CHAPTER 27

Indefinite Integrals

Exercise

1. $\int \frac{\sin x - \cos x}{\sqrt{1-\sin 2x}} e^{\sin x} \cos x \, dx$ is equal to

(a) $e^{\sin x} + c$ (b) $e^{\sin x - \cos x} + c$
(c) $e^{\sin x + \cos x} + c$ (d) $e^{\cos x - \sin x} + c$

2. $\int \frac{\sqrt{\tan x}}{\sin x \cos x} dx$ is equal to

(a) $2\sqrt{\tan x} + c$ (b) $2\sqrt{\cot x} + c$
(c) $\frac{\sqrt{\tan x}}{2} + c$ (d) None of these

3. $\int \frac{1}{(3+x)\sqrt{x}} dx$ is equal to

(a) $\frac{1}{\sqrt{3}} \sin^{-1} \sqrt{\frac{x}{3}} + c$ (b) $\frac{2}{\sqrt{3}} \tan^{-1} \sqrt{\frac{x}{3}} + c$
(c) $\frac{2}{\sqrt{3}} \sinh^{-1} \sqrt{\frac{x}{3}} + c$ (d) None of these

4. $\int \frac{1}{(x+1)\sqrt{x+2}} dx$ is equal to

(a) $\tan^{-1} \sqrt{x+2} + c$
(b) $\log\left(\frac{\sqrt{x+2}-1}{\sqrt{x+2}+1}\right) + c$
(c) $\log\left(\frac{\sqrt{x+2}+1}{\sqrt{x+2}-1}\right) + c$
(d) None of these

5. $\int \sin 2x \cdot \log \cos x \, dx$ is equal to

(a) $\cos^2 x\left(\frac{1}{2} + \log \cos x\right) + k$
(b) $\cos^2 x\left(\frac{1}{2} - \log \cos x\right) + k$
(c) $\cos^2 x \cdot \log \cos x + k$
(d) None of the above

6. If $\int \frac{1}{1+\sin x} dx = \tan\left(\frac{x}{2} + a\right) + b$, then

(a) $a = -\frac{\pi}{4}, b \in R$ (b) $a = \frac{\pi}{4}, b \in R$
(c) $a = \frac{5\pi}{4}, b \in R$ (d) None of these

7. $\int 5^{5^{5^x}} \cdot 5^{5^x} \cdot 5^x \, dx$ is equal to

(a) $\frac{5^{5^x}}{(\log 5)^3} + c$ (b) $5^{5^{5^x}} (\log 3)^5 + c$
(c) $\frac{5^{5^{5^x}}}{(\log 5)^3} + c$ (d) None of these

8. $\int \frac{\sin x}{\sin x - \cos x} dx$ is equal to

(a) $x + \log(\sin x - \cos x) + c$
(b) $\frac{1}{2}[x + \log(\sin x - \cos x)] + c$
(c) $x - \log \sin\left(x + \frac{\pi}{4}\right) + c$
(d) $x + \log \cos\left(x + \frac{\pi}{4}\right) + c$

9. $\int \frac{\sin x}{\sin(x-\alpha)} dx$ is equal to

(a) $x \cos\alpha + (\sin\alpha) \log \sin(x-\alpha) + c$
(b) $x \sin\alpha + (\sin\alpha) \log \sin(x-\alpha) + c$
(c) $x \cos\alpha - (\sin\alpha) \log \sin(x-\alpha) + c$
(d) $x \sin\alpha - (\sin\alpha) \log \sin(x-\alpha) + c$

10. $\int \frac{1}{\sin x+\sqrt{3}\cos x}dx$ is equal to

(a) $\frac{1}{2}\log\tan\left(\frac{1}{2}x-\frac{1}{6}\pi\right)+c$

(b) $\frac{1}{2}\log\left[\operatorname{cosec}\left(x+\frac{\pi}{3}\right)-\cot\left(x+\frac{\pi}{3}\right)\right]+c$

(c) $\frac{1}{2}\log\left[\sec\left(x-\frac{\pi}{6}\right)+\tan\left(x-\frac{\pi}{6}\right)\right]+c$

(d) $-\frac{1}{2}\log\left[\operatorname{cosec}\left(x+\frac{\pi}{3}\right)+\cot\left(x+\frac{\pi}{3}\right)\right]+c$

