CHAPTER 25

Differentiation

Exercise

1. $\frac{d}{dx}[\sin^{-1}(\cos x)]$ is
   (a) -1 (b) 1
   (c) 0 (d) None of these

2. If $y = \sqrt{\frac{x}{a}} + \sqrt{\frac{a}{x}}$, then $\frac{dy}{dx} \times 2xy$ is
   (a) $\frac{x^2}{a^2}$ (b) $\left(\frac{x}{a} - \frac{a}{x}\right)$
   (c) $\sqrt{a}$ (d) None of these

3. If $y = \operatorname{cosec}^{-1}\frac{\sqrt{x}+1}{\sqrt{x}-1} + \cos^{-1}\frac{\sqrt{x}-1}{\sqrt{x}+1}$, then $\frac{dy}{dx}$ is equal to
   (a) $\frac{-2}{(\sqrt{x}+1)^2}$ (b) $\frac{-2}{(\sqrt{x}+1)^2}$
   (c) $\frac{1}{x-1}$ (d) 0

4. $f(x) = \frac{x+e^x}{1+\log x}$, then $f'(x)$ is
   (a) $x \log x\,(1 + e^x)$
   (b) $\frac{x\log x \cdot (1+e^x) - e^x(1-x)}{x(1+\log x)^2}$
   (c) $\frac{-x\log x}{(1+\log x)^2}$
   (d) $\frac{-x\log x}{(1+\log x)^2}$

5. $f(x) = e^{\sin^{-1}2x}$, then $f'(x)$ is
   (a) $\frac{2}{\sqrt{1-4x^2}} . e^{\sin^{-1}2x}$ (b) $2 \cdot e^{\sin^{-1}2x}$
   (c) $\frac{2}{e^{\cos^{-1}x}}$ (d) None of these

6. If $y = \tan^{-1}\left(\frac{5x}{1-6x^2}\right)$, then $\frac{dy}{dx}$ is
   (a) $\frac{3}{1+9x^2} + \frac{2}{1+4x^2}$ (b) $\tan 5x$
   (c) $\frac{5}{1-6x^2}$ (d) None of these

7. If $y = \sec(\tan^{-1} x)$, then $\frac{dy}{dx}$ is
   (a) $\frac{x}{\sqrt{(1-x^2)}}$ (b) $x\sqrt{(1+x^2)}$
   (c) $\frac{1}{\sqrt{(1+x^2)}}$ (d) $\frac{1}{(1+x^2)}$

8. $\frac{d}{dx}\left[\tan^{-1}\left(\frac{\cos x}{1+\sin x}\right)\right]$ equals to
   (a) $\frac{1}{2}$ (b) $-\frac{1}{2}$
   (c) 1 (d) -1

9. If $f(x) = \log_{x^2}(\log x)$, then $f'(x)$ at $x = e$ is
   (a) 0 (b) 1
   (c) $\frac{1}{e}$ (d) $\frac{1}{2e}$

10. The differential coefficient of $f(\log x)$ with respect to x, where $f(x) = \log x$ is
    (a) $\frac{x}{\log x}$ (b) $\frac{\log x}{x}$
    (c) $(x \log x)^{-1}$ (d) None of these

11. The derivative of the function
    $f(x) = \cot^{-1}[(\cos 2x)^{1/2}]$ at $x = \frac{\pi}{6}$ is

(a) $\left(\frac{2}{3}\right)^{1/2}$ (b) $\left(\frac{1}{3}\right)^{1/2}$
(c) $3^{1/2}$ (d) $6^{1/2}$

12. If $\sin(x+y) = \log(x+y)$, then $\frac{dy}{dx}$ is equal to
(a) 2 (b) -2
(c) 1 (d) -1

13. If $x = at^2$, $y = 2at$, then $\frac{d^2y}{dx^2}$ is
(a) $-\frac{1}{t^2}$ (b) $\frac{1}{2at^3}$
(c) $-\frac{1}{t^3}$ (d) $-\frac{1}{2at^3}$

14. If $f(x) = |\cos x|$, then $f'\left(\frac{3\pi}{4}\right)$ is
(a) $-\frac{1}{\sqrt{2}}$ (b) $\frac{1}{\sqrt{2}}$
(c) 1 (d) None of these

15. If $y = \frac{1}{2}\log\left(\frac{1-\cos 2x}{1+\cos 2x}\right)$, then $\frac{dy}{dx}$ is equal to
(a) $\cos 2x$ (b) $2\,\text{cosec}\, 2x$
(c) $\log x$ (d) None of these

16. If $y = \frac{e^{2x}+e^{-2x}}{e^{2x}-e^{-2x}}$, then $\frac{dy}{dx}$ is
(a) $-\frac{8}{(e^{2x}-e^{-2x})^2}$ (b) $-e^{2x}$
(c) e^{2x} (d) None of these

17. The derivative of $\tan^{-1}\left\{\frac{1+x}{1-x}\right\}$ is
(a) $\frac{2}{1+x^2}$ (b) $\frac{2}{1-x^2}$
(c) $\frac{1}{\sqrt{(1-x^2)}}$ (d) $\frac{1}{(1+x^2)}$

18. If $y = a^{x^{a^{x\ldots\infty}}}$, then $x(1 - y\log x \log y)\frac{dy}{dx}$ is
(a) $y^2 \log y$ (b) $y \log y$
(c) $\frac{y^2}{\log y}$ (d) None of these

19. If $y = \sqrt{\log x + \sqrt{\log x + \sqrt{\log x + \ldots\infty}}}$, then $\frac{dy}{dx}$ is
(a) $x \log x$ (b) $2xy$
(c) $\frac{1}{x(2y-1)}$ (d) None of these

20. The value of $\frac{d}{dx}\left[\tan^{-1}\left\{\frac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}\right\}\right]$ is
(a) 0 (b) 1/2
(c) 1 (d) None of these

21. If $\sin y = x\cos(a+y)$, then the value of $\frac{dy}{dx}$ at $x = 0$, is
(a) $\cos a$ (b) $\cos y$
(c) $\sin^2 a$ (d) None of these

22. If $y = [x+\sqrt{x^2+a^2}]^n$, then $\frac{dy}{dx}$ is
(a) $n[x+\sqrt{x^2+a^2}]$ (b) $\frac{ny}{\sqrt{x^2+a^2}}$
(c) $x^{2n} + a^{2n}$ (d) None of these

23. The differential coefficient of $a^{\log_{10}\text{cosec}^{-1}x}$ is
(a) $\frac{a^{\log_{10}(\text{cosec}^{-1}x)}}{\text{cosec}^{-1}x}\cdot\frac{1}{x\sqrt{x^2-1}}\log_{10} a$
(b) $-\frac{a^{\log_{10}(\text{cosec}^{-1}x)}}{\text{cosec}^{-1}x}\cdot\frac{1}{|x|\sqrt{x^2-1}}\log_{10} a$
(c) $\frac{-a^{\log_{10}(\text{cosec}^{-1}x)}}{\text{cosec}^{-1}x}\cdot\frac{1}{|x|\sqrt{x^2-1}}\log_a 10$
(d) $\frac{a^{\log_{10}(\text{cosec}^{-1}x)}}{\text{cosec}^{-1}x}\cdot\frac{1}{x\sqrt{x^2-1}}\log_a 10$

24. If $x = \cos t$ and $y = \sin t$, then $\frac{dy}{dx}$ at $t = \frac{2\pi}{3}$, is equal to
(a) $\frac{1}{\sqrt{3}}$ (b) $\frac{\sqrt{3}}{2}$
(c) 0 (d) None of these

