$.

31. (d) $\because$ S is the median of ΔPQR

$\Rightarrow$ S is the mid point of QR

So, coordinates of S are



$\left(\frac{6+7}{2},\frac{-1+3}{2}\right)=\left(\frac{13}{2},1\right)$

Slope of $PS=\frac{1-2}{\frac{13}{2}-2}=\frac{-1}{\frac{9}{2}}=\frac{-2}{9}$

So, equation of a line passing through (1, –1) and parallel to PS is

$$y+1=-\frac{2}{9}(x-1)$$

$\Rightarrow 9y + 9 = -2x + 2$

$\Rightarrow 2x + 9y + 7 = 0$

32. (c) Let three are three points A, B and C having coordinates $A(5, 2)$, $B(6, -15)$ and $C(0, 0)$

Here, slope of $AC=\frac{2}{5}$

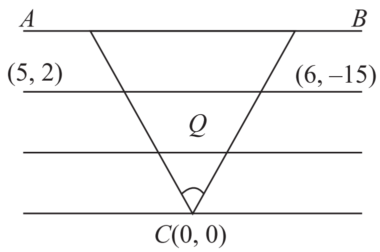
and slope of $BC=\frac{-15}{6}=-\frac{5}{2}$

Hence, $AC \perp BC$

Therefore, angle subtended by AB at C is $\pi/2$.

33. (c) $\because$ D is the middle point of BC.

$\therefore$ Coordinates of D are (1, 1).



Slope of line $AD=\frac{1-5}{1+1}=-2$

$\Rightarrow$ Slope of perpendicular from B on $AD=\frac{1}{2}$

Now, equation of line (BM) passing from (0,0) and having slope $\frac{1}{2}$ is $y=\frac{1}{2}x$ or $x-2y=0$

34. (b) Given two lines $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ are perpendicular to each other.

So, $a_1a_2 + b_1b_2 = 0$

35. (a) Slopes of the line $\frac{x}{a}+\frac{y}{b}=1$ and $\frac{x}{a}-\frac{y}{b}=1$ are

$m_1=-\frac{b}{a}$ and $m_2=\frac{b}{a}$ respectively.

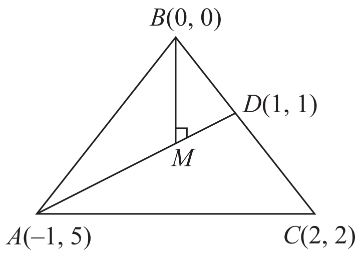
Angle between these lines

$$\tan\theta=\left|\frac{m_1-m_2}{1+m_1m_2}\right|=\left|\frac{-2b/a}{1-\frac{b^2}{a^2}}\right|$$

$$\Rightarrow \theta=\tan^{-1}\left(\frac{2b/a}{1-\frac{b^2}{a^2}}\right)=2\tan^{-1}\left(\frac{b}{a}\right)$$

36. (b) Let $A(x_1y_1)$, $B(x_2\,y_2)$ and $C(x_3y_3)$ be the vertices of ΔABC.

Then, mid point of AB, BC and CA are (– 5, 7), (1, 3) and (5, 7) respectively.



So, $\frac{x_1+x_2}{5} = -5$ & $\frac{y_1+y_2}{2} = 7$

$\frac{x_2+x_3}{2} = 1$ & $\frac{y_2+y_3}{2} = 3$

$\frac{x_3+x_1}{2} = 5$ & $\frac{y_3+y_1}{2} = 7$

$\Rightarrow x_1+x_2+x_3 = 1$ & $y_1+y_2+y_3 = 17$

$\Rightarrow x_1 = -1, x_2 = -9, x_3 = 11, y_1 = 11, y_2 = 3, y_3 = 3$

Hence, equation of line AB is

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

$$\Rightarrow y - 11 = \frac{3-11}{-9+1}(x+1) \Rightarrow x - y + 12 = 0$$

37. (c) Vertices of a parallelogram are $A(1, 2), B(4, y), C(x, 6)$ and $D(3, 5)$.

$\because$ Diagonals of a parallelogram bisect each other.