11. $\int(\sin 2x-\cos 2x)dx=\frac{1}{\sqrt{2}}\sin(2x-a)+b$, then

(a) $a=\frac{5\pi}{4}, b\in R$ (b) $a=-\frac{5\pi}{4}, b\in R$

(c) $a=\frac{\pi}{4}, b\in R$ (d) None of these

12. $\int\log[x+\sqrt{x^2+1}]\,dx$ is equal to

(a) $x\log(x+\sqrt{x^2+1})-\sqrt{x^2+1}+c$

(b) $\frac{x}{2}\log(x+\sqrt{x^2+1})+\frac{1}{2}\sqrt{x^2+1}+c$

(c) $x\sin h^{-1}x-\sqrt{x^2+1}+c$

(d) None of the above

13. $\int\sqrt{\frac{a+x}{a-x}}\,dx$ is equal to

(a) $a\sin^{-1}\frac{x}{a}-\sqrt{a^2-x^2}+c$

(b) $\frac{1}{a}\sin^{-1}\frac{x}{a}+\sqrt{a^2-x^2}+c$

(c) $a\sin h^{-1}\frac{x}{a}-\sqrt{a^2-x^2}+c$

(d) $a\cos h^{-1}\frac{x}{a}+\sqrt{a^2-x^2}+c$

14. The value of integral $\int\left(\frac{2+\sin 2x}{1+\cos 2x}\right)e^x dx$ is equal to

(a) $e^x\sin x+c$ (b) $e^x\cos x+c$

(c) $e^x\tan x+c$ (d) $e^x\cot x+c$

15. $\int\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)dx$ is equal to

(a) $2x\cos^{-1}x+\log(1-x^2)+c$

(b) $2x\sin^{-1}x-\log(1-x^2)+c$

(c) $2x\tan^{-1}x-\log(1+x^2)+c$

(d) None of the above

16. Consider the following expressions

1. $\int\frac{f'(x)}{[f(x)]^2}dx=-\frac{1}{f(x)}+c$

2. $\int e^{f(x)}\cdot f'(x)\,dx=e^{f(x)}+c$

3. $\int\log[f(x)]\,dx=\frac{1}{f(x)}+c$

Which of these are correct?

(a) 2 and 3 (b) 1 and 2

(c) 1 and 3 (d) 1, 2 and 3

17. $\int\frac{2^x}{\sqrt{1-4^x}}dx=K\sin^{-1}(2^x)+c$, then K is equal to

(a) log 2 (b) $\frac{1}{2}\log 2$

(c) $\frac{1}{2}$ (d) $\frac{1}{\log 2}$

18. $\int\frac{\cos 2x}{\cos^2 x\sin^2 x}dx$ is equal to

(a) $\tan x-\cot x+c$

(b) $\cot x-\tan x+c$

(c) $-\cot x-\tan x+c$

(d) $\cot x+\tan x+c$

19. $\int\frac{1-\tan(x/2)}{1+\tan(x/2)}dx$ is equal to

(a) $2\log\sec(x/2)+c$

(b) $2\log\cos(x/2)+c$

(c) $2\log\left[\sec\left(\frac{\pi}{4}-\frac{x}{2}\right)\right]+c$

(d) $2\log\left[\cos\left(\frac{\pi}{4}-\frac{x}{2}\right)\right]+c$

20. $\int\left\{\frac{\log x-1}{1+(\log x)^2}\right\}^2 dx$ is equal to

(a) $\frac{xe^x}{1+x^2}+c$ (b) $\frac{x}{(\log x)^2+1}+c$

(c) $\frac{\log x}{(\log x)^2+1}+c$ (d) $\frac{x}{x^2+1}+c$

21. $\int\frac{1}{\log x}dx-\int\frac{1}{(\log x)^2}dx$ is equal to **[NDA-II-2017]**