25. If $x = a(1-\cos\theta)$ and $y = a(\theta+\sin\theta)$, then $\frac{dy}{dx}$ at $\theta = \frac{\pi}{2}$ is
(a) 0 (b) 1
(c) -2 (d) None of these

26. The derivative of $\sin^{-1}\sqrt{1-x^2}$ with respect to $\cot^{-1}\left(\frac{x}{\sqrt{1-x^2}}\right)$ is
(a) $\sqrt{1-x^2}$ (b) 1
(c) x^3 (d) None of these

27. Derivative of $\tan^{-1}\left(\frac{\sqrt{1-x^2}}{x}\right)$ with respect to $\cos^{-1}(2x\sqrt{1-x^2})$ is
(a) 0 (b) $\frac{x}{2}$
(c) $-\frac{1}{2}$ (d) None of these

28. If $f(x) = x^{1/x}$, then $f''(e)$ is equal to
(a) $e^{1/e}$ (b) $e^{(1/3)-2}$
(c) $2^{1/(e-3)}$ (d) $-e^{(1/e)-3}$

29. If $y = ax^{n+1} + bx^{-n}$, then $x^2 \dfrac{d^2y}{dx^2}$ is
(a) $n(n-1)y$ (b) $n(n+1)y$
(c) ny (d) n^2y

30. The value of $\dfrac{dy}{dx}$, when $\cos x = \dfrac{1}{\sqrt{1+t^2}}$ and $\sin y = \dfrac{t}{\sqrt{1+t^2}}$, is
(a) -2 (b) -1
(c) 1 (d) 2

31. If $y = 3x - \dfrac{\cos x}{2}$, then $\dfrac{d^2x}{dy^2}$ is
(a) $\dfrac{-2\cos x}{(6+\sin x)^2}$ (b) $\dfrac{-4\cos x}{(6+\sin x)^2}$
(c) $\dfrac{-4\cos x}{(6+\sin x)^3}$ (d) $-\dfrac{4\sin x}{(6+\sin x)^3}$

32. If $y = x + e^x$, then $\dfrac{d^2x}{dy^2}$ is
(a) e^x (b) $-\dfrac{e}{(1+e^x)^3}$
(c) $-\dfrac{x}{(1+e^x)^2}$ (d) $\dfrac{1}{(1+e^x)^2}$

33. If $y = (1-x)(1+x^2)(1+x^4) \ldots\ldots (1+x^{2n})$, then find the value of $\dfrac{dy}{dx}$ at $x = 0$.
(a) -1 (b) $\dfrac{1}{(1+x)^2}$
(c) $\dfrac{x}{(1+x)^2}$ (d) $\dfrac{x}{(1-x)^2}$

34. If $3f(x) - 2f\left(\dfrac{1}{x}\right) = x$, then find the value of $f'(2)$.
(a) $\dfrac{2}{7}$ (b) $\dfrac{1}{2}$
(c) 2 (d) $\dfrac{7}{2}$

35. If $f(x) = |\cos x - \sin x|$, then $f'\left(\dfrac{\pi}{2}\right)$ is
(a) 1 (b) -1
(c) 0 (d) None of these

36. If $x = a \sin 2\theta (1 + \cos 2\theta)$,
$y = b \cos 2\theta (1 - \cos 2\theta)$, then $\dfrac{dy}{dx}$ is
(a) $\dfrac{b}{a}\tan\theta$ (b) $\dfrac{a}{b}\tan\theta$
(c) $\dfrac{b}{a}\cot\theta$ (d) None of these

37. If $x^{16} \cdot y^9 = (x^2 + y)^{17}$, tsyhen $x \cdot \dfrac{dy}{dx}$ is
(a) $2y$ (b) x^3y^7
(c) $-\dfrac{1}{2}$ (d) 0

38. If $y = x^y$, then $x(1 - y\log x) \cdot \dfrac{dy}{dx}$ is
(a) x^2 (b) y^2
(c) xy^2 (d) None of these

39. Find the derivative of
$y = (1-x)(2-x)(3-x) \ldots\ldots (n-x)$ at $x = 1$
(a) 0 (b) $n! - 1$
(c) $(-1)(n-1)!$ (d) $(-1)^{n-1}(n-1)!$

40. Differential coefficient of $\log_{10} x$ with respect to $\log_x 10$ is
(a) $-\dfrac{(\log x)^2}{(\log 10)^2}$ (b) $\dfrac{(\log_{10} x)^2}{(\log 10)^2}$
(c) $\dfrac{\log_x 10}{\log 10}$ (d) $-\dfrac{(\log 10)^2}{(\log x)^2}$

41. If $x\sqrt{(1+y)} + y\sqrt{(1+x)} = 0$, then dy/dx is
(a) $1/(1+x)^2$ (b) $-1/(1+x)^2$
(c) $1/(1+x)$ (d) None of these

42. If $y = f(x) = \left(\dfrac{\tan^m x}{\tan^n x}\right)^{m+n} \left(\dfrac{\tan^n x}{\tan^p x}\right)^{n+p} \left(\dfrac{\tan^p x}{\tan^m x}\right)^{p+m}$, then $\dfrac{dy}{dx}$ is
(a) 0 (b) 1
(c) $\tan^{m+n+p} x$ (d) None of these

43. If $y = \sin^n x \cos nx$, then $\dfrac{dy}{dx}$ is
(a) $n \sin^{n-1} x \cos(n+1)x$
(b) $n \sin^{n-1} x \sin(n+1)x$
(c) $n \sin^{n-1} x \cos(n-1)x$
(d) $n \sin^{n-1} x \cos nx$

44. If $2^x + 2^y = 2^{x+y}$, then $\dfrac{dy}{dx}$ is equal to
(a) $(2^x + 2^y)/(2^x - 2^y)$
(b) $(2^x + 2^y)/(1 + 2^{x+y})$
(c) $2^{x-y} \cdot \dfrac{2^y - 1}{1 - 2^x}$
(d) $(2^{x+y} - 2^x)/2^y$

45. If $f(x) = \cos x \cos 2x \cos 4x \cos 8x \cos 16x$, then $f'(\pi/4)$ is
(a) 1 (b) $\sqrt{2}$
(c) $1/\sqrt{2}$ (d) None of these

46. If $y = \log_7(\log_7 x)$, then $\dfrac{dy}{dx}$ is
(a) $\dfrac{1}{\log 7 \cdot x \log x}$ (b) $\dfrac{\log 7}{\log x}$
(c) $\dfrac{x \log x}{\log 7}$ (d) None of these

47. The value of $\frac{d}{dx}\left[\sin^2 \cot^{-1}\sqrt{\frac{1-x}{1+x}}\right]$ is

(a) $-\frac{1}{2}$ (b) $\frac{1}{2}$
(c) 1 (d) – 1

48. If $x = 2 \log \cot t$ and $y = \tan t + \cot t$, then $\frac{dy}{dx} \sin 2t + 1$ is
(a) $\cos^2 t$ (b) $\sin^2 t$
(c) $\cos 2t$ (d) $2 \cos^2 t$

49. If $x^y \cdot y^x = 1$, then $\frac{dy}{dx}$ is

(a) $\frac{y(y + x\log y)}{x(y\log x + x)}$ (b) $\frac{y(x + y\log x)}{x(y + x\log y)}$
(c) $-\frac{y(y + x\log y)}{x(x + y\log x)}$ (d) None of these