So, mid point of AC = mid point of BD

$$\Rightarrow \left(\frac{1+x}{2}, \frac{2+6}{2}\right) = \left(\frac{4+3}{2}, \frac{y+5}{2}\right)$$

$\Rightarrow x = 6$ and $y = 3$

Now, $AC^2 - BD^2$

$$= \left\{\sqrt{(6-1)^2 + (6-2)^2}\right\}^2 - \left\{\sqrt{(3-4)^2 + (5-3)^2}\right\}^2$$

$= 41 - 5 = 36$

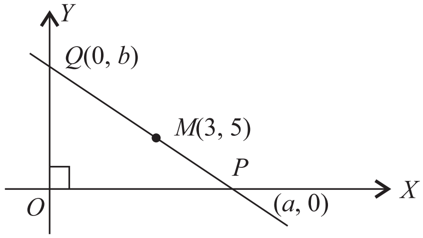
38. (a) From the solution of Q. 37.

Intersection point of diagonals

$$= \left(\frac{1+x}{2}, \frac{2+6}{2}\right) = \left(\frac{1+6}{2}, 4\right) = \left(\frac{7}{2}, 4\right)$$

39. (d) Let the coordinates of points P and Q are $(a, 0)$ and $(0, b)$ respectively.

Mid point of $PQ = M(3, 5)$



$$\Rightarrow \frac{0+a}{2} = 3 \text{ and } \frac{b+0}{2} = 5$$

$\Rightarrow a = 6$ and $b = 10$

So, coordinates of ΔPOQ are $P(6, 0)$, $O(0, 0)$ and $Q(0, 10)$.

$$\text{Then, area} = \frac{1}{2}\begin{vmatrix} 6 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 10 & 1 \end{vmatrix} = 30 \text{ sq. units.}$$

40. (d) Equation of a line passing through the intersection point of lines $x + y + 1 = 0$ and $3x + 2y + 1 = 0$ is given by

$(x + y + 1) + \lambda(3x + 2y + 1) = 0$

$\Rightarrow (1 + 3\lambda)x + (1 + 2\lambda)y + 1 + \lambda = 0$ ...(i)

This line is parallel to X axis. So, slope of line = 0

$\Rightarrow 1 + 3\lambda = 0 \Rightarrow \lambda = -1/3$

Put, in eq (i),

$$(1-1)x + \left(1 - \frac{2}{3}\right)y + 1 - \frac{2}{3} = 0$$

$\Rightarrow y + 2 = 0$

41. (b) Equation of line passing through intersection point of lines $x + y + 1 = 0$ and $3x + 2y + 1 = 0$ is

$(1 + 3\lambda)x + (1 + 2\lambda)y + 1 + \lambda = 0$ {From Q. 40}

Now, this line is parallel to Y-axis

So, slope of line $= \infty = \frac{1}{0}$

$$\Rightarrow \frac{-(1+3\lambda)}{1+2\lambda} = \frac{1}{0} \Rightarrow \lambda = -\frac{1}{2}$$

So, required line is

$$\left(1 - \frac{3}{2}\right)x + 0 + \left(1 - \frac{1}{2}\right) = 0 \quad \text{\{From (i)\}}$$

$\Rightarrow x - 1 = 0$

42. (a) $\because (a, 2b)$ is the mid point of the line joining $(10, -6)$ to $(k, 4)$

So, $a = \frac{10+k}{2}$ and $2b = -\frac{6+4}{2}$

$\Rightarrow 2b = -1$

$\because a - 2b = 7$

$$\Rightarrow \frac{10+k}{2} + 1 = 7$$

$\Rightarrow 10 + k = 12 \Rightarrow k = 2$

43. (a) Given lines are $y = \sqrt{3}x + 5$ and $y = \frac{x}{\sqrt{3}} - \frac{6}{\sqrt{3}}$

So, slopes of given lines are $m_1 = \sqrt{3}$ and $m_2 = \frac{1}{\sqrt{3}}$

Angle between the lines, $\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$

$$= \left| \frac{\sqrt{3} - 1/\sqrt{3}}{1 + \sqrt{3} \times \frac{1}{\sqrt{3}}} \right| = \left| \frac{2}{\sqrt{3} \times 2} \right| = \frac{1}{\sqrt{3}}$$

Hence, $\theta = 30°$

44. (c) Two vertices of ΔABC are given $A(0, 0)$ and $B(3, \sqrt{3})$.

