CHAPTER 21

Three Dimensional Geometry

Exercise

1. The line $\frac{x-2}{3}=\frac{y-3}{4}=\frac{z-4}{5}$ is parallel to the plane
   (a) $2x + y - 2z = 0$ (b) $3x + 4y + 5z = 7$
   (c) $x + y + z = 2$ (d) $2x + 3y + 4z = 0$
2. The line $\frac{x-4}{1}=\frac{y-2}{1}=\frac{z-k}{2}$ lies exactly on the plane $2x - 4y + z = 7$, then value of k is
   (a) 7 (b) -7
   (c) 1 (d) No real value
3. Distance between two parallel planes $2x + y + 2z - 8 = 0$ and $4x + 2y + 4z + 5 = 0$ is
   (a) $\frac{3}{2}$ (b) $\frac{5}{2}$
   (c) $\frac{7}{2}$ (d) $\frac{9}{2}$
4. The two points (1, 1, 1) and (– 3, 0, 1) with respect to the plane $3x + 4y - 12z + 13 = 0$ lie on
   (a) opposite side (b) same side
   (c) on the plane (d) None of these
5. The angle between the line $\frac{x-1}{2}=\frac{y-2}{1}=\frac{z+3}{-2}$ and the plane $x + y + 4 = 0$ is equal to
   (a) 0 (b) 30°
   (c) 45° (d) 90°
6. The distance of the point of intersection of the lines $\frac{x-2}{3}=\frac{y+1}{4}=\frac{z-2}{12}$ and the plane $x - y + z = 5$ from the point (– 1, – 5, – 10) is
   (a) 13 (b) 9
   (c) 5 (d) None of these
7. The points $A(1, -6, 10)$, $B(-1, -3, 4)$, $C(5, -1, 1)$ and $D(7, -4, 7)$ are the vertices of a
   (a) parallelogram (b) rhombus
   (c) rectangle (d) square
8. The equation of the plane containing the line $\frac{x-\alpha}{l}=\frac{y-\beta}{m}=\frac{z-\gamma}{n}$ is $A(x-\alpha) + B(y-\beta) + C(z-\gamma) = 0$, where
   (a) $A\alpha + B\beta + C\gamma = 0$ (b) $Al + Bm + Cn = 0$
   (c) $\frac{A}{l}=\frac{B}{m}=\frac{C}{n}$ (d) None of these
9. The locus of $x^2 + y^2 + z^2 = 0$ is
   (a) (0, 0, 0) (b) a sphere
   (c) a circle (d) None of these
10. The plane passing through the point (–2, –2, 2) and containing the line joining the points (1, 1, 1) and (1, –1, 2) makes intercepts a, b, c on the axes of coordinates. The value of $a + b + c$ is
    (a) 12 (b) 6
    (c) – 4 (d) – 3
11. The projections of a line segment on X, Y, Z-axes are 12, 4, 3. The length and the direction cosines of the line segments are
    (a) 13, < 12/13, 3/13, 3/13 >
    (b) 19, < 12/19, 4/19, 3/19 >
    (c) 11, < 12/11, 14/11, 3/11 >
    (d) 13, < 12/13, 4/13, 3/13 >
12. The equation of the plane through the line of intersection of planes $ax + by + cz + d = 0$, $a'x + b'y + c'z + d' = 0$ and parallel to the line $y = 0, z = 0$ is
    (a) $(ab' + a'b)x + (bc' - b'c)y + (ad' - dd) = 0$
    (b) $(ab' - a'b)x + (bc' - b'c)y + (ad' - a'd)z = 0$
    (c) $(ab' - a'b)y + (ac' - a'c)z + (ad' - a'd) = 0$
    (d) None of the above

13. If the direction ratios of two lines are given by $mn - 4nl + 3lm = 0$ and $l + 2m + 3n = 0$, then the angle between the lines is

(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{4}$

(c) $\frac{\pi}{3}$ (d) $\frac{\pi}{2}$

14. If the line $x = ay + b$, $z = cy + d$ and the line $x = a'y + b'$, $z = c'y + d'$ are perpendicular, then

(a) $aa' + cc' + 1 = 0$ (b) $aa' + bb' = 1$

(c) $aa' + bb' = 0$ (d) None of these

15. If the plane $2ax - 3ay + 4az + 6 = 0$ passes through the mid-point of the line joining the centres of the spheres $x^2 + y^2 + z^2 + 6x - 8y - 2z = 13$ and $x^2 + y^2 + z^2 - 10x + 4y - 2z = 8$, then a is

(a) – 2 (b) 2

(c) – 1 (d) 1

16. The lines $\frac{x-2}{1} = \frac{y-3}{1} = \frac{z-4}{-\lambda}$ and $\frac{x-2}{\lambda} = \frac{y-4}{2} = \frac{z-5}{1}$ are coplanar, if λ is

(a) 1, – 1 (b) 3, – 3

(c) 0, – 3 (d) 0, – 1

17. The plane $\frac{x}{a} + \frac{y}{b} + \frac{z}{c}$ meets the coordinate axes in points A, B, C respectively. The equation of sphere $OABC$ is

(a) $x^2 + y^2 + z^2 + ax + by + cz = 0$

(b) $x^2 + y^2 + z^2 - ax - by - cz = 0$

(c) $x^2 + y^2 + z^2 + 2ax + 2by + 2cx = 0$

(d) $x^2 + y^2 + z^2 - 2ax - 2by - 2cz = 0$

18. A line passes through the points (6, –7, –1) and (2, –3, 1). The direction cosines of the line so directed that the angle made by it with the positive direction of X-axis is acute, are

(a) $\frac{2}{3}, -\frac{2}{3}, -\frac{1}{3}$ (b) $-\frac{2}{3}, \frac{2}{3}, \frac{1}{3}$

(c) $\frac{2}{3}, -\frac{2}{3}, \frac{1}{3}$ (d) $\frac{2}{3}, \frac{2}{3}, \frac{1}{3}$

19. The equation of a line passing through origin and parallel to the planes $(q + r)x + (r + p)y + (p + q)z = k$ and $(q - r)x + (r - p)y + (p - q)z = k$ is