(a) $x\log x+c$ (b) $\frac{x}{\log x}+c$

(c) $x+\frac{1}{\log x}+c$ (d) None of these

22. $\int\frac{\log(x+1)-\log x}{x(x+1)}dx$ equals to

(a) $-\log\left(\frac{x+1}{x}\right)+c$

(b) $-\log\left[\log\left(\frac{x+1}{x}\right)\right]+c$

(c) $-\frac{1}{2}\left[\log\left(\frac{x+1}{x}\right)\right]^2+c$

(d) $c-\frac{1}{2}[\log(x+1)^2-(\log x)^2]$

23. $\int \frac{1+x^4}{(1-x^4)^{3/2}}dx$ is equal to

(a) $\frac{1}{\sqrt{x^2-1/x^2}}+c$ (b) $\frac{1}{\sqrt{1/x^2-x^2}}+c$

(c) $\frac{1}{\sqrt{x^2+1/x^2}}+c$ (d) None of these

24. $\int x\frac{(\sec 2x-1)}{(\sec 2x+1)}dx$ is equal to

(a) $x\tan x-\log\sec x-\frac{x^2}{2}+c$

(b) $x\tan x+\log\sec x+\frac{x^2}{2}+c$

(c) $x\sec^2 x+\log\tan x-\frac{x^2}{2}+c$

(d) None of the above

25. Integral of $f(x)=\sqrt{1+x^2}$ with respect to x^2 is

(a) $\frac{2}{3}x(1+x^2)^{3/2}+c$ (b) $\frac{2}{3x}(1+x^2)^{3/2}+c$

(c) $\frac{2}{3}(1+x^2)^{3/2}+c$ (d) None of the above

26. $\int \sin 3x\sin 2x\,dx$ is equal to

(a) $-\frac{1}{5}\cos 5x+c$

(b) $\frac{1}{2}\sin x+\frac{1}{10}\sin 5x+c$

(c) $\frac{1}{2}\sin x-\frac{1}{10}\sin 5x+c$

(d) $-\frac{1}{3}\cos 3x-\frac{1}{2}\sin 2x+c$

27. $\int \frac{x^6(x+1)}{\sqrt{(5x^{10}+6x^9+x^4)}}dx$ is equal to

(a) $\frac{\sqrt{(5x^6+6x^5+1)}}{15}+c$

(b) $\frac{\sqrt{(5x^8+6x^7+x^2)}}{30}+c$

(c) $\frac{\sqrt{(5x^8+6x^7+x^2)}}{15}+c$

(d) $\sqrt{(5x^{10}+6x^5+x^4)}+c$

28. If $\int \frac{1}{(x^2+1)(x^2+4)}dx = A\tan^{-1}x+B\tan^{-1}\frac{x}{2}+C,$ then

(a) $A=\frac{1}{3}, B=-\frac{1}{6}$ (b) $B=\frac{2}{3}, A=\frac{1}{3}$

(c) $A=-\frac{1}{3}, B=\frac{1}{6}$ (d) $B=-\frac{1}{6}, A=-\frac{1}{3}$

29. $\int \frac{1}{\sqrt{x^2+4x+2}}dx$ is equal to

(a) $\sin^{-1}\left(\frac{x+2}{\sqrt{2}}\right)+c$

(b) $\log(x+2\sqrt{x^2+4x+2})+c$

(c) $\log(x+2+\sqrt{x^2+4x+2})+c$

(d) None of these

30. The value of the integral $\int \frac{1+x^2}{1+x^4}dx$ is equal to

(a) $\tan^{-1}x^2+c$

(b) $\frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{x^2-1}{\sqrt{2}x}\right)+c$

(c) $\frac{1}{2\sqrt{2}}\log\left(\frac{x^2+\sqrt{2}x+1}{x^2-\sqrt{2}x+1}\right)+c$

