CHAPTER 2

Relation

Exercise

1. A and B are two sets having 3 and 5 elements respectively and having 2 elements in common. Then the number of elements in A × B is
(a) 6 (b) 36
(c) 15 (d) None of these

2. If A = {2, 4} and B = {3, 4, 5}, then $(A \cap B) \times (A \cup B)$ is
(a) {(2, 2), (3, 4), (4, 2), (5, 4)}
(b) {(2, 3), (4, 3), (4, 5)}
(c) {(2, 4), (3, 4), (4, 4), (4, 5)}
(d) {(4, 2), (4, 3), (4, 4), (4, 5)}

3. If A and B are two sets such that $n(A \cap \overline{B}) = 9$, $n(\overline{A} \cap B) = 10$ and $n(A \cap B) = 24$, then $n(A \times B)$ is equal to
(a) 105 (b) 210
(c) 70 (d) None of these

4. If A = {1, 2, 4}, B = {2, 4, 5}, C = {2, 5}, then (A – B) × (B – C) is
(a) {(1, 2), (1, 5), (2, 5)} (b) {(1, 4)}
(c) {(1, 3)} (d) None of these

5. Let A and B are two sets given in such a way that A × B contains 6 elements. If 3 elements of A × B be (1, 3), (2, 5) and (3, 3), then its remaining elements are
(a) (1, 1), (2, 3), (3, 5)
(b) (1, 2), (2, 3), (3, 5)
(c) (1, 5), (2, 2), (3, 5)
(d) (1, 5), (2, 3), (3, 5)

6. A relation ϕ from C to R is defined by $x \,\phi\, y \Leftrightarrow |x| = y$. Which one of the following is correct ?
(a) $(2 + 3i)\,\phi\, 13$ (b) $3\,\phi(-3)$
(c) $(1 + i)\,\phi\, 2$ (d) $i\,\phi\, 1$

7. If A and B have n elements in common, then the number of elements common to (A × B) and (B × A) is
(a) n (b) $n!$
(c) $n/2$ (d) n^2

8. If R is a relation from a finite set A having m elements to a finite set B having n elements, then the number of relations from A to B is
(a) 2^{mn} (b) $2^{mn} - 1$
(c) $2mn$ (d) m^n

9. Let R be a relation on a set A such that $R = R^{-1}$, then R is
(a) reflexive (b) symmetric
(c) transitive (d) None of these

10. The void relation on a set A is
(a) reflexive
(b) symmetric and transitive
(c) transitive
(d) reflexive and transitive

11. If R is a relation on a finite set having n elements, then the number of relations on A is
(a) 2^n (b) 2^{n^2}
(c) n^2 (d) n^n

12. A relation R is defined on the set Z of integers as follows : ${}_mR_n \Leftrightarrow m + n$ is odd. Then R is
(a) reflexive (b) symmetric
(c) transitive (d) all of these

13. Which of the following statements is not correct for the relation R defined by ${}_aR_b$ if and only if b lives within one kilometre from a ?
(a) R is reflexive
(b) R is symmetric
(c) R is anti-symmetric
(d) None of the above

14. If A = {1, 2, 3}, then a relation R = {(2, 3), (2, 1), (3, 1)} on A is
(a) symmetric and transitive
(b) symmetric only
(c) transitive only
(d) None of the above