(a) $\frac{x}{q^2 - r^2} = \frac{y}{r^2 - p^2} = \frac{z}{p^2 - q^2}$

(b) $\frac{x}{p^2 - qr} = \frac{y}{q^2 - pr} = \frac{z}{r^2 - pq}$

(c) $\frac{x}{q} = \frac{y}{r} = \frac{z}{p}$

(d) None of these

20. The ratio in which the YZ-plane divides the join of the points (– 2, 4, 7) and (3, – 5, 8) is

(a) 2 : 3 (b) 3 : 2

(c) –2 : 3 (d) 4 : –3

21. A straight line which makes an angle of 60° with each of X-and Y-axes, is inclined with Z-axis at an angle of

(a) 45° (b) 30°

(c) 75° (d) 60°

22. The vertices of a ΔABC are $A(-1, -2, -3)$, $B(-1, 2, 3)$ and $C(0, 0, 0)$. Then, the direction ratios of internal bisector of $\angle C$ are

(a) 0, 0, 1 (b) 1, – 1, 1

(c) – 1, 0, 0 (d) None of these

23. The plane $ax + by + cz = 1$ meets the coordinate axes in A, B, C. The centroid of the triangle is

(a) $(3a, 3b, 3c)$ (b) $\left(\frac{a}{3}, \frac{b}{3}, \frac{c}{3}\right)$

(c) $\left(\frac{3}{a}, \frac{3}{b}, \frac{3}{c}\right)$ (d) $\left(\frac{1}{3a}, \frac{1}{3b}, \frac{1}{3c}\right)$

24. A line passes through two points $A(2, -3, -1)$ and $B(8, -1, 2)$. The coordinates of a point on this line at a distance of 14 units from A are

(a) (14, 1, 5) (b) (– 10, – 7, – 7)

(c) (86, 25, 41) (d) None of these

25. The coordinates of the foot of the perpendicular drawn from the point $A(1, 0, 3)$ to the join of the points $B(4, 7, 1)$ and $C(3, 5, 3)$ are

(a) $\left(\frac{5}{3}, \frac{7}{3}, \frac{17}{3}\right)$ (b) (5, 7, 17)

(c) $\left(\frac{5}{3}, -\frac{7}{3}, \frac{17}{3}\right)$ (d) $\left(-\frac{5}{3}, \frac{7}{3}, -\frac{17}{3}\right)$

26. The image of the point (– 1, 3, 4) in the plane $x - 2y = 0$ is

(a) (15, 11, 4) (b) $\left(-\frac{17}{3}, -\frac{19}{3}, 1\right)$

(c) $\left(\frac{9}{5}, \frac{-13}{5}, 4\right)$ (d) $\left(-\frac{17}{3}, -\frac{19}{3}, 4\right)$

27. The direction ratios of the line given by planes $x - y + z - 5 = 0$ and $x - 3y - 6 = 0$ are

(a) 3, 1, – 2 (b) 2, – 4, 1

(c) $\frac{3}{\sqrt{14}}, \frac{1}{\sqrt{14}}, -\frac{2}{\sqrt{14}}$ (d) $\frac{2}{\sqrt{41}}, -\frac{4}{\sqrt{41}}, \frac{1}{\sqrt{41}}$

28. The equation of a plane through the line of intersection of planes $2x + 3y + z - 1 = 0$ and $x + 5y - 2z + 7 = 0$ and parallel to line $y = 0 = z$ is

(a) $4x + 7y - 5z + 15 = 0$

(b) $13y - 3z + 13 = 0$

(c) $7x - 5y + 15 = 0$

(d) $7y - 5z + 15 = 0$

29. If O is the origin and A is the point (a, b, c), then the equation of the plane through A and at right angles to OA is
(a) $a(x-a) - b(x-b) - c(x-c) = 0$
(b) $a(x+a) + b(x+b) + c(x+c) = 0$
(c) $a(x-a) + b(x-b) + c(x-c) = 0$
(d) None of the above

30. Equation of the line passing through the point (1, 2, 3) and parallel to the plane $2x + 3y + z + 5 = 0$ is
(a) $\frac{x-1}{-1} = \frac{y-2}{1} = \frac{z-3}{-1}$
(b) $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{1}$
(c) $\frac{x-1}{1} = \frac{y-2}{4} = \frac{z-3}{7}$
(d) $\frac{x-1}{3} = \frac{y-2}{4} = \frac{z-3}{2}$

31. Equation of the plane through the points (2, 1, – 1) and (– 1, 3, 4) and perpendicular to the plane $x - 2y + 4z = 0$ is given by
(a) $18x + 17y + 4z = 49$
(b) $18x - 17y + 4z = 49$
(c) $18x + 17y - 4z + 49 = 0$
(d) None of these

32. The angle between the lines $2x = 3y = -z$ and $6x = -y = -4z$ is equal to
(a) 0° (b) 30°
(c) 45° (d) 90°

33. If the straight lines $\frac{x-1}{k} = \frac{y-2}{2} = \frac{z-3}{3}$ and $\frac{x-2}{3} = \frac{y-3}{k} = \frac{z-1}{2}$ intersect at a point, then the integer k is equal to
(a) –5 (b) 5
(c) 2 (d) –2

34. In what condition do the planes $bx - ay = n$, $cy - bz = l$ and $az - cx = m$ intersect in a line?
(a) $a + b + c = 0$ (b) $a = b = c$
(c) $al + bm + cn = 0$ (d) $l + m + n = 0$

35. If (2, 3, 5) is one end of a diameter of the sphere $x^2 + y^2 + z^2 - 6x - 12y - 2z + 20 = 0$, then coordinates of the other end of the diameter are
(a) (4, 9, –3) (b) (4, –3, 3)
(c) (4, 3, 5) (d) (4, 3, 3)

36. The plane $2x - 2y + z + 12 = 0$ touches the sphere $x^2 + y^2 + z^2 - 2x - 4y + 2z - 3 = 0$ at the point
(a) (4, –4, –2) (b) (–1, 4, –2)
(c) (–1, –4, 2) (d) (1, 4, –2)

37 The four points (0, 4, 3), (– 1, –5, –3), (–2, –2, 1) and (1, 1, – 1) lie in the plane
(a) $4x + 3y + 2z - 9 = 0$ (b) $9x - 5y + 6z + 2 = 0$
(c) $3x + 4y + 7z - 5 = 0$ (d) None of these