(d) None of the above

31. If $\int \frac{\cos^4 x}{\sin^2 x}dx = A\cot x+B\sin 2x+C\frac{x}{2}+D,$ then

(a) $A=-2$ (b) $B=-\frac{1}{4}$

(c) $C=3$ (d) $A=1$

32. $\int \frac{dx}{1+3\sin^2 x+8\cos^2 x}$ is equal to

(a) $\frac{1}{6}\tan^{-1}\left(\frac{2\tan x}{3}\right)+c$

(b) $\frac{1}{6}\tan^{-1}\left(\frac{3\tan x}{2}\right)+c$

(c) $\frac{1}{8}\tan^{-1}\left(\frac{2\tan x}{3}\right)+c$

(d) None of these

33. $\int \frac{x}{x^4+x^2+1}dx$ is equal to

(a) $\frac{1}{\sqrt{3}}\log\left(\frac{\sqrt{3}x^2+1}{\sqrt{3}x^2-1}\right)+c$

(b) $\frac{1}{\sqrt{3}}\log\left(\frac{\sqrt{3}x^2-1}{\sqrt{3}x^2+1}\right)+c$

(c) $\frac{1}{\sqrt{3}}\tan^{-1}\left(\frac{2x^2+1}{\sqrt{3}}\right)+c$

(d) None of these

34. If $\int\frac{dx}{5+4\cos x}=A\tan^{-1}\left(B\tan\frac{x}{2}\right)+C,$ then

(a) $A=1$ (b) $A=-\frac{2}{3}$

(c) $B=\frac{1}{3}$ (d) $B=\frac{2}{3}$

35. $\int x\,e^x\cos x\,dx$ is equal to

(a) $\frac{e^x}{2}\{(1-x)\sin x-x\cos x\}+c$

(b) $\frac{e^x}{2}\{(1+x)\sin x-x\cos x\}+c$

(c) $\frac{e^x}{2}\{(x-1)\sin x+x\cos x\}+c$

(d) None of the above

36. $\int\frac{1}{(9x^2-12x+8)}dx$ is equal to

(a) $\frac{1}{6}\tan^{-1}\left(\frac{3x-2}{2}\right)+c$

(b) $\frac{1}{6}\sin^{-1}\left(\frac{3x-2}{2}\right)+c$

(c) $\frac{1}{6}\cos^{-1}\left(\frac{3x-2}{2}\right)+c$

(d) None of these

37. If $\int\frac{2\sin x+3\cos x}{(3\sin x+4\cos x)}dx$

$= Ax + B\log(3\sin x + 4\cos x) + c$, then

(a) $A=\frac{1}{25},B=\frac{8}{25}$ (b) $A=\frac{18}{25},B=\frac{1}{25}$

(c) $A=\frac{8}{25},B=\frac{1}{25}$ (d) $A=\frac{9}{25},B=\frac{4}{25}$

38. $\int\frac{dx}{3+4\cot x}=Ax+B\log(3\sin x+4\cos x)+c$

(a) $A=\frac{3}{25},B=\frac{4}{25}$ (b) $A=-\frac{3}{25},B=\frac{4}{25}$

(c) $A=\frac{3}{25},B=-\frac{4}{25}$ (d) $A=\frac{-3}{25},B=\frac{-4}{25}$

39. $\int\frac{e^{\log(1+1/x^2)}}{x^2+1/x^2}dx$ is

(a) $\frac{1}{\sqrt{2}}\tan^{-1}\left(x-\frac{1}{x}\right)+c$

(b) $-\frac{1}{\sqrt{2}}\tan^{-1}\left(x-\frac{1}{x}\right)+c$

(c) $\frac{1}{\sqrt{2}}\log\left(\frac{x^2+1}{x\sqrt{2}}\right)+c$

(d) $\frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{x^2-1}{x\sqrt{2}}\right)+c$

40. $\int\frac{\cos 4x-1}{\cot x-\tan x}dx$ is equal to

(a) $-\frac{1}{2}\cos 4x+c$ (b) $-\frac{1}{4}\cos 4x+c$

(c) $-\frac{1}{2}\sin 2x+c$ (d) None of these

ANSWERS

1.	(a)	**2.**	(a)	**3.**	(b)	**4.**	(b)	**5.**	(b)	**6.**	(c)	**7.**	(c)	**8.**	(b)	**9.**	(a)	**10.**	(b)
11.	(b)	**12.**	(a)	**13.**	(a)	**14.**	(c)	**15.**	(c)	**16.**	(d)	**17.**	(d)	**18.**	(c)	**19.**	(d)	**20.**	(b)
21.	(b)	**22.**	(c)	**23.**	(b)	**24.**	(a)	**25.**	(c)	**26.**	(a)	**27.**	(a)	**28.**	(a)	**29.**	(c)	**30.**	(b)
31.	(b)	**32.**	(a)	**33.**	(c)	**34.**	(c)	**35.**	(c)	**36.**	(b)	**37.**	(b)	**38.**	(c)	**39.**	(d)	**40.**	(d)

Exercise

1. (a) $I=\int\frac{\sin x-\cos x}{\sqrt{1-\sin 2x}}e^{\sin x}\cdot\cos x\,dx$

$\because$ $1-\sin 2x=(\sin x-\cos x)^2$

So, $I=\int e^{\sin x}\cdot\cos x\,dx$

Let $\sin x=t \Rightarrow \cos x\,dx=dt$

So, $I=\int e^t dt=e^t+c=e^{\sin x}+c$

2. (a) $\int \frac{\sqrt{\tan x}}{\sin x \cos x} \times \frac{\sqrt{\tan x}}{\sqrt{\tan x}} dx$

$= \int \frac{1}{\cos^2 x \sqrt{\tan x}} dx = \int \frac{\sec^2 x\, dx}{\sqrt{\tan x}} = 2\sqrt{\tan x} + c$

3. (b) Let $x = t^2 \Rightarrow dx = 2t\, dt$

$\int \frac{dx}{(x+3)\sqrt{x}} = \int \frac{2t\, dt}{(t^2+3)t}$

$= 2\int \frac{dt}{t^2 + (\sqrt{3})^2} = \frac{2}{\sqrt{3}} \tan^{-1} \frac{t}{\sqrt{3}} + c$

$= \frac{2}{\sqrt{3}} \tan^{-1} \sqrt{\frac{x}{3}} + c$

4. (b) Let $x + 2 = t^2 \Rightarrow dx = 2t\, dt$

$\int \frac{dx}{(x+1)\sqrt{(x+2)}} = \int \frac{2t\, dt}{(t^2-1)t}$

$= \int \left(\frac{1}{t-1} - \frac{1}{t+1} \right) dt$

$= \log (t - 1) - \log (t + 1)$

$= \log \frac{t-1}{t+1} = \log \frac{\sqrt{x+2}-1}{\sqrt{x+2}+1} + c$

5. (b) $I = \int 2 \sin x \cos x \cdot \log \cos x\, dx$

Put $\log \cos x = t \Rightarrow \cos x = e^t$

$\Rightarrow -\frac{\sin x}{\cos x} dx = dt$

$I = -2\int \cos^2 x \cdot t\, dt = -2\int te^{2t} dt$

$= -2\left[t \cdot \frac{e^{2t}}{2} - \int \frac{e^{2t}}{2} \cdot dt \right] = -t \cdot e^{2t} + \frac{1}{2} e^{2t} + k$