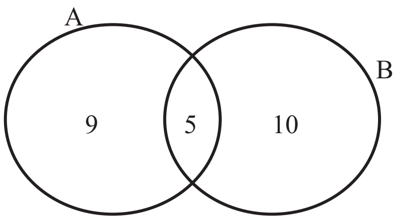
15. If $A = \{1, 2, 3\}$, $B = \{1, 4, 6, 9\}$ and R is a relation from A to B defined by 'x is greater than y'. Then the range of R is
   (a) $\{1, 4, 6, 9\}$ (b) $\{4, 6, 9\}$
   (c) $\{1\}$ (d) None of these
16. Given the relation $R = \{(1, 2), (2, 3)\}$ is defined on the set $A = \{1, 2, 3\}$ then minimum number of ordered pairs which when added to R make it an equivalence relation is
   (a) 5 (b) 6
   (c) 7 (d) 8
17. Let R be an equivalence relation on a finite set A having n elements. Then the number of ordered pairs in R is
   (a) less than n
   (b) greater than or equal to n
   (c) less than or equal to n
   (d) None of the above
18. Let S be the set of all real numbers. A relation R has been defined on S by ${}_aR_b : |a - b| \le 1$. Then R is
   (a) reflexive and symmetric but not transitive
   (b) reflexive and transitive but not symmetric
   (c) symmetric and transitive but not reflexive
   (d) an equivalence relation
19. Let S be the set of all real numbers. Then, the relation $R = \{(a, b) : 1 + ab > 0\}$ on S is
   (a) reflexive and symmetric but not transitive
   (b) reflexive and transitive but not symmetric
   (c) symmetric and transitive but not reflexive
   (d) symmetric, transitive and reflexive
20. The relation "less than" in the set of natural numbers is
   (a) only symmetric (b) only transitive
   (c) only reflexive (d) None of these

ANSWERS

1.	(c)	**2.**	(d)	**3.**	(b)	**4.**	(b)	**5.**	(d)	**6.**	(d)	**7.**	(d)	**8.**	(a)	**9.**	(b)	**10.**	(b)
11.	(b)	**12.**	(b)	**13.**	(c)	**14.**	(c)	**15.**	(c)	**16.**	(c)	**17.**	(b)	**18.**	(a)	**19.**	(a)	**20.**	(b)

Explanations

1. (c) $n(A) = 3$ and $n(B) = 5$
   $\Rightarrow n(A \times B) = 3 \times 5 = 15$
2. (d) $A = \{2, 4\}$ and $B = \{3, 4, 5\}$
   $\Rightarrow A \cap B = \{4\}$ and $A \cap B = \{2, 3, 4, 5\}$
   $\Rightarrow (A \cap B) \times (A \cap B) = \{(4, 2), (4, 3), (4, 4), (4, 5)\}$
3. (b) $n(A \cap \overline{B}) = 9$, $n(\overline{A} \cap B) = 10$ and $n(A \cap B) = 24$



   $n(A \cap B) = n(A \cap \overline{B}) + n(\overline{A} \cap B) + n(A \cap B)$
   $\Rightarrow n(A \cap B) = 5$
   so $n(A) = 14$ and $n(B) = 15$
   and $n(A \times B) = 14 \times 15 = 210$
4. (b) $A = \{1, 2, 4\}$, $B = \{2, 4, 5\}$, $C = \{2, 5\}$
   $A - B = \{1\}$ and $B - C = \{4\}$
   then $(A - B) \times (B - C) = \{(1, 4)\}$
5. (d) $A = \{1, 2, 3\}$, $B = \{3, 5\}$
   $A \times B = \{(1, 3), (1, 5), (2, 3), (2, 5), (3, 3), (3, 5)\}$
   So, remaining elements of $A \times B$ are $(1, 5)$, $(2, 3)$, $(3, 5)$.
6. (d) $\phi : C \to R$ such that $x \,\phi\, y \Leftrightarrow |x| = y$
   (i) $|2 + 3i| = \sqrt{4+9} = \sqrt{13} \ne 13$
   (ii) $3 \ne -3$
   (iii) $|1 + i| = \sqrt{1+1} = \sqrt{2} \ne 2$
   (iv) $|i| = \sqrt{(1)} = 1$
   So, $i \,\phi\, 1$ is correct.
7. (d) Let $A = \{1, 2, 3\}$ and $B = \{2, 3, 4\}$
   i.e., $n(A \cap B) = 2$
   then $A \times B = \{(1, 2) (1, 3) (1, 4) (2, 2) (2, 3) (2, 4) (3, 2) (3, 3) (3, 4)\}$
   and $B \times A = \{(2, 1) (2, 2) (2, 3) (3, 1) (3, 2) (3, 3) (4, 1) (4, 2) (4, 3)\}$
   $(A \times B) \cap (B \times A) = \{(2, 2) (2, 3) (3, 3) (3, 2)\}$
   i.e., $n(A \times B) \cap (B \times A)\} = 4$
   So, if $n(A \cap B) = n$
   then $n\{(A \times B) \cap (B \times A)\} = n^2$
8. (a) $|A| \to m$, $|B| \to n$
   $\Rightarrow |A \times B| = mn$
   and $R \subseteq (A \times B)$
   So, number of relations from A to B $= 2^{mn}$
9. (b) Let $(a, b) \in R$.
   Then $(a, b) \in R \Rightarrow (b, a) \in R^{-1}$
   But given $R = R^{-1} \Rightarrow (b, a) \in R$
   Hence, R is symmetric.