50. If $y = \sqrt{x}^{\sqrt{x}^{\sqrt{x}^{\sqrt{x}\ldots\infty}}}$, then $(2 - y \log x)\frac{dy}{dx}$ is

(a) xy^2 (b) $\frac{y^2}{x}$
(c) y^2 (d) None of these

51. If $2f(\sin x) + f(\cos x) = x$, then $\frac{d}{dx}f(x)$ is

(a) $\frac{1}{\sqrt{1-x^2}}$ (b) $\sin x + \cos x$
(c) 2 (d) None of these

52. If $\sqrt{(1-x^2)} + \sqrt{(1-y^2)} = a(x - y)$, then $\frac{dy}{dx}$ is

(a) $\sqrt{\frac{1-y^2}{1-x^2}}$ (b) $\sqrt{\frac{1-x^2}{1-y^2}}$
(c) $\sqrt{(1-x^2)(1-y^2)}$ (d) None of these

53. $\frac{d}{dx}\left[\tan^{-1}\left\{\frac{a\cos x - b\sin x}{b\cos x + a\sin x}\right\}\right]$ is
(a) –1 (b) 0
(c) $\frac{1}{2}$ (d) None of these

54. If $y = e^{2x}$, then $\frac{d^2y}{dx^2}\cdot\frac{d^2x}{dy^2}$ is equal to
(a) e^{-2x} (b) $-2e^{-2x}$
(c) $2e^{-2x}$ (d) 1

55. If $y = \tan^{-1}[\{\sqrt{1+x^2} - 1\}/x]$, then
(a) $y'(0) = 1$ (b) $y'(0) = 1/2$
(c) $y'(0) = 0$ (d) None of these

56. Let $f(x)$ and $g(x)$ be twice differentiable functions on $[0, 2]$ satisfying $f''(x) = g''(x)$, $f'(1) = 4$, $g'(1) = 6$, $f(2) = 3$ and $g(2) = 9$, then what is $f(x) - g(x)$ at $x = 4$ equal to? **[NDA-I 2016]**
(a) –10 (b) – 6
(c) – 4 (d) 2

Directions (Q. 57 and Q. 58):
Consider the function

$$f(x) = \begin{vmatrix} x^3 & \sin x & \cos x \\ 6 & -1 & 0 \\ p & p^2 & p^3 \end{vmatrix}, \text{ where } p \text{ is constant.}$$

57. What is the value of $f'(0)$? **[NDA-I 2016]**
(a) p^3 (b) $3p^3$
(c) $6p^3$ (d) $-6p^3$

58. What is the value of p for which $f''(0) = 0$? **[NDA-I 2016]**

(a) $-\frac{1}{6}$ or 0 (b) – 1 or 0
(c) $-\frac{1}{6}$ or 1 (d) – 1 or 1

Directions (Q. 59 to Q. 62):
Let $f: R \to R$ be a function such that
$f(x) = x^3 + x^2 f'(1) + x f''(2) + f'''(3)$ for $x \in R$

59. What is $f(1)$ equal to? **[NDA-I 2016]**
(a) – 2 (b) – 1
(c) 0 (d) 4

60. What is $f'(1)$ equal to? **[NDA-I 2016]**
(a) – 6 (b) – 5
(c) 1 (d) 0

61. What is $f'''(10)$ equal to? **[NDA-I 2016]**
(a) 1 (b) 5
(c) 6 (d) 8

62. Consider the following **[NDA-I 2016]**
1. $f(2) = f(1) - f(0)$
2. $f''(2) - 2f'(1) = 12$

Which of the above is/are correct?
(a) Only 1 (b) Only 2
(c) Both 1 and 2 (d) Neither 1 nor 2

Directions (Q. 63 and Q. 64):
Consider the function $f(x) = |x^2 - 5x + 6|$

63. What is $f'(4)$ equal to? **[NDA-I 2016]**
(a) – 4 (b) – 3
(c) 3 (d) 2

64. What is $f''(2.5)$ equal to? **[NDA-I 2016]**
(a) – 3 (b) – 2
(c) 0 (d) 2

65. If $y = \log_{10} x + \log_x 10 + \log_x x + \log_{10} 10$, then what is $\left(\frac{dy}{dx}\right)_{x=10}$ equal to? **[NDA-I 2016]**
(a) 10 (b) 2
(c) 1 (d) 0

Directions (Q. 66 to Q. 68):
Let $f(x) = \{|x| - |x - 1|\}^2$

66. What is $f'(x)$ equal to when $x > 1$? **[NDA-II 2016]**
(a) 0 (b) $2x - 1$
(c) $4x - 2$ (d) $8x - 4$

67. What is $f'(x)$ equal to when $0 < x < 1$? **[NDA-II 2016]**
(a) 0 (b) $2x - 1$
(c) $4x - 2$ (d) $8x - 4$

68. Which of the following equations is/are correct?
1. $f(-2) = f(5)$
2. $f''(-2) + f''(0.5) + f''(3) = 4$
Select the correct answer using the code given below.
(a) Only 1 (b) Only 2
(c) Both 1 and 2 (d) Neither 1 nor 2

69. Let $f(x+y) = f(x)f(y)$ for all x and y. Then, what is $f'(5)$ equal to [where $f'(x)$ is the derivative of $f(x)$]? **[NDA-I 2017]**
(a) $f(5)f'(0)$ (b) $f(5) - f'(0)$
(c) $f(5)f(0)$ (d) $f(5) + f''(0)$

70. What is the derivative of $\log_{10}(5x^2+3)$ with respect to x? **[NDA-I 2017]**
(a) $\frac{x\log_{10} e}{5x^2+3}$ (b) $\frac{2x\log_{10} e}{5x^2+3}$
(c) $\frac{10x\log_{10} e}{5x^2+3}$ (d) $\frac{10x\log_{10} 10}{5x^2+3}$

ANSWERS

1.	(a)	**2.**	(b)	**3.**	(d)	**4.**	(b)	**5.**	(a)	**6.**	(a)	**7.**	(a)	**8.**	(b)	**9.**	(d)	**10.**	(c)
11.	(a)	**12.**	(d)	**13.**	(d)	**14.**	(b)	**15.**	(b)	**16.**	(a)	**17.**	(d)	**18.**	(a)	**19.**	(c)	**20.**	(b)
21.	(a)	**22.**	(b)	**23.**	(b)	**24.**	(a)	**25.**	(b)	**26.**	(b)	**27.**	(c)	**28.**	(d)	**29.**	(b)	**30.**	(c)
31.	(c)	**32.**	(b)	**33.**	(a)	**34.**	(b)	**35.**	(a)	**36.**	(a)	**37.**	(a)	**38.**	(b)	**39.**	(c)	**40.**	(a)
41.	(b)	**42.**	(a)	**43.**	(b)	**44.**	(c)	**45.**	(b)	**46.**	(a)	**47.**	(b)	**48.**	(d)	**49.**	(c)	**50.**	(b)
51.	(a)	**52.**	(a)	**53.**	(a)	**54.**	(b)	**55.**	(b)	**56.**	(a)	**57.**	(d)	**58.**	(a)	**59.**	(d)	**60.**	(b)
61.	(c)	**62.**	(c)	**63.**	(c)	**64.**	(b)	**65.**	(d)	**66.**	(a)	**67.**	(d)	**68.**	(a)	**69.**	(a)	**70.**	(c)