Let the third vertex be $C(x, y)$.

Then, $AB^2 = BC^2 = CA^2$

$\Rightarrow 9 + 3 = (3 - x)^2 + (\sqrt{3} - y)^2 = x^2 + y^2$

$\Rightarrow 12 = 12 + x^2 + y^2 - 6x - 2\sqrt{3}y = x^2 + y^2$

$\Rightarrow x^2 + y^2 = 12$ …(i)

and $6x + 2\sqrt{3}y = 12$ …(ii)

On solving both the equations

$x = 0, 3$ and $y = 2\sqrt{3}, -\sqrt{3}$

Hence, third vertex

$= C(0, 2\sqrt{3})$ or $C(3, -\sqrt{3})$

45. (a) Let the ends of the line segment are $A(1, 1)$ and $B(2, 3)$

Slope of $AB = \frac{3-1}{2-1} = 2$

$\Rightarrow$ Slope of $\perp$ bisector of $AB = -\frac{1}{2}$

Mid point of $AB = \left(\frac{1+2}{2}, \frac{1+3}{2}\right) = \left(\frac{3}{2}, 2\right)$

Now, equation of right bisector is

$y - y_1 = m(x - x_1)$

$\Rightarrow y - 2 = -\frac{1}{2}\left(x - \frac{3}{2}\right)$

$\Rightarrow 2y - 4 = -x + \frac{3}{2}$

$\Rightarrow 2x + 4y - 11 = 0$

46. (a) Given lines are $|x + y| = 2$

i.e., $x + y = 2$ and $x + y = -2$

Now, the distance between these two parallel lines

$= \left| \frac{2+2}{\sqrt{1+1}} \right| = 2\sqrt{2}$

$\because (a, a)$ lies between these two parallel lines.

So, distance of (a, a) from $(0, 0) < 2\sqrt{2}$

$\Rightarrow \sqrt{a^2 + a^2} < 2\sqrt{2} \Rightarrow |a| < 2$

47. (d) Equation of the line passing through the intersection point of the lines $x + 2y - 5 = 0$ and $3x + 7y - 17 = 0$ is $(x + 2y - 5) + \lambda(3x + 7y - 17) = 0$

i.e., $(1 + 3\lambda)x + (2 + 7\lambda)y + (-5 - 17\lambda) = 0$ ... (i)

This line is perpendicular to $3x + 4y = 10$

$\Rightarrow -\frac{(1+3\lambda)}{(2+7\lambda)} = \frac{4}{3} \Rightarrow \lambda = \frac{-11}{37}$

Hence, required line is

$4x - 3y + 2 = 0$ {From eq. (i)}

48. (b) Given, distance of (a, b) from $8x + 6y + 1 = 0$ is 1

$\Rightarrow \left| \frac{8a + 6b + 1}{\sqrt{64 + 36}} \right| = 1$

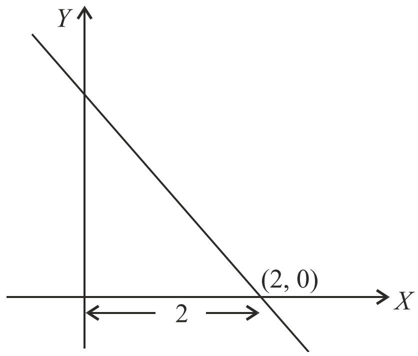
$\Rightarrow 8a + 6b + 1 = \pm 10$

$8a + 6b + 1 = 10$ or $8a + 6b + 1 = -10$

$\Rightarrow 8a + 6b - 9 = 0$ or $8a + 6b + 11 = 0$

Hence, II and III conditions are correct.