38. $A(3, 2, 0)$, $B(5, 3, 2)$, $C(-9, 6, -3)$ are the vertices of a ΔABC. If the bisector $\angle BAC$ meets BC at D, then coordinates of D are
(a) $\left(\frac{19}{8}, \frac{57}{16}, \frac{17}{16}\right)$ (b) $\left(-\frac{19}{8}, \frac{57}{16}, \frac{17}{16}\right)$
(c) $\left(\frac{17}{8}, -\frac{57}{16}, \frac{17}{16}\right)$ (d) $\left(\frac{19}{8}, \frac{57}{16}, -\frac{17}{16}\right)$

39. What is the angle between two lines having direction ratios $(\sqrt{3}-1, -\sqrt{3}-1, 4)$ and $(-\sqrt{3}-1, \sqrt{3}-1, 4)$?
(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{4}$
(c) $\frac{\pi}{3}$ (d) $\frac{\pi}{2}$

40. Consider the following relations among the angles α, β and γ made by a vector with coordinate axes.
I. $\cos 2\alpha + \cos 2\beta + \cos 2\gamma = 1$
II. $\sin^2 \alpha + \sin^2 \beta + \sin^2 \gamma = 1$
Which of the above statement(s) is/are correct ?
(a) Only I (b) Only II
(c) Both 1 and II (d) Neither I nor II

Directions (Q. Nos. 41-43):

A plane P passes through the line of intersection of the planes $2x - y + 3z = 2$, $x + y - z = 1$ and the point (1, 0, 1).

41. What are the direction ratios of the line of intersection of the given planes? **[NDA-I 2016]**
(a) < 2, – 5, – 3 > (b) < 1, – 5, – 3 >
(c) < 2, 5, 3 > (d) < 1, 3, 5 >

42. What is the equation of the plane P ? **[NDA-I 2016]**
(a) $2x + 5y - 2 = 0$ (b) $5x + 2y - 5 = 0$
(c) $x + z - 2 = 0$ (d) $2x - y - 2z = 0$

43. If the plane P touches the sphere $x^2 + y^2 + z^2 = r^2$, then what is r equal to? **[NDA-I 2016]**
(a) $\frac{2}{\sqrt{29}}$ (b) $\frac{4}{\sqrt{29}}$
(c) $\frac{5}{\sqrt{29}}$ (d) 1

Directions (Q. Nos. 44-45):

Let Q be the image of the point P (–2, 1, –5) in the plane $3x - 2y + 2z + 1 = 0$.

44. Consider the following statements. **[NDA-II2016]**
I. The coordinates of Q are (4, –3, –1).
II. PQ is of length more than 8 units.
III. The point (1, – 1, –3) is the mid-point of the line segment PQ and lies on the given plane.

Which of the above statements are correct?

(a) I and II (b) II and III

(c) I and III (d) I, II and III

45. Consider the following statements. **[NDA-II 2016]**

I. The direction ratios of the segment PQ are < 3, – 2, 2 >.

II. The sum of the squares of direction cosines of the line segment PQ is unity.

Which of the above statements is/are correct?

(a) I only (b) II only

(c) Both I and II (d) Neither I nor II

Directions (Q. Nos. 46-47):

A line L passes through the point $P(5, -6, 7)$ and is parallel to the planes $x + y + z = 1$ and $2x - y - 2z = 3$.

46. What are the direction ratios of the line of intersection of the given planes? **[NDA-II 2016]**

(a) < 1, 4, 3 > (b) < – 1, – 4, 3 >

(c) < 1, – 4, 3 > (d) < 1, – 4, – 3 >

47. What is the equation of the line L? **[NDA-II 2016]**

(a) $\frac{x-5}{-1}=\frac{y+6}{4}=\frac{z-7}{-3}$ (b) $\frac{x+5}{-1}=\frac{y-6}{4}=\frac{z+7}{-3}$

(c) $\frac{x-5}{-1}=\frac{y+6}{-4}=\frac{z-7}{3}$ (d) $\frac{x-5}{-1}=\frac{y+6}{-4}=\frac{z-7}{-3}$

48. A straight line with direction cosines 0, 1, 0 is **[NDA-I 2017]**

(a) parallel to X-axis

(b) parallel to Y-axis

(c) parallel to Z-axis

(d) equally inclined to all the axes

49. (0, 0, 0), (a, 0, 0), (0, b, 0) and (0, 0, c) are four distinct points. What are the coordinates of the point which is equidistant from the four points? **[NDA-I-2017]**

(a) $\left(\frac{a+b+c}{3},\frac{a+b+c}{3},\frac{a+b+c}{3}\right)$

(b) (a, b, c)

(c) $\left(\frac{a}{2},\frac{b}{2},\frac{c}{2}\right)$

(d) $\left(\frac{a}{3},\frac{b}{3},\frac{c}{3}\right)$

50. The points $P(3, 2, 4)$, $Q(4, 5, 2)$, $R(5, 8, 0)$ and $S(2, -1, 6)$ are **[NDA-I 2017]**

(a) vertices of a rhombus which is not a square

(b) non-coplanar

(c) collinear

(d) coplanar but not collinear

ANSWERS

1.	(a)	**2.**	(a)	**3.**	(c)	**4.**	(a)	**5.**	(c)	**6.**	(a)	**7.**	(a)	**8.**	(b)	**9.**	(b)	**10.**	(a)
11.	(a)	**12.**	(c)	**13.**	(d)	**14.**	(a)	**15.**	(a)	**16.**	(c)	**17.**	(b)	**18.**	(a)	**19.**	(b)	**20.**	(a)
21.	(a)	**22.**	(c)	**23.**	(d)	**24.**	(a)	**25.**	(a)	**26.**	(c)	**27.**	(a)	**28.**	(d)	**29.**	(c)	**30.**	(a)
31.	(a)	**32.**	(d)	**33.**	(a)	**34.**	(c)	**35.**	(a)	**36.**	(b)	**37.**	(b)	**38.**	(a)	**39.**	(c)	**40.**	(a)
41.	(a)	**42.**	(b)	**43.**	(c)	**44.**	(d)	**45.**	(c)	**46.**	(c)	**47.**	(a)	**48.**	(b)	**49.**	(c)	**50.**	(c)

Explanations

1. (a) Given line is $\frac{x-2}{3}=\frac{y-3}{4}=\frac{z-4}{5}$

Its direction ratios are 3, 4, 5.

Let it be parallel to the plane $ax + by + cz + d = 0$

So, $3a + 4b + 5c = 0$

By hit and trail method, $a = 2, b = 1, c = -2$

Hence the required plane is $2x + y - 2z = 0$.