Explanations

1. (a) $\frac{d}{dx}\sin^{-1}(\cos x) = \frac{(-\sin x)}{\sqrt{1-\cos^2 x}} = -1$

2. (b) $y = \sqrt{\frac{x}{a}} + \sqrt{\frac{a}{x}}$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{2\sqrt{ax}} - \frac{1}{2x}\sqrt{\frac{a}{x}}$$

$$\Rightarrow 2xy\frac{dy}{dx} = \left[\frac{x}{\sqrt{ax}} - \sqrt{\frac{a}{x}}\right]\left[\sqrt{\frac{x}{a}} + \sqrt{\frac{a}{x}}\right]$$

$$= \left[\sqrt{\frac{x}{a}} - \sqrt{\frac{a}{x}}\right]\left[\sqrt{\frac{x}{a}} + \sqrt{\frac{a}{x}}\right] = \frac{x}{a} - \frac{a}{x}$$

3. (d) $y = \operatorname{cosec}^{-1}\frac{\sqrt{x}+1}{\sqrt{x}-1} + \cos^{-1}\frac{\sqrt{x}-1}{\sqrt{x}+1}$

$$\Rightarrow y = \sin^{-1}\frac{\sqrt{x}-1}{\sqrt{x}+1} + \cos^{-1}\frac{\sqrt{x}-1}{\sqrt{x}+1} = \frac{\pi}{2} \Rightarrow \frac{dy}{dx} = 0$$

4. (b) $f(x) = \frac{x+e^x}{1+\log x}$

$$f'(x) = \frac{(1+\log x)(1+e^x) - (x+e^x)\left(\frac{1}{x}\right)}{(1+\log x)^2}$$

$$= \frac{x\log x(1+e^x) - e^x(1-x)}{x(1+\log x)^2}$$

5. (a) $f(x) = e^{\sin^{-1}2x}$

$$\Rightarrow f'(x) = e^{\sin^{-1}2x}.\left(\frac{1}{\sqrt{1-4x^2}}\right)(2) = \frac{2e^{\sin^{-1}2x}}{\sqrt{1-4x^2}}$$

6. (a) $y = \tan^{-1}\left(\frac{5x}{1-6x^2}\right) = \tan^{-1}\left[\frac{3x+2x}{1-(3x)(2x)}\right]$

$$\Rightarrow y = \tan^{-1}(3x) + \tan^{-1}(2x)$$

$$\Rightarrow \frac{dy}{dx} = \frac{3}{1+9x^2} + \frac{2}{1+4x^2}$$

7. (a) $y = \sec(\tan^{-1}x)$

Let $x = \tan\theta \Rightarrow \frac{dx}{d\theta} = \sec^2\theta$

So, $y = \sec\theta$

$$\Rightarrow \frac{dy}{dx} = \sec\theta\tan\theta\cdot\frac{d\theta}{dx} = \sec\theta, \tan\theta\cdot\frac{1}{\sec^2\theta}$$

$$= \frac{\tan\theta}{\sec\theta} = \frac{x}{\sqrt{1+x^2}}$$

8. (b) $\frac{d}{dx}\left[\tan^{-1}\left(\frac{\cos x}{1+\sin x}\right)\right]$

$$= \frac{d}{dx}\left[\tan^{-1}\left\{\frac{\cos^2\frac{x}{2}-\sin^2\frac{x}{2}}{\left(\cos\frac{x}{2}+\sin\frac{x}{2}\right)^2}\right\}\right]$$

$$= \frac{d}{dx}\left[\tan^{-1}\left\{\frac{1-\tan\frac{x}{2}}{1+\tan\frac{x}{2}}\right\}\right]$$

$$= \frac{d}{dx}\left[\tan^{-1}\left\{\tan\left(\frac{\pi}{4}-\frac{x}{2}\right)\right\}\right]$$

$$= \frac{d}{dx}\left(\frac{\pi}{4}-\frac{x}{2}\right) = -\frac{1}{2}$$

9. (d) $f(x) = \log_{x^2}(\log x) = \frac{1}{2}\log_x(\log x)$

$$\Rightarrow f(x) = \frac{1}{2}\frac{\log(\log x)}{\log x}$$

$$\Rightarrow f'(x) = \frac{1}{2}\left[\frac{\log x\cdot\left(\frac{1}{\log x}\right)\left(\frac{1}{x}\right)-\frac{1}{x}\log(\log x)}{(\log x)^2}\right]$$

$$= \frac{1}{2}\left[\frac{\frac{1}{x}-\frac{1}{x}\log(\log x)}{(\log x)^2}\right]$$

$$f'(e) = \frac{1}{2e}$$

10. (c) $y = f(\log x) \Rightarrow \frac{dy}{dx} = \frac{f'(\log x)}{x}$ $\left\{\begin{array}{l}\because f(x) = \log x\\ \text{So, } f'(x) = \frac{1}{x}\end{array}\right.$

$$\Rightarrow \frac{dy}{dx} = (x\log x)^{-1}$$

11. (a) $y = \cot^{-1}[(\cos 2x)^{1/2}]$

$$\Rightarrow \frac{dy}{dx} = -\left(\frac{1}{1+\cos 2x}\right)\cdot\frac{(-2\sin 2x)}{2\sqrt{\cos 2x}}$$

Now, $\left(\frac{dy}{dx}\right)_{\left(x=\frac{\pi}{6}\right)} = \frac{1\sin\pi/3}{\left(1+\cos\frac{\pi}{3}\right)\sqrt{\cos\frac{\pi}{3}}} = \sqrt{\frac{2}{3}}$

12. (d) $\sin(x+y) = \log(x+y)$

$$\cos(x+y)\left[1+\frac{dy}{dx}\right] = \frac{1}{x+y}\left(1+\frac{dy}{dx}\right)$$

$$\frac{dy}{dx}\left[\cos(x+y)-\frac{1}{x+y}\right] = \frac{1}{x+y}-\cos(x+y)$$

$$\frac{dy}{dx} = -1$$

13. (d) $x = at^2 \Rightarrow dx/dt = 2at$

$y = 2at \Rightarrow dy/dt = 2a$

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{1}{t}$$

and $\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{1}{t}\right) = -\frac{1}{t^2}\cdot\frac{dx}{dt} = -\frac{1}{2at^3}$

14. (b) $f(x) = |\cos x|$

$$f(x) = \begin{cases}\cos x, & 0 < x \le \frac{\pi}{2}\\ -\cos x; & \frac{\pi}{2} < x \le \frac{3\pi}{2}\end{cases}$$

So, $f'(x) = \begin{cases}-\sin x; & 0 < x \le \frac{\pi}{2}\\ \sin x; & \frac{\pi}{2} < x \le \frac{3\pi}{2}\end{cases}$

$$\therefore f'\left(\frac{3\pi}{4}\right) = \sin\frac{3\pi}{4} = \frac{1}{\sqrt{2}}$$

15. (b) $y = \frac{1}{2}\log\left[\frac{1-\cos 2x}{1+\cos 2x}\right]$

$$y = \frac{1}{2}\log\left[\frac{2\sin^2 x}{2\cos^2 x}\right] = \log\tan x$$

$$\Rightarrow \frac{dy}{dx} = \frac{\sec^2 x}{\tan x} = \frac{1}{\sin x\cos x} = \frac{2}{\sin 2x} = 2\operatorname{cosec} 2x$$