10. (b) Void relation, i.e., null relation is symmetric and transitive but not reflexive because there is a relation of equality in each element.

11. (b) $|A| \to n$

$\Rightarrow |A \times A| = n \times n = n^2$

and $R \subseteq A \times A$

So, number of relations on A = 2^{n^2}

12. (b) $\because$ Sum of two odd and even numbers is an even numbers. So, R is not reflexive.

If $m + n$ is odd then $n + m$ is also odd.

$\Rightarrow {}_mR_n$ and ${}_nR_m$

$\therefore$ R is symmetric.

13. (c) ${}_aR_b \Rightarrow b$ lives within one kilometre from a.

R is reflexive and symmetric but not anti-symmetric.

14. (c) A = {1, 2, 3} and R = {(2, 3) (2, 1) (3, 1)}

$(2, 3) \in R$ and $(3, 1) \in R \Rightarrow (2, 1) \in R$

$\Rightarrow$ R is transitive only.

15. (c) Range (R) = $\{y : (x, y) \in R\}$

A = {1, 2, 3} and B = {1, 4, 6, 9}

So, Range = {1}

$\because$ 2 > 1 and 3 > 1 only

16. (c) R is reflexive if it contains (1, 1), (2, 2) and (3, 3)

$\because (1, 2) \in R, (2, 3) \in R$

$\therefore$ R is symmetric, if $(2, 1), (3, 2) \in R$

Now, R = {(1, 1) (2, 2) (3, 3) (2, 1) (3, 2) (2, 3) (1, 2)}

R will be transitive if $(3, 1) (1, 3) \in R$.

Thus, R becomes an equivalence relation by adding (1, 1), (2, 2), (3, 3), (2, 1) (3, 2), (1, 3), (3, 1)

So, the total number of ordered pairs to be added = 7

17. (b) $\because$ R is an equivalence relation on a set A, therefore $(a, a) \in R \ \forall \ a \in A$.

Hence, R has atleast n ordered pairs.

18. (a)

(i) $\because |a - a| \le 1$, so ${}_aR_a$

$\therefore$ R is reflexive.

(ii) ${}_aR_b \Rightarrow |a - b| \le 1$

$\Rightarrow |b - a| \le 1 \Rightarrow {}_bR_a$

$\therefore$ R is symmetric.

(iii) ${}_2R_1$ and ${}_1R_{1/2}$

But 2 is not related to 1/2.

$\therefore$ R is not transitive.

Thus, R is reflexive and symmetric but not transitive.

19. (a)

(i) ${}_aR_a \because 1 + a^2 > 0$

$\Rightarrow$ R is reflexive.

(ii) ${}_aR_b \Rightarrow 1 + ab > 0 \Rightarrow 1 + ba > 0 \Rightarrow {}_bR_a$

$\Rightarrow$ R is symmetric.

(iii) If $a = -\frac{1}{2}, b = \frac{1}{2}$ and $c = 4$, then

${}_aR_b$ and ${}_bR_c$ but a is not related to c.

So, R is not transitive.

Thus, R is reflexive and symmetric but not transitive.

20. (b) ${}_aR_b : a < b \ \forall \ a, b \in N$

$\because a \nless a$

$\Rightarrow$ Relation is not reflexive.

$\because a < b \not\Rightarrow b < a$

$\Rightarrow$ Relation is not symmetric.

$\because a < b$ and $b < c \Rightarrow a < c$

$\Rightarrow$ Relation is transitive only.