2. (a) Given, line $\frac{x-4}{1}=\frac{y-2}{1}=\frac{z-k}{2}$ lies exactly on the plane $2x - 4y + z = 7$.

$\Rightarrow$ Normal (2, –4, 1) is perpendicular to the line whose direction ratios are < 1, 1, 2 > and the point (4, 2, k) will lie on the plane.

So, $2(4) - 4(2) + k = 7 \Rightarrow k = 7$

3. (c) Distance between two parallel planes $2x + y + 2z - 8 = 0$ and $4x + 2y + 4z + 5 = 0$ is given by

$$d=\left|\frac{-8-\left(\frac{5}{2}\right)}{\sqrt{4+1+4}}\right|=\frac{21}{2\times 3}=\frac{7}{2}$$

4. (a) Let $S = 3x + 4y - 12z + 13$

For point (1, 1, 1),

$S = 3(1) + 4(1) - 12(1) + 13 = 8 > 0$

For point (–3, 0,1),

$S = 3(-3) + 4(0) - 12(1) + 13 = -8 < 0$

Hence, both the given points lies on the opposite sides of the plane.

5. (c) Angle between the line $\frac{x-1}{2}=\frac{y-2}{1}=\frac{z+3}{-2}$ and the plane $x + y + 4 = 0$ is given by

$$\sin\theta=\frac{aa_1+bb_1+cc_1}{\sqrt{\Sigma a^2}\sqrt{\Sigma a_1^2}}$$

$$=\frac{1(2)+1(1)+(-2)(0)}{(\sqrt{1+1+0})(\sqrt{4+1+4})}=\frac{3}{3\sqrt{2}}=\frac{1}{\sqrt{2}}$$

$\Rightarrow\ \theta = 45°$

6. (a) Let $\frac{x-2}{3}=\frac{y+1}{4}=\frac{z-2}{12}=k$

$\Rightarrow P(3k+2, 4k-1, 12k+2)$ be any point on the line.

It also lies on the plane $x - y + z = 5$

$\Rightarrow 3k + 2 - 4k + 1 + 12k + 2 = 5$

$\Rightarrow k = 0$

So, coordinates of P are $(2, -1, 2)$.

Now, distance between $P(2, -1, 2)$ and point $(-1, -5, -10)$

$=\sqrt{(-1-2)^2+(-5+1)^2+(-10-2)^2}$

$=\sqrt{9+16+144}=13$

7. (a) Given points are $A(1, -6, 10)$, $B(-1, -3, 4)$, $C(5, -1, 1)$ and $D(7, -4, 7)$.

Here, $AB = BC = CD = DA = 7$

$\Rightarrow ABCD$ can be a square or a rhombus.

Direction ratios of $AB = (-1 - 1, -3 + 6, 4 - 10)$ $= (-2, 3, -6)$

and direction ratios of $AD = (7 - 1, -4 + 6, 7 - 10)$ $= (6, 2, -3)$

$\because\ a_1a_2 + b_1b_2 + c_1c_2$

$= (-2)(6) + (3)(2) + (-6)(-3) = -12 \neq 0$

$\Rightarrow \angle A \neq 90°$

Hence, $ABCD$ is a rhombus.

8. (b) Plane $A(x - \alpha) + B(y - \beta) + C(z - \gamma) = 0$ contains the line $\frac{x-\alpha}{l}=\frac{y-\beta}{m}=\frac{z-\gamma}{n}$.

$\Rightarrow Al + Bm + Cn = 0$

9. (b) Given equation $x^2 + y^2 + z^2 = 0$ represents a point $(0, 0, 0)$.

10. (a) Equation of a plane passing through $(-2, -2, 2)$ is given by $a(x + 2) + b(y + 2) + c(z - 2) = 0$ …(i)

Equation of a line joining (1, 1, 1) and(1, –1, 2) is

$$\frac{x-1}{0}=\frac{y-1}{-2}=\frac{z-1}{1} \quad \text{…(ii)}$$

This lies lie on the plane given in eq.

$\therefore\ 0(a) + (-2)(b) + 1(c) = 0$

$\Rightarrow 0a + 2b - c = 0$ …(iii)

Now, point (1, 1, 1) lies on the plane also.

So, $3a + 3b - c = 0$ …(iv)

Solving eqs (iii) and (iv),

$$\frac{a}{-1}=\frac{b}{3}=\frac{c}{6}$$

Hence, equation of plane is

$-1(x + 2) + 3(y + 2) + 6(z - 2) = 0$

$\Rightarrow x - 3y - 6z + 8 = 0$

Intercepts on X, Y and Z axis are

$a=-8,\ b=\frac{8}{3},\ c=\frac{8}{6}$ respectively.

So, $a+b+c=-8+\frac{8}{3}+\frac{8}{6}=-4.$

11. (a) Given projections of a line segment on X, Y and Z axis are $a = 12$, $b = 4$ and $c = 3$ respectively.

Length of line segment $=\sqrt{a^2+b^2+c^2}=13$ and direction cosines of line segment are

$$=\left\langle\frac{12}{13},\frac{4}{13},\frac{3}{13}\right\rangle$$

12. (c) Equation of plane passing through the intersection of the given planes is

$(ax + by + cz + d) + \lambda(a'x + b'y + c'z + d') = 0$

$\Rightarrow x(a + a'\lambda) + y(b + b'\lambda) + z(c + c'\lambda) + (d + d') = 0$ ...(i)

This plane is parallel to the lines $y = 0$ and $z = 0$

$\Rightarrow (a + \lambda a') \times 1 = 0 \Rightarrow \lambda=\frac{-a}{a'}$

Put in eq (i),

$$0+\left(b-\frac{b'a}{a'}\right)y+\left(c-\frac{ac'}{a'}\right)z+\left(d-\frac{d'a}{a'}\right)=0$$

or $(ab' - a'b)\,y + (ac' - a'c)\,z + (ad' - a'd) = 0$

13. (d) Direction ratios of two lines are given by

$mn - 4nl + 3lm = 0$ …(i)

and $l + 2m + 3n = 0$ …(ii)

From eq (ii), $l = -(2m + 3n)$

Put in (i), $mn + 4n(2m + 3n) - 3(2m + 3n)m = 0$

$\Rightarrow 12n^2 - 6m^2 = 0$

$\Rightarrow \frac{m}{n}=\sqrt{2}$ or $\frac{m}{n}=-\sqrt{2}$

Let two roots of the above equation be $\frac{m_1}{n_1}$ and $\frac{m_2}{n_2}$.