16. (a) $y = \frac{e^{2x}+e^{-2x}}{e^{2x}-e^{-2x}}$

$$\Rightarrow \frac{dy}{dx} = \frac{(e^{2x}-e^{-2x})(2e^{2x}-2e^{-2x}) - (e^{2x}+e^{-2x})(2e^{2x}+2e^{-2x})}{(e^{2x}-e^{-2x})^2}$$

$$= \frac{-8}{(e^{2x}-e^{-2x})^2}$$

17. (d) $y = \tan^{-1}\left(\frac{1+x}{1-x}\right)$ {Let $x = \tan\theta$}

$$\Rightarrow y = \tan^{-1}\left(\frac{1+\tan\theta}{1-\tan\theta}\right)$$

$$= \left(\tan\left(\frac{\pi}{4}+\theta\right)\right) = \frac{\pi}{4}+\theta$$

$$\Rightarrow \frac{dy}{dx} = \frac{d}{dx}\tan^{-1}x = \frac{1}{1+x^2}$$

18. (a) $y = a^{x^y} \Rightarrow \log y = x^y\log a$

$\Rightarrow \log(\log y) = y\log x + \log(\log a)$

$$\Rightarrow \frac{1}{\log y}\frac{1}{y}\frac{dy}{dx} = \frac{y}{x} + \log x\frac{dy}{dx}$$

$$\frac{dy}{dx}\left[\frac{1}{y\log y}-\log x\right] = \frac{y}{x}$$

or $x(1 - y\log x\log y)\frac{dy}{dx} = y^2\log y$

19. (c) $y = \sqrt{\log x + y} \Rightarrow y^2 = \log x + y$

$$2y\frac{dy}{dx} = \frac{1}{x} + \frac{dy}{dx} \Rightarrow \frac{dy}{dx} = \frac{1}{x(2y-1)}$$

20. (b) $\frac{d}{dx}\left[\tan^{-1}\left\{\frac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}\right\}\right]$

$$= \frac{d}{dx}\tan^{-1}\left\{\frac{\sin\frac{x}{2}+\cos\frac{x}{2}+\sin\frac{x}{2}-\cos\frac{x}{2}}{\sin\frac{x}{2}+\cos\frac{x}{2}-\sin\frac{x}{2}+\cos\frac{x}{2}}\right\}$$

$$= \frac{d}{dx}\tan^{-1}\left(\tan\frac{x}{2}\right) = \frac{d}{dx}\left(\frac{x}{2}\right) = \frac{1}{2}$$

21. (a) $\sin y = x\cos(a+y) \Rightarrow x = \frac{\sin y}{\cos(a+y)}$

$$\Rightarrow \frac{dx}{dy} = \frac{\cos(a+y)\cos y + \sin(a+y)\sin y}{\cos^2(a+y)}$$

$$= \frac{\cos a}{\cos^2(a+y)} \Rightarrow \frac{dy}{dx} = \frac{\cos^2(a+y)}{\cos a}$$

So, $\left(\frac{dy}{dx}\right)_{(x=0)} = \frac{\cos^2 a}{\cos a} = \cos a$

22. (b) $y = [x+\sqrt{x^2+a^2}]^n$

$$\Rightarrow \frac{dy}{dx} = n[x+\sqrt{x^2+a^2}]^{n-1}\left[1+\frac{2x}{2\sqrt{x^2+a^2}}\right]$$

$$= \frac{n[x+\sqrt{x^2+a^2}]^n}{\sqrt{x^2+a^2}} = \frac{ny}{\sqrt{x^2+a^2}}$$

23. (b) $y = a^{\log_{10}\operatorname{cosec}^{-1}x}$

$$\frac{dy}{dx} = a^{\log_{10}\operatorname{cosec}^{-1}x}.\log a\frac{d}{dx}(\log_{10}\operatorname{cosec}^{-1}x)$$

$$=_a \log_{10}\operatorname{cosec}^{-1}x.\log a\frac{d}{dx}\left(\frac{\log\operatorname{cosec}^{-1}x}{\log 10}\right)$$

$$= a^{\log_{10}\operatorname{cosec}^{-1}x}.\left(\frac{\log a}{\log 10}\right)\left(\frac{1}{\operatorname{cosec}^{-1}x}\right)\left(\frac{-1}{|x|\sqrt{x^2-1}}\right)$$

$$= \frac{-a^{\log_{10}\operatorname{cosec}^{-1}x}}{\operatorname{cosec}^{-1}x}.\frac{1}{|x|\sqrt{x^2-1}}.\log_{10}a$$

24. (a) $x = \cos t, y = \sin t$

$$\Rightarrow \frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{\cos t}{-\sin t}$$

$$\left(\frac{dy}{dx}\right)_{\left(t=\frac{2\pi}{3}\right)} = -\cot\frac{2\pi}{3} = \frac{1}{\sqrt{3}}$$

25. (b) $x = a(1-\cos\theta)$ and $y = a(\theta+\sin\theta)$

$$\Rightarrow \frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = \frac{1+\cos\theta}{\sin\theta}$$

$$\left(\frac{dy}{dx}\right)_{\left(\theta=\frac{\pi}{2}\right)} = 1$$

26. (b) $u = \sin^{-1}\sqrt{1-x^2}, v = \cot^{-1}\left(\frac{x}{\sqrt{1-x^2}}\right)$

Let $x = \cos\theta$, then

$u = \sin^{-1}(\sin\theta) = \theta$ and $v = \cot^{-1}\left(\frac{\cos\theta}{\sin\theta}\right) = \theta$

$$\frac{du}{dv} = \frac{du/d\theta}{dv/d\theta} = 1$$

27. (c) $u = \tan^{-1}\left(\frac{\sqrt{1-x^2}}{x}\right), v = \cos^{-1}(2x\sqrt{1-x^2})$

Let $x = \cos\theta$

$\Rightarrow u = \tan^{-1}\left(\frac{\sin\theta}{\cos\theta}\right)$, $v = \cos^{-1}\{2\cos\theta\sin\theta\}$

$\Rightarrow u = \theta$, $v = \cos^{-1}\{(\cos(90^\circ - 2\theta)\}$

$\Rightarrow v = 90^\circ - 2\theta$

So, $\frac{du}{dv} = \frac{\frac{du}{d\theta}}{\frac{dv}{d\theta}} = -\frac{1}{2}$

28. (d) $y = f(x) = x^{1/x} \Rightarrow \log y = \frac{\log x}{x}$

$$\Rightarrow \frac{1}{y}\frac{dy}{dx} = \frac{1-\log x}{x^2} \Rightarrow \frac{dy}{dx} = y\left[\frac{1-\log x}{x^2}\right]$$

$$\frac{d^2y}{dx^2} = y\left[\frac{x^2\left(-\frac{1}{x}\right)-(1-\log x)(2x)}{x^4}\right] + \left(\frac{1-\log x}{x^2}\right)\frac{dy}{dx}$$

$$= y\left[\frac{2\log x - 3}{x^3}\right] + \left[\frac{1-\log x}{x^2}\right]^2 \cdot y$$

$$f''(e) = e^{1/e}\left[\frac{2\log e - 3}{e^3}\right] + \left[\frac{1-\log e}{e^2}\right].e^{1/e}$$