$= -e^{2t}\left(\frac{1}{2} - t \right) + k = \cos^2 x \left(\frac{1}{2} - \log \cos x \right) + k$

6. (c) $\int \frac{1}{1+\sin x} dx = \int \frac{1}{1+\cos\left(\frac{\pi}{2} - x \right)} dx$

$= \frac{1}{2} \int \sec^2 \left(\frac{\pi}{4} - \frac{x}{2} \right) dx = -\tan\left(\frac{\pi}{4} - \frac{x}{2} \right) + c$

$= \tan\left(\frac{x}{2} - \frac{\pi}{4} \right) + c$

On comparing, $a = \frac{-\pi}{4}$ and $b \in R$

7. (c) Let $5^{5^{5^x}} = t$

$\Rightarrow 5^{5^{5^x}} \cdot 5^{5^x} \cdot 5^x (\log 5)^3 dx = dt$

$\Rightarrow 5^{5^{5^x}} \cdot 5^{5^x} \cdot 5^x dx = \int \frac{1}{(\log 5)^3} dt = \frac{5^{5^{5^x}}}{(\log 5)^3} + c$

8. (b) Let $I = \int \frac{\sin x}{\sin x - \cos x} dx$

Put $\sin x = A (\sin x - \cos x) + B (\cos x + \sin x)$

$\Rightarrow A + B = 1$ and $B - A = 0$

$\Rightarrow A = \frac{1}{2}$ and $B = \frac{1}{2}$

So, $\int \frac{\sin x}{\sin x - \cos x} dx = \frac{1}{2} \int \frac{\sin x - \cos x}{\sin x - \cos x} dx + \frac{1}{2} \int \frac{\cos x + \sin x}{\sin x - \cos x} dx$

$= \frac{1}{2} x + \frac{1}{2} \log (\sin x - \cos x) + c$

9. (a) $I = \int \frac{\sin x}{\sin (x - \alpha)} dx = \int \frac{\sin (x - \alpha + \alpha)}{\sin (x - \alpha)}$

$= \int \cos \alpha\, dx + \sin \alpha \int \cot (x - \alpha)\, dx$

$= x \cos \alpha + \sin \alpha \log \sin (x - \alpha) + c$

10. (b) $I = \int \frac{1}{\sin x + \sqrt{3} \cos x} dx$

$I = \int \frac{1}{2\left(\frac{1}{2} \sin x + \frac{\sqrt{3}}{2} \cos x \right)} dx$

$= \frac{1}{2} \int \frac{1}{\sin\left(x + \frac{\pi}{3} \right)} dx$

$= \frac{1}{2} \int \operatorname{cosec}\left(x + \frac{\pi}{3} \right) dx$

$= \frac{1}{2} \log \left\{ \operatorname{cosec}\left(x + \frac{\pi}{3} \right) - \cot\left(x + \frac{\pi}{3} \right) \right\} + c$

11. (b) $\int (\sin 2x - \cos 2x) dx$

$= -\sqrt{2} \int \left(\frac{1}{\sqrt{2}} \cos 2x - \frac{1}{\sqrt{2}} \sin 2x \right) dx$

$= -\sqrt{2} \int \cos\left(2x + \frac{\pi}{4} \right) dx$

$= \frac{-1}{\sqrt{2}} \sin\left(2x + \frac{\pi}{4} \right) + c$

$= \frac{-1}{\sqrt{2}} \sin\left(2x + \frac{5\pi}{4} \right) + c$

On comparing, $a = -\frac{5\pi}{4}$ and $b \in R$.

12. (a) $I = \int \log(x + \sqrt{x^2+1}) \cdot 1\, dx$

$= x \log (x + \sqrt{x^2+1}) - \int \frac{x\left(1 + \frac{2x}{2\sqrt{x^2+1}} \right)}{(x + \sqrt{x^2+1})} dx$

$$I = x\log(x+\sqrt{x^2+1}) - \int \frac{x}{\sqrt{x^2+1}}dx$$

$$= x\log(x+\sqrt{x^2+1}) - \frac{1}{2}\int \frac{dt}{\sqrt{t}}$$

$\{$Let $x^2 + 1 = t \Rightarrow 2x\,dx = dt\}$

$$= x\log(x+\sqrt{x^2+1}) - \sqrt{x^2+1} + c$$

13. (a) $I = \int \frac{\sqrt{a+x}}{\sqrt{a-x}} \times \frac{\sqrt{a+x}}{\sqrt{a+x}}dx = \int \frac{a+x}{\sqrt{a^2-x^2}}dx$

$$= a\int \frac{dx}{\sqrt{a^2-x^2}} - \frac{1}{2}\int \frac{-2x}{\sqrt{a^2-x^2}}$$

$$= a\sin^{-1}\frac{x}{a} - \frac{1}{2}\int \frac{dt}{\sqrt{t}},\ \text{where } a^2 - x^2 = t^2$$

$$= a\sin^{-1}\frac{x}{a} - \sqrt{t} + c = a\sin^{-1}\frac{x}{a} - \sqrt{a^2-x^2} + c$$