49. (d) Interception form of a line is $\frac{x}{a} + \frac{y}{b} = 1$



Intercept on X axis, $a = 2$

$\Rightarrow \frac{x}{2} + \frac{y}{b} = 1$ ...(i)

It passes through point $(-3, 5)$

So, $-\frac{3}{2} + \frac{5}{b} = 1 \Rightarrow b = 2$

Hence, equation of a line is $x + y = 2$ {From eqn. (i)}

Let foot of the perpendicular from point $(3, 3)$ on the line $x + y - 2 = 0$ is (h, k)

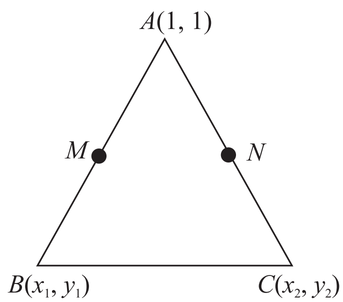
Then, $\frac{h-3}{1} = \frac{k-3}{1} = -\frac{(1 \times 3 + 1 \times 3 - 2)}{1+1}$

$\Rightarrow h = 1$ and $k = 1$

Hence, foot of perpendicular $= (1, 1)$

50. (d) Let the vertices of ΔABC be $A(1, 1)$, $B(x_1, y_1)$ and $C(x_2, y_2)$

Given mid points of AB and AC are $M(-1, 2)$ and $N(3, 2)$ respectively.



$\Rightarrow \dfrac{x_1+1}{2} = -1, \dfrac{y_1+1}{1} = 2$

$\Rightarrow x_1 = -3$ and $y_1 = 3$

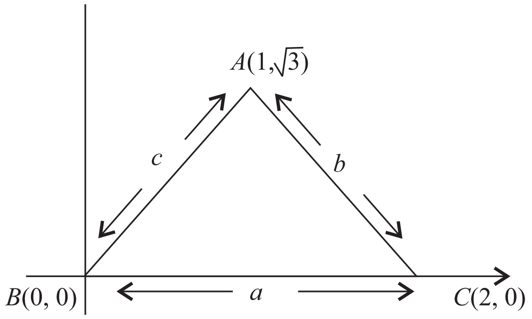
and $\dfrac{x_2+1}{2} = 3$ and $\dfrac{y_2+1}{2} = 2$

$\Rightarrow x_2 = 5$ and $y_2 = 3$

Other two vertices are $B(-3, 3)$ and $C(5, 3)$

Hence, centroid $= \left(\dfrac{1+(-3)+5}{3}, \dfrac{1+3+3}{3}\right) = \left(1, \dfrac{7}{3}\right)$

51. (d) Vertices of ΔABC are $A(1, \sqrt{3})$, $B(0, 0)$ and $C(2, 0)$.
So, $a = 2$, $b = 2$ and $c = 2$
$\Rightarrow$ ΔABC is an equilateral triangle.



Hence, incentre of triangle = centroid of triangle

$= \left(\dfrac{1+0+2}{3}, \dfrac{\sqrt{3}+0+0}{3}\right) = \left(1, \dfrac{1}{\sqrt{3}}\right)$

52. (a) Given three vertices of a parallelogram $ABCD$ are $A(-2, -1)$, $B(1, 0)$, $C(4, 3)$ and $D(x, y)$.
Then, mid point of AC = mid point of BD

$\left(\dfrac{-2+4}{2}, \dfrac{-1+3}{2}\right) = \left(\dfrac{1+x}{2}, \dfrac{0+y}{2}\right)$

$\Rightarrow (1,1) = \left(\dfrac{1+x}{2}, \dfrac{y}{2}\right)$

$\Rightarrow 1 + x = 2$ and $y = 2$

$\Rightarrow x = 1$ and $y = 2$

Hence, fourth vertex = (1, 2)

53. (b) Let the point $C\left(\dfrac{-2}{7}, \dfrac{-20}{7}\right)$ divides the join of the points $A(-2, -2)$ and $B(2, -4)$ in the ratio $m : 1$.