$= -e^{1/e-3}$

29. (b) $y = ax^{n+1} + bx^{-n}$

$$\Rightarrow \frac{dy}{dx} = (n+1)ax^n - nbx^{-n-1}$$

$$\Rightarrow \frac{d^2y}{dx^2} = n(n+1)ax^{n-1} + n(n+1)bx^{-n-2}$$

$$\Rightarrow x^2\frac{d^2y}{dx^2} = n(n+1)ax^{n+1} + n(n+1)bx^{-n}$$

$= n(n+1)y$

30. (c) $\sin y = \dfrac{t}{\sqrt{1+t^2}}, \cos x = \dfrac{1}{\sqrt{1+t^2}}$

Put $t = \tan\theta$

$\Rightarrow \sin y = \dfrac{\tan\theta}{\sec\theta}, \cos x = \dfrac{1}{\sec\theta}$

$\Rightarrow \sin y = \sin\theta, \cos x = \cos\theta$

$\Rightarrow y = \theta, x = \theta$

$\Rightarrow \dfrac{dy}{dx} = \dfrac{dy/d\theta}{dx/d\theta} = 1$

31. (c) $y = 3x - \dfrac{\cos x}{2}$

$\Rightarrow \dfrac{dy}{dx} = 3 + \dfrac{\sin x}{2} \Rightarrow \dfrac{dy}{dx} = \dfrac{2}{6+\sin x}$

$\Rightarrow \dfrac{d^2x}{dy^2} = \dfrac{-2\cos x}{(6+\sin x)^2}\cdot\dfrac{dx}{dy} = \dfrac{-4\cos x}{(6+\sin x)^3}$

32. (b) $y = x + e^x$

$\dfrac{dy}{dx} = 1 + e^x \Rightarrow \dfrac{dx}{dy} = \dfrac{1}{1+e^x}$

$\dfrac{d^2x}{dy^2} = \dfrac{d}{dy}\left(\dfrac{dx}{dy}\right) = \dfrac{d}{dx}\left(\dfrac{1}{1+e^x}\right)\left(\dfrac{dx}{dy}\right)$

$= \dfrac{-e^x}{(1+e^x)^2} \times \dfrac{1}{1+e^x}$

$= \dfrac{-e^x}{(1+e^x)^3}$

33. (a) $y = (1-x)(1+x^2)(1+x^4).....(1+x^{2n})$

or $y = \dfrac{\{(1+x)(1-x)\}(1+x^2)(1+x^4)...(1+x^{2n})}{(1+x)}$

$\Rightarrow y = \dfrac{(1-x^2)(1+x^2)(1+x^4)...(1+x^{2n})}{1+x}$

So, $y = \dfrac{1-x^{4n}}{1+x}$

$\Rightarrow \dfrac{dy}{dx} = \dfrac{(1+x)(-4nx^{4n-1}) - (1-x)^{4n}}{(1+x)^2}$

At $x = 0$,

$\dfrac{dy}{dx} = \dfrac{-1}{1} = -1$

34. (b) $3f(x) - 2f\left(\dfrac{1}{x}\right) = x$ …(i)

Replace x by $\dfrac{1}{x}$

$3f\left(\dfrac{1}{x}\right) - 2f(x) = \dfrac{1}{x}$ …(ii)

On solving eqs. (i) and (ii),

$f(x) = \dfrac{1}{5}\left\{3x + \dfrac{2}{x}\right\}$

Then, $f'(x) = \dfrac{1}{5}\left\{3 - \dfrac{2}{x^2}\right\}$

So, $f'(2) = \dfrac{1}{5}\left(3 - \dfrac{1}{2}\right) = \dfrac{1}{2}$

35. (a) $f(x) = |\cos x - \sin x|$

$$f(x) = \begin{cases} \cos x - \sin x, 0 \le x < \dfrac{\pi}{4} \\ \sin x - \cos x, \dfrac{\pi}{4} \le x \le \dfrac{\pi}{2} \end{cases}$$

Then, $f(x) = \sin x - \cos x$ at $x = \dfrac{\pi}{2}$

$\Rightarrow f'(x) = \cos x + \sin x$

$\Rightarrow f'(\pi/2) = \cos\dfrac{\pi}{2} + \sin\dfrac{\pi}{2} = 1$

36. (a) Given $x = a\sin 2\theta(1 + \cos 2\theta) = 2a\sin 2\theta\cos^2\theta$ and $y = b\cos 2\theta(1 - \cos 2\theta) = 2b\cos 2\theta\sin^2\theta$

$\Rightarrow \dfrac{dy}{dx} = \dfrac{dy/d\theta}{dx/d\theta}$

$= \dfrac{2b[2\cos 2\theta\sin\theta\cos\theta - 2\sin 2\theta\sin^2\theta]2a}{2a[-2\sin 2\theta\cos\theta\sin\theta + 2\cos 2\theta\cos^2\theta]}$

$= \dfrac{b}{a}\tan\theta$

37. (a) $x^{16}\cdot y^9 = (x+y)^{17}$

$\Rightarrow 16\log x + 9\log y = 17\log(x^2+y)$

$\Rightarrow \dfrac{16}{x} + \dfrac{9}{y}\dfrac{dy}{dx} = \dfrac{17}{x^2+y}\left[2x + \dfrac{dy}{dx}\right]$

$\Rightarrow \dfrac{dy}{dx}\left[\dfrac{9}{y} - \dfrac{17}{x^2+y}\right] = \dfrac{34x}{x^2+y} - \dfrac{16}{x}$

$\Rightarrow \dfrac{dy}{dx} = \dfrac{2y}{x}$ or $x\dfrac{dy}{dx} = 2y$

38. (b) $y = x^y \Rightarrow \log y = y\log x$

$\Rightarrow \dfrac{1}{y}\dfrac{dy}{dx} = \dfrac{y}{x} + \log x\dfrac{dy}{dx}$

$\Rightarrow \dfrac{dy}{dx}\left[\dfrac{1}{y} - \log x\right] = \dfrac{y}{x}$

$\Rightarrow x(1 - y\log x)\dfrac{dy}{dx} = y^2$

39. (c) $y = (1-x)\{(2-x)(3-x)....(n-x)\}$

$\Rightarrow \dfrac{dy}{dx} = (1-x)\dfrac{d}{dx}\{(2-x)(3-x)...(n-x)\}$

$+ \{(2-x)(3-x)...(n-x)\}\dfrac{d}{dx}(1-x)$

At $x = 1$,

$\dfrac{dy}{dx} = 0 + \{1.2.3. ....(n-1)\}(-1) = (-1)(n-1)!$

40. (a) Let $f = \log_{10} x$ and $g = \log_x 10$

$$\Rightarrow \frac{df}{dg} = \frac{df/dx}{dg/dx} = \frac{\frac{d}{dx}\left(\frac{\log x}{\log 10}\right)}{\frac{d}{dx}\left(\frac{\log 10}{\log x}\right)}$$

$$= \frac{1(\log x)^2 . x}{-x \log 10 \times \log 10} = \frac{-(\log x)^2}{(\log 10)^2}$$

41. (b) Given $x\sqrt{1+y} = -y\sqrt{1+x}$

or $x^2 + x^2y = y^2 + y^2x$

or $(x^2 - y^2) + (x^2y - y^2x) = 0$

or $(x - y)(x + y + xy) = 0$

$\because x \neq y \Rightarrow x + y + xy = 0$

or $y = \frac{-x}{1+x}$

$$\frac{dy}{dx} = \frac{-1}{(1+x)^2}$$

42. (a) $y = \left(\frac{\tan^m x}{\tan^n x}\right)^{m+n}\left(\frac{\tan^n x}{\tan^p x}\right)^{n+p}\left(\frac{\tan^p x}{\tan^m x}\right)^{p+m}$