$\Rightarrow$ Product of roots $\frac{m_1m_2}{n_1n_2}=-2$

or $\frac{m_1m_2}{-2}=\frac{n_1n_2}{1}$ …(iii)

Similarly, eliminating n, we get

$\frac{n_1n_2}{1}=\frac{l_1l_2}{1}$ …(iv)

Therefore form eqs. (iii) and (iv),

$$\frac{m_1m_2}{-2} = \frac{n_1n_2}{1} = \frac{l_1l_2}{1} = k$$

So, Angle between both the line is

$$\cos\theta = l_1l_2 + m_1m_2 + n_1n_2$$
$$= k + (-2k) + k = 0$$
$$\Rightarrow \theta = \frac{\pi}{2}$$

14. (a) Given lines $x = ay + b$, $z = cy + d$ and $x = a'y + b'$, $z = c'y + d'$ can be written as

$$\frac{x-b}{a} = \frac{y-0}{1} = \frac{z-d}{c} \quad \ldots(i)$$

and $$\frac{x-b'}{a'} = \frac{y-0}{1} = \frac{z-d'}{c'} \quad \ldots(ii)$$

$\because$ Both the lines are perpendicular.

So, $aa' + (1)(1) + cc' = 0$

15. (a) Centre of sphere

$x^2 + y^2 + z^2 + 6x - 8y - 2z - 13 = 0$ is $C_1(-3, 4, 1)$

and centre of sphere

$x^2 + y^2 + z^2 - 10x + 4y - 2z = 8$ is $C_2(5, -2, 1)$

So, mid-point of $C_1C_2 = \left(\frac{-3+5}{2}, \frac{4-2}{2}, \frac{1+1}{2}\right)$,

i.e., (1, 1, 1).

This, point lies on the plane

$2ax - 3ay + 4az + 6 = 0$

$\Rightarrow 2a - 3a + 4a + 6 = 0$

$\Rightarrow a = -2$

16. (c) Given lines $\frac{x-2}{1} = \frac{y-3}{1} = \frac{z-4}{-\lambda}$ and

$\frac{x-1}{\lambda} = \frac{y-4}{2} = \frac{z-5}{1}$ are coplanar.

$$\text{So, } \begin{vmatrix} 2-1 & 3-4 & 4-5 \\ 1 & 1 & -\lambda \\ \lambda & 2 & 1 \end{vmatrix} = 0 \Rightarrow \begin{vmatrix} 1 & -1 & -1 \\ 1 & 1 & -\lambda \\ \lambda & 2 & 1 \end{vmatrix} = 0$$

$\Rightarrow \lambda^2 + 3\lambda = 0$

$\Rightarrow \lambda = 0$ or $\lambda = -3$

17. (b) The plane $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$ meets the coordinate axes in points A, B, C respectively.

So, coordinates of A, B and C are $A(a, 0, 0)$, $B(0, b, 0)$ and $C(0, 0, c)$ respectively.

Centre of sphere $OABC$ is a point equidistance from all the four points. So, centre is $\left(\frac{a}{2}, \frac{b}{2}, \frac{c}{2}\right)$.

Hence, equation of sphere is

$x^2 + y^2 + z^2 - ax - by - cz = 0$.

18. (a) Equation of a line passing through the points (6, –7, –1) and (2, –3, 1) is

$$\frac{x-6}{2-6} = \frac{y+7}{-3+7} = \frac{z+1}{1+1},$$

$$i.e., \frac{x-6}{4} = \frac{y+7}{-4} = \frac{z+1}{-2}$$

Now, direction ratios of line

$= <4, -4, -2> = <-2, 2, 1>$

So, direction cosines $= <-\frac{2}{3}, \frac{2}{3}, \frac{1}{3}>$

Since, the lines makes an acute angle with X-axis.

So, $\cos\alpha > 0$

Hence, direction cosines are $<\frac{2}{3}, \frac{-2}{3}, \frac{-1}{3}>$.

19. (b) Let the direction ratios of the required line are l, m and n.

Since, the line is parallel to the planes

$(q + r)x + (r + p)y + (p + q)z - k = 0$ and

$(q - r)x + (r - p)y + (p - q)z - k = 0$

So, $l(q + r) + m(r + p) + n(p + q) = 0$ …(i)

and $l(q - r) + m(r - p) + n(p - q) = 0$ …(ii)

On solving, $\frac{l}{p^2 - qr} = \frac{m}{q^2 - pr} = \frac{n}{r^2 - pq}$

Now this line also passes through the origin.

So, $\frac{x-0}{l} = \frac{y-0}{m} = \frac{z-0}{n}$

or $\frac{x}{p^2 - qr} = \frac{y}{q^2 - pr} = \frac{z}{r^2 - pq}$

20. (a) Let yz plane divides the join of the points (–2, 4, 7) and (3, –5, 8) at M in ratio $\lambda : 1$.

Then, coordinates of M are

$$\left(\frac{3\lambda-2}{\lambda+1}, \frac{-5\lambda+4}{\lambda+1}, \frac{8\lambda+7}{\lambda+1}\right)$$

In YZ plane, x coordinate is 0.

So, $\frac{3\lambda-2}{\lambda+1} = 0 \Rightarrow \lambda = \frac{2}{3}$

Hence, required ratio = 2 : 3

21. (a) $\because$ Line makes an angle of 60° with each X and Y axes.

So, $l = \cos 60° = \frac{1}{2}$ and $m = \cos 60° = \frac{1}{2}$

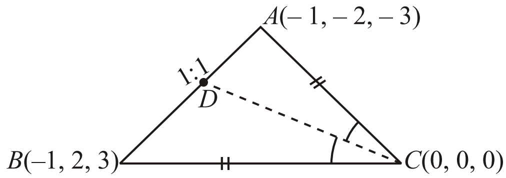
Let the line makes an angle X with Y axis.