14. (c) $\int e^x\left\{\frac{2+\sin 2x}{1+\cos 2x}\right\}dx$

$$= \int e^x\left\{\frac{2}{2\cos^2 x} + \frac{2\sin x\cos x}{2\cos^2 x}\right\}dx$$

$$= \int e^x\{\tan x + \sec^2 x\}dx = e^x\tan x + c$$

15. (c) $I = \int 1\cdot\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)dx$

$$= x\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right) - \int \frac{2x}{1+x^2}dx$$

$$= 2x\tan^{-1}x - \log(1+x^2) + c$$

16. (d) $\frac{d}{dx}\left[\frac{1}{f(x)}\right] = \frac{-f'(x)}{[f(x)]^2}$

$$\therefore \quad \int \frac{-f'(x)}{[f(x)]^2}dx = \frac{-1}{f(x)} + c$$

$$\frac{d}{dx}[e^{f(x)}] = e^{f(x)}\cdot f'(x)$$

$$\Rightarrow \quad \int e^{f(x)}f'(x)\,dx = e^{f(x)} + c$$

Hence, 1 and 2 are correct.

17. (d) Put $2^x = t \Rightarrow 2^x \log 2\,dx = dt$

$$I = \frac{1}{\log 2}\int \frac{dt}{\sqrt{1-t^2}} = k\sin^{-1}(2^x) + c$$

$$\Rightarrow \quad \frac{1}{\log 2}\sin^{-1}(2^x) + c = k\sin^{-1}(2^x) + c$$

$$\Rightarrow \quad k = \frac{1}{\log 2}$$

18. (c) $I = \int \frac{\cos 2x}{\cos^2 x\sin^2 x}dx = \int \frac{(\cos^2 x - \sin^2 x)}{\cos^2 x\sin^2 x}dx$

$$= \int(\text{cosec}^2 x - \sec^2 x)\,dx = -\cot x - \tan x + c.$$

19. (d) $I = \int \frac{1-\tan\frac{x}{2}}{1+\tan\frac{x}{2}}dx = \int \tan\left(\frac{\pi}{4} - \frac{x}{2}\right)dx$

$$= -2\int \tan t\,dt \quad \left(\text{where, } \frac{\pi}{4} - \frac{x}{2} = t\right)$$

$$= 2\log(\cos t) + c = 2\log\left(\cos\left(\frac{\pi}{4} - \frac{x}{2}\right)\right) + c$$

20. (b) $I = \int\left\{\frac{\log x - 1}{1+(\log x)^2}\right\}^2 dx$

$$I = \int e^t\left\{\frac{t-1}{1+t^2}\right\}^2 dt$$

$\left\{\text{Let } \log x = t \text{ or } x = e^t \Rightarrow \frac{1}{x}dx = dt\right\}$

$$= \int e^t\left\{\frac{t^2+1}{(1+t^2)^2} - \frac{2t}{(1+t^2)}\right\}dt$$

$$= e^t\left(\frac{1}{1+t^2}\right) + c = \frac{x}{1+(\log x)^2} + c$$

21. (b) $I = \int \frac{1}{\log x}\cdot 1\,dx - \int \frac{1}{\log(x)^2}dx$

$$I = \frac{x}{\log x} - \int \frac{-1}{(\log x)^2}\cdot\frac{1}{x}\cdot x\,dx - \int \frac{1}{(\log x)^2}dx$$

$$= \frac{x}{\log x} + c$$

22. (c) $I = \int \frac{\log(x+1) - \log x}{x(x+1)}dx = \int \frac{\log\left(\frac{x+1}{x}\right)}{\left(\frac{x+1}{x}\right)}\cdot\frac{1}{x^2}dx$

Let $\frac{x+1}{x} = t$ or $1 + \frac{1}{x} = t$

$$\Rightarrow \quad -\frac{1}{x^2}dx = dt$$

So, $I = -\int \frac{\log t}{t}dt = -\frac{1}{2}\left[\log\left(\frac{x+1}{x}\right)\right]^2 + c$

23. (b) $I = \int \frac{1+x^4}{(1-x^4)^{3/2}}dx = \int \frac{x+\frac{1}{x^3}}{\left(\frac{1}{x^2} - x^2\right)^{3/2}}dx$

Let $\frac{1}{x^2} - x^2 = t \Rightarrow -2\left[x + \frac{1}{x^3}\right]dx = dt$