$\Rightarrow y = (\tan x)^{m^2-n^2}(\tan x)^{n^2-p^2}(\tan x)^{p^2-m^2}$

$\Rightarrow y = (\tan x)^{m^2-n^2+n^2-p^2+p^2-m^2} = (\tan x)^{0} = 1$

$\Rightarrow \frac{dy}{dx} = 0$

43. (b) $y = \sin^n x \cos nx$

$\Rightarrow \frac{dy}{dx} = -n\sin^n x \sin nx + n\sin^{n-1} x \cos x \cos nx$

$= n\sin^{n-1} x[\cos nx \cos x - \sin nx \sin x]$

$= n \sin^{n-1} x \cos(n+1)x$

44. (c) $2^x + 2^y = 2^{x+y}$

$\Rightarrow 2^x \log 2 + 2^y \log 2 \frac{dy}{dx} = 2^{x+y} \log 2\left\{1 + \frac{dy}{dx}\right\}$

$\Rightarrow 2^x - 2^{x+y} = (2^{x+y} - 2y)\frac{dy}{dx}$

$\Rightarrow \frac{dy}{dx} = \frac{2^x[1-2^y]}{2^y[2^x-1]} = \frac{2^{x-y}(2^y-1)}{(1-2^x)}$

45. (b) $f(x) = \cos x \cos 2x \cos 4x \cos 8x \cos 16x$

$= \frac{\sin\{2^5 x\}}{2^5 \sin x} = \frac{\sin 32x}{32 \sin x}$

$$f'(x) = \frac{1}{32}\left[\frac{32\sin x \cos 32x - \sin 32x \cos x}{\sin^2 x}\right]$$

$$f'\left(\frac{\pi}{4}\right) = \frac{1}{32}\left[\frac{32 \times \frac{1}{\sqrt{2}} - 0}{\frac{1}{2}}\right] = \sqrt{2}$$

46. (a) $y = \log_7(\log_7 x) = \frac{\log(\log_7 x)}{\log 7}$

$$\Rightarrow y = \frac{\log\left\{\frac{\log x}{\log 7}\right\}}{\log 7}$$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{\log 7} \times \frac{\log 7}{\log x} \times \frac{1}{x \log 7} = \frac{1}{x \log x \log 7}$$

47. (b) $\frac{d}{dx}\left[\sin^2 \cot^{-1}\sqrt{\frac{1-x}{1+x}}\right]$

Let $x = \cos\theta \Rightarrow \theta = \cos^{-1}x$

So, $\frac{d}{dx}[\sin^2 \cot^{-1}(\tan\theta/2)]$

$$= \frac{d}{dx}\left[\sin^2\left(\frac{\pi}{2} - \frac{\theta}{2}\right)\right] = \frac{d}{dx}\left[\cos^2\frac{\theta}{2}\right]$$

$$= -\frac{2}{2}\cos\frac{\theta}{2}\sin\frac{\theta}{2}\cdot\frac{d\theta}{dx} = -\frac{1}{2}\sin\theta\frac{d}{dx}\cos^{-1}x$$

$$= -\frac{1}{2}\sqrt{1-x^2} \times \left\{\frac{-1}{\sqrt{1-x^2}}\right\} = \frac{1}{2}$$

48. (d) $x = 2 \log \cot t$

$\Rightarrow \frac{dx}{dt} = \frac{-2\,\text{cosec}^2 t}{\cot t} = \frac{-4}{\sin 2t}$

and $y = \tan t + \cot t$

$\Rightarrow \frac{dy}{dt} = \sec^2 t - \text{cosec}^2 t = \frac{-4\cos 2t}{\sin^2 2t}$

$\Rightarrow \frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{\cos 2t}{\sin 2t}$

$\sin 2t\frac{dy}{dx} + 1 = 1 + \cos 2t = 2\cos^2 t$

49. (c) $x^y . y^x = 1$

$\Rightarrow y \log x + x \log y = 0$

$\Rightarrow \frac{y}{x} + \log x\frac{dy}{dx} + \frac{x}{y}\frac{dy}{dx} + \log y = 0$

$$\Rightarrow \frac{dy}{dx} = \frac{-\left(\frac{y}{x} + \log y\right)}{\left(\log x + \frac{x}{y}\right)} = -\frac{y}{x}\left[\frac{y + x\log y}{x + y\log x}\right]$$

50. (b) $y = \sqrt{x}^{\sqrt{x}^{\sqrt{x}\ldots\infty}}$

$\Rightarrow y = (\sqrt{x})^y$

$\Rightarrow \log y = y \log\sqrt{x}$

$\Rightarrow \frac{1}{y}\frac{dy}{dx} = \frac{y}{\sqrt{x}}\cdot\frac{1}{2\sqrt{x}} + \log\sqrt{x}\frac{dy}{dx}$

$\Rightarrow \frac{dy}{dx}\left[\frac{1}{y} - \frac{1}{2}\log x\right] = \frac{y}{2x}$

$\Rightarrow (2 - y\log x)\frac{dy}{dx} = \frac{y^2}{x}$

51. (a) $2f(\sin x)+f(\cos x)=x$ ...(i)

Replace x by $\frac{\pi}{2}-x$

$2f(\cos x)+f(\sin x)=\frac{\pi}{2}-x$ ...(ii)

From Eqs. (i) and (ii),

$f(\sin x)=x-\frac{\pi}{6}$

Let $\sin x=t$

$f(t)=\sin^{-1}t-\frac{\pi}{6}$ or $f(x)=\sin^{-1}(x)-\frac{\pi}{6}$

$\therefore f'(x)=\frac{1}{\sqrt{1-x^2}}$

52. (a) $\sqrt{1-x^2}+\sqrt{1-y^2}=a(x-y)$

Let $x=\sin\alpha$ and $y=\sin\beta$

$\Rightarrow \cos\alpha+\cos\beta=a(\sin\alpha-\sin\beta)$

or $\frac{\cos\alpha+\cos\beta}{\sin\alpha-\sin\beta}=a$ or $\cot\frac{\alpha-\beta}{2}=a$

$\Rightarrow \alpha-\beta=2\cot^{-1}a$

or $\sin^{-1}x-\sin^{-1}y=2\cot^{-1}a$

$\Rightarrow \frac{1}{\sqrt{1-x^2}}-\frac{1}{\sqrt{1-y^2}}\frac{dy}{dx}=0$

$\Rightarrow \frac{dy}{dx}=\frac{\sqrt{1-y^2}}{\sqrt{1-x^2}}$

53. (a) $y=\sec^{-1}\left(\frac{x+1}{x-1}\right)+\sin^{-1}\left(\frac{x-1}{x+1}\right)$

$\Rightarrow y=\cos^{-1}\left(\frac{x-1}{x+1}\right)+\sin^{-1}\left(\frac{x-1}{x+1}\right)$

$\Rightarrow y=\frac{\pi}{2}\Rightarrow \frac{dy}{dx}=0$

54. (b) $y=e^{2x}$

$\Rightarrow \frac{dy}{dx}=2e^{2x}\Rightarrow \frac{dx}{dt}=\frac{1}{2}e^{-2x}$

$\Rightarrow \frac{d^2y}{dx^2}=4e^{2x}$ ...(i)

Now, $\frac{d^2x}{dy^2}=\frac{1}{2}\frac{d}{dy}(e^{-2x})=\frac{1}{2}(-2)e^{-2x}\cdot\frac{dx}{dy}$

$=-e^{-2x}\cdot\frac{1}{2}\cdot e^{-2x}$

$\frac{d^2x}{dy^2}=-\frac{1}{2}e^{-4x}$ ...(ii)

From Eqs. (i) and (ii),

$\frac{d^2y}{dx^2}\times\frac{d^2x}{dy^2}=4e^{2x}\times\frac{-1}{2}e^{-4x}=-2e^{-2x}$

55. (b) $y=\tan^{-1}\frac{\sqrt{1+x^2}-1}{x}$

Let $x=\tan\theta$, $y=\tan^{-1}\left(\frac{\sec\theta-1}{\tan\theta}\right)$

$=\tan^{-1}\left(\tan\frac{\theta}{2}\right)=\frac{\theta}{2}$

$\Rightarrow \frac{dy}{dx}=\frac{1}{2}\frac{d}{dx}(\tan^{-1}x)=\frac{1}{2(1+x^2)}$

$\left(\frac{dy}{dx}\right)_{\text{at } x=0}=\frac{1}{2}$

56. (a) Given $f(x)$ and $g(x)$ be twice differentiable function.