Then $l^2 + m^2 + n^2 = 1$

$\Rightarrow \frac{1}{4} + \frac{1}{4} + \cos^2 y = 1$

$\Rightarrow \cos^2 y = \frac{1}{2} \Rightarrow \cos y = \frac{1}{\sqrt{2}} \Rightarrow y = 45°$

22. (c) Length of $AC = \sqrt{1^2 + 2^2 + 3^2} = \sqrt{14}$



Length of $BC = \sqrt{1^2 + 2^2 + 3^2} = \sqrt{14}$

$\because AC = BC$

So, the bisector of $\angle C$, divides the line AB in 1 : 1.

$\Rightarrow$ Coordinates of $D = \left(\frac{-1-1}{2}, \frac{2-2}{2}, \frac{3-3}{2}\right)$

$= (-1, 0, 0)$

Hence, direction ratio of $CD = (-1, 0, 0)$

23. (d) Let the plane $ax + by + cz - 1 = 0$ meets the coordinates axes at points A, B and C.

Then, coordinates of these points are

$A\left(\frac{1}{a}, 0, 0\right), B\left(0, \frac{1}{b}, 0\right), C\left(0, 0, \frac{1}{c}\right).$

So, centroid of triangle

$= \left(\frac{\frac{1}{a}+0+0}{3}, \frac{0+\frac{1}{b}+0}{3}, \frac{0+0+\frac{1}{c}}{3}\right) = \left(\frac{1}{3a}, \frac{1}{3b}, \frac{1}{3c}\right)$

24. (a) Equation of a line passing through the points $A(2, -3, -1)$ and $B(8, -1, 2)$ is

$\frac{x-2}{6} = \frac{y+3}{2} = \frac{z+1}{3}.$

Directions cosines of the line are $< \frac{6}{7}, \frac{2}{7}, \frac{3}{7} >$

So, equation of a line can be written as

$\frac{x-2}{6/7} = \frac{y+3}{2/7} = \frac{z+1}{3/7} = \lambda$

where λ is the distance of any point P on the line from A. $\Rightarrow \lambda = 14$

So, coordinates of P are $\left(\frac{2+6\lambda}{7}, \frac{-3+2\lambda}{7}, \frac{-1+3\lambda}{7}\right)$

(14, 1, 5).

25. (a) Equation of a line joining the points $B(4, 7, 1)$ and $C(3, 5, 3)$ is $\frac{x-4}{-1} = \frac{y-7}{-2} = \frac{z-1}{2} = \lambda$ (say)

Let point $M(4 - \lambda, 7 - 2\lambda, 1 + 2\lambda)$ be the foot of the perpendicular drawn from $A(1, 0, 3)$ on the line BC.

Then, direction ratios of

$AM = (4 - \lambda - 1, 7 - 2\lambda - 0, 2\lambda + 1 - 3)$

$= (3 - \lambda, 7 - 2\lambda, 2\lambda - 2)$

$\because AM \perp BC$

$\Rightarrow -1(3 - \lambda) - 2(7 - 2\lambda) + 2(2\lambda - 2) = 0$

$\Rightarrow \lambda = 7/3$

Hence, M is $\left(4-\frac{7}{3}, 7-\frac{14}{3}, \frac{14}{3}+1\right) = \left(\frac{5}{3}, \frac{7}{3}, \frac{17}{3}\right)$

26. (c) Let Q be the foot of the perpendicular drawn from point $P(-1, 3, 4)$ in the plane $x - 2y = 0$.

So, equation of line PQ is

$\frac{x+1}{1} = \frac{y-3}{-2} = \frac{z-4}{0} = \lambda$ (say)

Let coordinates of Q are $(\lambda - 1, -2\lambda + 3, 4)$ and this point Q lies on the plane $x - 2y = 0$.

$\Rightarrow \lambda - 1 + 4\lambda - 6 = 0$ or $\lambda = \frac{7}{5}$

So, Q is $\left(\frac{2}{5}, \frac{1}{5}, 4\right)$.

Let $M(x_1, y_1, z_1)$ be the image of $P(-1, 3, 4)$.

Then, Q is the mid point of PM.

$\Rightarrow \frac{x_1-1}{2} = \frac{2}{5}, \frac{y_1+3}{2} = \frac{1}{5}$ and $\frac{z_1+4}{2} = 4$

$\Rightarrow x_1 = \frac{9}{5}, y_1 = \frac{-13}{5}$ and $z_1 = 4$

Hence, M is $\left(\frac{9}{5}, \frac{-13}{5}, 4\right)$.

27. (a) Let the direction cosines of the line are l, m and n.

Given planes are $x - y + z - 5 = 0$

and $x - 3y - 6 = 0$

So, $l - m + n = 0$ and $l - 3m + 0m = 0$

On solving, $\frac{l}{3} = \frac{-m}{-1} = \frac{n}{-2} = \frac{\Sigma l^2}{\sqrt{9+1+4}}$

$\Rightarrow l = \frac{m}{\sqrt{14}}, m = \frac{1}{\sqrt{14}}$ and $n = \frac{-2}{\sqrt{14}}$

So, direction ratios are $(3, 1, -2)$.

28. (d) Equation of plane through the line of intersection of two planes is given by

$(2x + 3y + z - 1) + \lambda(x + 5y - 2z + 7) = 0$

$\Rightarrow (2 + \lambda)x + (3 + 5\lambda)y + (1 - 2\lambda)z - 1 + 7\lambda = 0$

This plane is parallel to line $y = 0 = z$, i.e., (1, 0, 0).

So, $1(2 + \lambda) + 0(3 + 5\lambda) + 0(1 - 2\lambda) = 0$

$\Rightarrow \lambda = -2$

So, required plane is $-7y + 5z - 15 = 0$

or $7y - 5z + 15 = 0$

29. (c) Given plane is a right angles to point $A(a, b, c)$.

OA is the normal to the plane.

Direction ratio of $OA = (a - 0, b - 0, c - 0)$

$= (a, b, c)$

Then, equation of plane passing through A is

$a(x - a) + b(x - b) + c(x - c) = 0$

30. (a) Equation of the line passing through the point (1, 2, 3) is given by

$\frac{x-1}{l}=\frac{y-2}{m}=\frac{z-3}{n}$ ...(i)

This line is parallel to the plane $2x+3y+z+5=0$

$\Rightarrow 2l+3m+n=0$

It is satisfied when $l=-1$, $m=1$ and $n=-1$

So, equation of line is $\frac{x-1}{-1}=\frac{y-2}{1}=\frac{z-3}{-1}$.