So, $I = -\frac{1}{2}\int \frac{dt}{t^{3/2}} = \frac{1}{t^{1/2}} + c = \frac{1}{\sqrt{\frac{1}{x^2} - x^2}} + c$

24. (a) $I = \int \frac{x(\sec 2x - 1)}{(\sec 2x + 1)} dx$

$= \int x \frac{(1-\cos 2x)}{(1+\cos 2x)} dx$

$= \int x \tan^2 x\, dx = \int x \sec^2 x\, dx - \int x\, dx$

$= x \tan x - \int \tan x\, dx - \frac{x^2}{2}$

$= x \tan x - \log \sec x - \frac{x^2}{2} + c$

25. (c) $f(x) = \sqrt{1+x^2}$

Integration of $f(x)$ w.r. to x^2

$I = \int \sqrt{1+x^2}\, dx^2 = \frac{(1+x^2)^{3/2}}{3/2} + c$

26. (a) $I = \frac{1}{2} \int 2 \sin 3x \sin 2x\, dx$

$= \frac{1}{2} \int (\cos x - \cos 5x) dx$

$= \frac{1}{2} \sin x - \frac{1}{10} \sin 5x + c$

27. (a) $I = \int \frac{x^6(x+1)}{\sqrt{5x^{10} + 6x^9 + x^4}} dx = \int \frac{x^4(x+1)}{\sqrt{5x^6 + 6x^5 + 1}} dx$

Let $5x^6 + 6x^5 + 1 = t \Rightarrow 30(x^5 + x^4)\, dx = dt$

$I = \frac{1}{30} \int \frac{dt}{\sqrt{t}} = \frac{1}{30} 2\sqrt{t} + c$

$= \frac{1}{15} \sqrt{5x^6 + 6x^5 + 1} + c$

28. (a) $\int \frac{1}{(x^2+1)(x^2+4)} dx = A \tan^{-1} x + B \tan^{-1} \frac{x}{2} + c$

LHS $= \frac{1}{3} \int \left(\frac{1}{x^2+1} - \frac{1}{x^2+4} \right) dx$

$= \frac{1}{3} \left[\tan^{-1} x - \frac{1}{2} \tan^{-1} \frac{x}{2} \right] + c$

On comparing, $A = \frac{1}{3}$ and $B = \frac{-1}{6}$

29. (c) $\int \frac{1}{\sqrt{x^2 + 4x + 2}} = \int \frac{1}{\sqrt{(x+2)^2 - \sqrt{(\sqrt{2})^2}}} dx$

$= \log (x + 2 + \sqrt{x^2 + 4x + 2}) + c$

30. (b) $I = \int \frac{1+x^2}{1+x^4} dx$

Divide N^r and D^r by x^2

$= \int \frac{\frac{1}{x^2} + 1}{\frac{1}{x^2} + x^2} dx$

Let $x - \frac{1}{x} = z \Rightarrow x^2 + \frac{1}{x^2} = z^2 + 2$

and $\left(1 + \frac{1}{x^2}\right) dx = dz$

$I = \int \frac{dz}{z^2 + (\sqrt{2})^2} = \frac{1}{\sqrt{2}} \tan^{-1} \frac{z}{\sqrt{2}} + c$

$= \frac{1}{\sqrt{2}} \tan^{-1} \left(\frac{x^2 - 1}{\sqrt{2}x} \right) + c$

31. (b) $\int \frac{\cos^4 x}{\sin^4 x} dx = \int \frac{(1 - \sin^2 x)^2}{\sin^2 x} dx$

$= \int \text{cosec}^2 x\, dx + \int \sin^2 x\, dx - 2 \int dx$

$= -\cot x + \int \frac{1 - \cos 2x}{2} dx - 2x + c$

$= -\cot x + \frac{x}{2} - \frac{\sin 2x}{4} - 2x + c$

$= -\cot x - \frac{1}{4} \sin 2x - \frac{3}{2} x + c$

On comparing, $A = -1, B = -\frac{1}{4}, C = -3$

32. (a) Let $I = \int \frac{dx}{1 + 3\sin^2 x + 8\cos^2 x}$

Divide N^r and D^r by $\cos^2 x$

$I = \int \frac{\sec^2 x\, dx}{\sec^2 x + 3\tan^2 x + 8} = \int \frac{\sec^2 x\, dx}{4\tan^2 x + 9}$

Let $\tan x = t \Rightarrow \sec^2 x\, dx = dt$

$I = \int \frac{dt}{4t^2 + 9} = \frac{1}{6} \tan^{-1} \left(\frac{2 \tan x}{3} \right) + c$

33. (c) Let $x^2 = t \Rightarrow 2x\, dx = dt$

$\int \frac{x}{x^4 + x^2 + 1} dx = \frac{1}{2} \int \frac{dt}{t^2 + t + 1}$

$= \frac{1}{2} \int \frac{dt}{\left(t + \frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2}$