So, let $f(x)=ax+b$ and $g(x)=cx+d$

Given $f'(1)=4$ and $g'(1)=6$

$\Rightarrow a=4$ and $c=6$

$f(2)=3$ and $g(2)=9$

$\Rightarrow 2a+b=3$ and $2c+d=9$

$\Rightarrow b=-5$ and $d=-3$

So, $f(x)=4x-5$ and $g(x)=6x-3$

Then, $f(x)-g(x)=-2x-2$

at $x=4$, $f(x)-g(x)=-10$

57. (d) $f(x)=\begin{vmatrix} x^3 & \sin x & \cos x \\ 6 & -1 & 0 \\ p & p^2 & p^3 \end{vmatrix}$

$$f'(x)=\begin{vmatrix} 3x^2 & \sin x & \cos x \\ 0 & -1 & 0 \\ 0 & p^2 & p^3 \end{vmatrix}+\begin{vmatrix} x^3 & \cos x & \cos x \\ 6 & 0 & 0 \\ p & 0 & p^3 \end{vmatrix}+\begin{vmatrix} x^3 & \sin x & -\sin x \\ 6 & -1 & 0 \\ p & p^2 & 0 \end{vmatrix}$$

So, $f'(x)=-3p^3x^2-6p^3\cos x-6p^2\sin x-p\sin x$

$\Rightarrow f'(0)=-6p^3$

58. (a) $f''(x)=-6p^3x+6p^3\sin x-6p^2\cos x-p\cos x$

$f''(0)=0$

$\Rightarrow -6p^2-p=0$

$\Rightarrow p=0$ or $\frac{-1}{6}$

(Q. Nos. 59-62):

$f(x)=x^3+x^2f'(1)+xf''(2)+f'''(3)$ …(i)

$\Rightarrow f'(x)=3x^2+2xf'(1)+f''(2)$ …(ii)

$\Rightarrow f''(x)=6x+2f'(1)$ …(iii)

$\Rightarrow f'''(x)=6$

$\Rightarrow f'''(3)=6$ and $f'''(10)=6$

So, from eq. (i), $f(x)=x^3+x^2f'(1)+xf''(2)+6$

From eq. (ii), $f'(1)=3+2f'(1)+f''(2)$

or $f''(2)=-3-f'(1)$

From eq. (iii), $f''(2)=12+2f'(1)$

$\Rightarrow -3 - f'(1) = 12 + 2f'(1)$

or $f'(1) = -5$

and then $f''(2) = 12 - 10 = 2$

So, $f(x) = x^3 - 5x^2 + 2x + 6$

$f(1) = 1 - 5 + 2 + 6 = 4$

$\Rightarrow f(2) = 2^3 - 5\cdot 2^2 + 2\cdot 2 + 6 = -2$

and $f'''(10) = 6$

So, $f(1) - f(0) = 4 - 6 = -2 = f(2)$

And $f''(2) - 2f'(1) = 2 - 2 \times (-5) = 12$

(Q. No. Nos. 63 and 64):

$f(x) = |x^2 - 5x + 6| = |(x-2)(x-3)|$

$$f(x) = \begin{cases} x^2 - 5x + 6; & x < 2 \text{ or } x > 3 \\ -x^2 + 5x - 6; & 2 \le x \le 3 \end{cases}$$

$$f'(x) = \begin{cases} 2x - 5; & x < 2 \text{ or } x > 3 \\ 5 - 2x; & 2 \le x \le 3 \end{cases}$$

So, $f'(4) = 2(4) - 5 = 3$

and $f''(x) = -2;\ 2 \le x \le 3$

So, $f''(2.5) = -2$

65. (d) $y = \log_{10} x + \log_x 10 + \log_x x + \log 10x$

$$y = \frac{\log x}{\log 10} + \frac{\log 10}{\log x} + 2$$

$$\frac{dy}{dx} = \frac{1}{\log 10}\cdot\frac{1}{x} + \log 10\left[-\frac{1}{x}\cdot\frac{1}{(\log x)^2}\right] + 0$$

$$= \frac{1}{x\log 10} - \frac{\log 10}{x(\log x)^2}$$

$$\left(\frac{dy}{dx}\right)_{x=10} = \frac{1}{10\log 10} - \frac{\log 10}{10(\log 10)^2}$$

$$= \frac{1}{10\log 10} - \frac{1}{10\log 10} = 0$$

66. (a) $f(x) = \{|x| - |x-1|\}^2$

$$f(x) = \begin{cases} 1 & , \quad x \le 0 \\ (2x-1)^2 & , \quad 0 < x \le 1 \\ 1 & , \quad 1 \le x \end{cases}$$

$$\Rightarrow f'(x) = \begin{cases} 0 & , \quad x \le 0 \\ 4(2x-1)^2 & , \quad 0 < x \le 1 \\ 0 & , \quad 1 < x \end{cases}$$

So, $f'(x) = 0$ when $x > 1$

67. (d) Same as solved in Q. 66

$f'(x) = 4(2x - 1)$ when $0 < x < 1$

$f'(x) = 8x - 4$

68. (a) Same as solved in Q. 66.

$$f''(x) = \begin{cases} 0 & , \quad x \le 0 \\ 8 & , \quad 0 < x \le 1 \\ 0 & , \quad 1 < x \end{cases}$$

$\Rightarrow f(-2) = 1$ and $f(5) = 1$

$\Rightarrow f(-2) = f(5)$

and $f''(-2) = 0, f''(0.5) = 8, f''(3) = 0$

So, $f''(-2) + f''(0.5) + f''(3) = 8 \ne 4$

So, only Statement 1 is correct.

69. (a) $f'(5) = \lim\limits_{h\to 0} \dfrac{f(5+h) - f(5)}{h} = \lim\limits_{h\to 0} \dfrac{f(5)f(h) - f(5)}{h}$

$\{f(x + y) = f(x)\cdot f(y) \Rightarrow f(0) = 1$, so applying L'Hospital rule$\}$

$$\therefore f'(5) = \lim_{h\to 0} \frac{f(5)f'(h)}{1}$$

$$= f(5)f'(0).$$

70. (c) Let $y = \log_{10}(5x^2 + 3)$

$$= \frac{\log(5x^2+3)}{\log 10}$$

$$\frac{dy}{dx} = \frac{1}{\log 10} \times \frac{10x}{5x^2+3}$$

$$= \frac{10x}{(5x^2+3)\log 10} = \frac{10x\log_{10} e}{5x^2+3}$$