31. (a) Equation of the plane passing through (2, 1, – 1) is

$a(x-2)+b(y-1)+c(z+1)=0$ ...(i)

It passes through point (– 1, 3, 4)

$\Rightarrow -3a+2b+5c=0$ ...(ii)

This plane is also perpendicular to the plane

$x-2y+4z=0 \Rightarrow a-2b+4c=0$ ...(iii)

Solving eqs. (ii) and (iii), $\frac{a}{18}=\frac{b}{17}=\frac{c}{4}$

Hence, equation of plane is

$18(x-2)+17(y-1)+4(z+1)=0$

or $18x+17y+4z=49$

32. (d) Given lines $2x=3y=-z$ and $6x=-y=-4z$ can be written as $\frac{x}{3}=\frac{y}{2}=\frac{z}{-6}$ and $\frac{x}{2}=\frac{y}{-12}=\frac{z}{-3}$.

Angle between the lines is

$$\cos\theta=\frac{a_1a_2+b_1b_2+c_1c_2}{\sqrt{a_1^2+b_1^2+c_1^2}\sqrt{a_2^2+b_2^2+c_2^2}}$$

$$\Rightarrow \cos\theta=\frac{(3)(2)+(2)(-12)+(-6)(-3)}{\sqrt{(9+4+36)(4+144+9)}}=0$$

$\Rightarrow \theta=90°$

33. (a) The lines $\frac{x-1}{k}=\frac{y-2}{2}=\frac{z-3}{3}$ and

$\frac{x-2}{3}=\frac{y-3}{k}=\frac{z-1}{2}$ intersect at a point.

This implies that lines are coplanar.

So, $\begin{vmatrix} 1 & 1 & -2 \\ k & 2 & 3 \\ 3 & k & 2 \end{vmatrix}=0$

$\Rightarrow 2k^2+5k-25=0$

$\Rightarrow (k+5)(2k-5)=0$ {$\because$ k is an integer.}

$\Rightarrow k=-5$

34. (c) Given planes are $bx-ay=n$, $cy-bz=l$ and $az-cx=m$.

Let $z=0$ and solving the first two equations

$y=\frac{l}{c}$ and $x=\frac{m}{b}+\frac{al}{cb}$

$\because$ Point $\left(\frac{n}{b}+\frac{al}{cb}+\frac{l}{c},0\right)$ intersect in a line. So it will satisfy the third plane.

$\Rightarrow a\times 0-c\left(\frac{n}{b}+\frac{al}{cb}\right)=m \Rightarrow al+mb+nc=0$

35. (a) Given sphere is $x^2+y^2+z^2-6x-12y-2z+20=0$

Its centre is (3, 6, 1).

Let the other end of the diameter be (x_1, y_1, z_1) and the given end is (2, 3, 5).

$\because$ Centre is the middle point of the diameter.

So, $3=\frac{x_1+2}{2}$, $6=\frac{y_1+3}{2}$, $1=\frac{z_1+5}{2}$

$\Rightarrow x_1=4, y_1=9$ and $z_1=-3$

Hence, other end is (4, 9, – 3).

36. (b) Centre of the sphere

$x^2+y^2+z^2-2x-4y+2z-3=0$ is (1, 2, – 1).

Given plane $2x-2y+z+12=0$ touches the sphere.

$\Rightarrow$ This is a tangent plane.

$\Rightarrow$ To find the contact point we have to find the foot of the perpendicular drawn from (1, 2, – 1) to the plane $2x-2y+z+12=0$.

Equation of a line passing from (1, 2, – 1) and $\perp$ to the given plane is

$\frac{x-1}{2}=\frac{y-2}{-2}=\frac{z+1}{1}$

Its direction cosines are $<\frac{2}{3},\frac{-2}{3},\frac{1}{3}>$

So, line is $\frac{x-1}{2/3}=\frac{y-2}{-2/3}=\frac{z+1}{1/3}=\lambda$ (let)

Let point $Q\left(\frac{2\lambda}{3}+1, 2-\frac{2\lambda}{3},\frac{\lambda}{3}-1\right)$ be the foot of perpendicular and lies on the plane.

So, $2\left(\frac{2\lambda}{3}+1\right)-2\left(2-2\frac{\lambda}{3}\right)+\left(\frac{\lambda}{3}-1\right)+12=0$

$\Rightarrow \lambda=-3$

Hence foot of perpendicular = (– 1, 4, – 2)

37. (b) Equation of a plane passing though (0, 4, 3) is

$a(x-0)+b(y-4)+c(z-3)=0$ ...(i)

Points (– 1, – 5, – 3), (– 2, – 2, 1) and (1, 1, – 1) lie on this plane.

$a+9b+6c=0$, ...(ii)

So, $a+3b+c=0$ ...(iii)

and $a-3b-4c=0$ ...(iv)

On solving eqs. (ii), (iii) and (iv),

$a=-9$, $b=5$ and $c=-6$

Hence, equation of plane is

$-9(x)+5(y-4)-6(z-3)=0$

i.e., $9x-5y+6z+2=0$

38. (a) Given vertices of ΔABC are $A(3, 2, 0)$, $B(5, 3, 2)$ and $C(-9, 6, -3)$.

So, $AB=\sqrt{(5-3)^2+(3-2)^2+(2-0)^2}=3$

$AC=\sqrt{(-9-3)^2+(6-2)^2+(-3-0)^2}=13$

$\because$ AD is the bisector of $\angle BAC$.

$\therefore$ AD, divides line BC in the ratio of its adjacent sides, i.e., $\frac{BD}{DC}=\frac{AB}{AC}=\frac{3}{13}$

Coordinates of D are

$$\left(\frac{13\times5+3\times-9}{13+3},\frac{13\times3+3\times6}{13+3},\frac{13\times2+3\times-3}{3+13}\right)$$

$$=\left(\frac{19}{8},\frac{57}{16},\frac{17}{16}\right).$$

39. (c) Angle between two lines

$$\cos\theta=\frac{(\sqrt{3}-1)(-\sqrt{3}-1)+(-\sqrt{3}-1)(\sqrt{3}-1)+(4)(4)}{\sqrt{(\sqrt{3}-1)^2+(-\sqrt{3}-1)^2+4^2}\sqrt{(-\sqrt{3}-1)^2+(\sqrt{3}-1)^2+4^2}}$$

$$=\frac{-(3-1)-(3-1)+16}{24}=\frac{12}{24}=\frac{1}{2}$$

$\Rightarrow \theta = 60°$

40. (a) $\because l^2 + m^2 + n^2 = 1$

$\Rightarrow \cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$ ...(i)

$$\Rightarrow \frac{1+\cos2\alpha}{2}+\frac{1+\cos2\beta}{2}+\frac{1+\cos2\gamma}{2}=1$$

$\Rightarrow \cos 2\alpha + \cos 2\beta + \cos 2\gamma = 2 - 3 = -1$

Now, $1 - \sin^2\alpha + 1 - \sin^2\beta + 1 - \sin^2\gamma = 1$

$\Rightarrow \sin^2\alpha + \sin^2\beta + \sin^2\gamma = 2$

Hence, only statement 1 is correct.