Then, $-\dfrac{2}{7} = \dfrac{m \times 2 + 1 \times -2}{m+1} \Rightarrow -2m - 2 = 14m - 14$

$\Rightarrow m = 3/4$

Hence, required ratio $= m : 1 = 3 : 4$

54. (a) Equation of a straight line parallel to $2x + 3y + 1 = 0$ is $2x + 3y + k = 0$ …(i)
This line passes through $(-1, 2)$
$\Rightarrow 2(-1) + 3(2) + k = 0$ {From eq. (i)}
$\Rightarrow k = -4$
Hence, eq. of required line is
$2x + 3y - 4 = 0$ {From eq. (i)}

55. (a) Slope of line $\sqrt{2}x + \sqrt{3}y = 1$ is $m_1 = -\dfrac{\sqrt{2}}{\sqrt{3}}$

Slope of line $\sqrt{3}x + \sqrt{2}y = 2$ is $m_2 = -\dfrac{\sqrt{3}}{\sqrt{2}}$

Acute angle between the lines $\tan\theta = \left|\dfrac{m_1 - m_2}{1 + m_1 m_2}\right|$

$= \left|\dfrac{-\sqrt{\dfrac{2}{3}} + \sqrt{\dfrac{3}{2}}}{1 + \sqrt{\dfrac{2}{3}}\sqrt{\dfrac{3}{2}}}\right| = \dfrac{1}{2\sqrt{6}}$

$\Rightarrow \theta = \tan^{-1}\left(\dfrac{1}{2\sqrt{6}}\right)$

56. (a) Centroid of a triangle having vertices $(7, x)$, $(y, -6)$ and $(9, 10)$ is $(6, 3)$

So, $\dfrac{7+y+9}{3} = 6$ and $\dfrac{x-6+10}{3} = 3$

$\Rightarrow y = 2$ and $x = 5$

57. (d) Let the four points be $O(0, 0)$, $A(-a, -b)$, $C(a, b)$ and $B(a^2, ab)$
Then, slope of $OA = b/a$
Slope of $OB = b/a$
Slope of $OC = b/a$
$\because$ Slopes of OA, OB and OC are same.
Hence, O, A, B and C are collinear.

58. (b) Slope of line $x + y - 3 = 0$ is $m_1 = -1$
Slope of line $x - y + 3 = 0$ is $m_2 = 1$
$\because m_1 m_2 = -1$
$\Rightarrow$ Angle between the lines = 90°
So, $\alpha = 90°$

Slope of line $x - \sqrt{3}y + 2\sqrt{3} = 0$ is $m_3 = \dfrac{1}{\sqrt{3}}$

Slope of line $3x - y + 1 = 0$ is $m_4 = \sqrt{3}$

Angle between these two lines $\tan\beta$

$= \left|\dfrac{\dfrac{1}{\sqrt{3}} - \sqrt{3}}{1 + \dfrac{1}{\sqrt{3}} \cdot \sqrt{3}}\right| = \dfrac{1}{\sqrt{3}}$

$\Rightarrow \beta = 30°$

Clearly, $\alpha > \beta$

59. (d) Slope of line $4x + y = 4$ is $m = -4$
Equation of line parallel to $4x + y = 4$ and passing from $(1, 3)$ is $y - 3 = -4(x - 1)$
$\Rightarrow 4x + y = 7$ …(i)
Now intersection point of lines $4x + y = 7$ and $2x + 3y = 6$ is $\left(\dfrac{3}{2}, 1\right)$

Hence, distance between the points (1, 3) and $\left(\frac{3}{2}, 1\right)$ is

$$= \sqrt{\left(\frac{3}{2} - 1\right)^2 + (1 - 3)^2} = \frac{\sqrt{17}}{2}$$

60. (a) Given, angle made by line with positive X-axis = 120°

$\Rightarrow$ $m = \tan 120° = \tan (180° - 60°)$

$= -\tan 60° = -\sqrt{3}$

Slope form of line is $y = mx + c$

where c = intercept on Y-axis

Here, $c = -5$

So, required line is $y = -\sqrt{3}x - 5$

or $\sqrt{3}x + y + 5 = 0$