41. (a) Let the direction cosines of the line of intersection of the planes $2x - y + 3z = 2$ and $x + y + z = 1$ are l, m, n.

$\because$ This line is perpendicular to the normal of the both planes.

$\Rightarrow 2l - m + 3n = 0$ ...(i)

$l + m + n = 0$ ...(ii)

On solving this equations

$$\frac{l}{-2}=\frac{-m}{-5}=\frac{n}{3}=k(\text{say})$$

$\Rightarrow l = -2k, m = 5k, n = 3k$

So, direction cosines $= < -2k, 5k, 3k >$

and direction ratios $= < -2, 5, 4 >$ or $< 2, -5, -3 >$

42. (b) Equation of plane passing through the intersection of the given planes is

$(2x - y + 3z - 2) + \lambda(x + y - z - 1) = 0$

$\Rightarrow (2 + \lambda)x + (\lambda - 1)y + (3 - \lambda)z - 2 - \lambda = 0$ ...(i)

This plane passes through point (1, 0, 1)

$\Rightarrow 2 + \lambda + 3 - \lambda - 2 - \lambda = 0$

Hence, required plane (P) is

$5x + 2y - 5 = 0$ {From (i)}

43. (c) Centre of sphere $x^2 + y^2 + z^2 = r^2$ is (0, 0, 0) and radius $= r$

Since, sphere touches the plane P.

$\therefore$ Perpendicular drawn from the centre of sphere on the plane is equal to the radius of the sphere.

$$\Rightarrow r=\left|\frac{5(0)+2(0)-5}{\sqrt{5^2+2^2}}\right|=\frac{5}{\sqrt{29}}$$

44. (d) Given, Q is the image of $P(-2, 1, -5)$ in the plane $3x - 2y + 2z + 1 = 0$

So, direction ratio of PQ are $< 3, -2, 2 >$

Equation of line PQ is

$$\frac{x+2}{3}=\frac{y-1}{-2}=\frac{z+5}{2}=k(\text{say})$$

Let $(3k - 2, -2k + 1, 2k - 5)$ be any point on the PQ.

Now, mid-point of PQ is M

$$\left(\frac{3k-2-2}{2},\frac{-2k+1+1}{2},\frac{2k-5-5}{2}\right)$$

$$=M\left(\frac{3k}{2}-2,-k+1,k-5\right)$$

This point lies on the given plane.

So, $3\left(\frac{3k}{2}-2\right)-2(-k+1)+2(k-5)$

$$\Rightarrow \frac{17}{2}k-17=0\Rightarrow k=2$$

So, coordinates of $Q = (4, -3, -1)$

Mid-point = M $(1, -1, -3)$

Length of $PQ = \sqrt{(-2-4)^2+(1+3)^2+(-5+1)^2}$

$=2\sqrt{17}>8$

Hence, all the three statements are correct.

45. (c) From the solution of Q 44.

Direction ratio of $PQ = < 3, -2, 2 >$

$\because l^2 + m^2 + n^2 = 1$

$\Rightarrow$ Sum of the square of direction cosines of the line segment PQ is unity.

Hence, both statements are correct.

46. (c) Let the direction ratios of the line of intersection of given planes $x + y + z = 1$ and $2x - y - 2z = 3$ are $< a, b, c >$

$\Rightarrow a + b + c = 0$ ...(i)

and $2a - b - 2c = 0$ ...(ii)

Solving (i) and (ii),

$$\frac{a}{-2+1}=\frac{-b}{-2-2}=\frac{c}{-1-2}$$

$$\Rightarrow \frac{a}{-1}=\frac{-b}{-4}=\frac{c}{-3}$$

$\Rightarrow < a, b, c > = < 1, -4, 3 >$

47. (a) Now equation of line L having direction ratios $< 1, -4, 3 >$ and passing from $P(5, -6, 7)$ is

$$\frac{x-5}{1}=\frac{y+6}{-4}=\frac{z-7}{3}.$$

48. (b) The given line has direction cosines $< 0, 1, 0 >$.

Clearly, the given line is parallel to γ-axis.

49. (c) Let the required point is $P(x, y, z)$.

Given, points are $O(0, 0, 0)$, $A(a, 0, 0)$, $B(0, b, 0)$ and $(0, 0, c)$.

Now, $\because PO = PA = PB = PC$

Let $PO^2 = PA^2$

$\Rightarrow x^2 + y^2 + z^2 = (a - x)^2 + y^2 + z^2$

$\Rightarrow x^2 = a^2 + x^2 - 2ax$

$\Rightarrow x = \dfrac{a}{2}$

Similarly from $PO = PB$ and $PO = PC$, we get

$y = \dfrac{b}{2}$ and $z = \dfrac{c}{2}$

So, required point is $\left(\dfrac{a}{2}, \dfrac{b}{2}, \dfrac{c}{2}\right)$.

50. (c) Given points are $P(3, 2, 4)$, $Q(4, 5, 2)$, $r(5, 8, 0)$ and $S(2, -1, 6)$.

Direction ratios of $PQ = < 4 - 3, 5 - 2, 2 - 4 >$
$= < 1, 3, -2 >$

Direction ratios of $QP = < 5 - 4, 8 - 5, 0 - 2 >$
$= < 1, 3, -2 >$

Direction ratios of $RS = < 2 - 5, -1 - 8, 6 - 9 >$
$= < 1, 3, -2 >$

Direction ratios of $SP = < 3 - 2, 2 + 1, 4 - 6 >$
$= < 1, 3, -2 >$

$\because$ Direction ratios of all the lines are same.

Hence, all points are collinear.