$= \frac{1}{\sqrt{3}} \tan^{-1} \left(\frac{2x^2 + 1}{\sqrt{3}} \right) + c$

34. (c) Let $I = \int \frac{dx}{5 + 4\cos x} = \int \frac{\sec^2 \frac{x}{2} dx}{9 + \tan^2 \frac{x}{2}}$

Let $\tan \frac{x}{2} = t \Rightarrow \frac{1}{2} \sec^2 \frac{x}{2} dx = dt$

So, $I = 2 \int \frac{dt}{9 + t^2} = \frac{2}{3} \tan^{-1} \left(\frac{1}{3} \tan \frac{x}{2} \right) + c$

On comparing with $A\tan^{-1}\left(B\tan\frac{x}{2}\right)+c$

We get $A=\frac{2}{3}$ and $B=\frac{1}{3}$

35. (c) $I=\int x\,e^x\cos x\,dx$

$\because$ $e^{ix}=\cos x+i\sin x$

$\Rightarrow$ $\cos x$ is real part of e^{ix}.

So, $I=$ Real part of $\int xe^x\cdot e^{ix}\,dx$

$=$ R.P. of $\int xe^{(1+i)x}dx$

$=$ R.P. $\left\{x\cdot\frac{e^{(1+i)x}}{1+i}-\frac{1}{1+i}\int e^{(1+i)x}dx\right\}$

$=$ R.P. $\left\{\frac{x\cdot e^{(1+i)x}}{1+i}-\frac{e^{(1+i)x}}{(1+i)^2}\right\}$

$=$ R.P. $\left\{\frac{\{x(1+i)-1\}\cdot e^{(1+i)x}}{(1+i)^2}\right\}$

$=$ R.P. $\left\{\left(\frac{(x-1)+ix}{2i}\right)e^x\cdot(\cos x+i\sin x)\right\}$

$=\frac{e^x}{2}[(x-1)\sin x+x\cos x]+c$

36. (b) $\int\frac{dx}{9x^2-12x+8}$

$=\int\frac{dx}{(3x-2)^2+(2)^2}=\frac{1}{6}\tan^{-1}\left(\frac{3x-2}{2}\right)+c$

37. (b) $2\sin x+3\cos x=A(3\sin x+4\cos x)$

$+B\frac{d}{dx}(3\sin x+4\cos x)$

$\Rightarrow$ $2\sin x+3\cos x=A(3\sin x+4\cos x)$

$+B(3\cos x-4\sin x)$

$\Rightarrow$ $3A-4B=2$ and $4A+3B=3$

$\Rightarrow$ $A=\frac{18}{25}$ and $B=\frac{1}{25}$

38. (c) $\int\frac{dx}{3+4\cot x}=\int\frac{\sin x\,dx}{3\sin x+4\cos x}$ ...(i)

$\Rightarrow$ $\sin x=A(3\sin x+4\cos x)$

$+B\frac{d}{dx}(3\sin x+4\cos x)$

$\Rightarrow$ $\sin x=A(3\sin x+4\cos x)$

$+B(3\cos x-4\sin x)$

$3A-4B=1$ and $4A+3B=0$

On solving,

$A=3/25$ and $B=-4/25$

On putting in eq. (i)

$\int\frac{\sin x\,dx}{3\sin x+4\cos x}=\frac{3}{25}\int dx-\frac{4}{25}\int\frac{3\cos x-4\sin x}{3\sin x+4\cos x}$

$=\frac{3}{25}x-\frac{4}{25}\log(3\sin x+4\cos x)+c$

On comparing, $A=\frac{3}{25}$, $B=\frac{-4}{25}$

39. (d) $I=\int\frac{e^{\log\left(1+\frac{1}{x^2}\right)}}{x^2+\frac{1}{x^2}}dx$

Let $x-\frac{1}{x}=t$

or $x^2+\frac{1}{x^2}=t^2+2$

$\Rightarrow$ $\left(1+\frac{1}{x^2}\right)dx=dt$

So, $I=\int\frac{1+\frac{1}{x^2}}{x^2+\frac{1}{x^2}}dx=\int\frac{dx}{t^2+2}$

$=\frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{t}{\sqrt{2}}\right)+c=\frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{x^2-1}{x\sqrt{2}}\right)+c$

40. (d) $I=\int\frac{\cos 4x-1}{\cot x-\tan x}dx=\int\frac{-2\sin^2x}{\frac{\cos 2x}{\sin x\cos x}}dx$

$=\int\frac{-\sin^3 2x}{\cos 2x}dx=\int\frac{-(1-\cos^2 2x)}{\cos 2x}\cdot\sin 2x\,dx$

Let $\cos 2x=t$ $\Rightarrow$ $-2\sin 2x\,dx=dt$

$\therefore$ $I=\frac{1}{2}\int\frac{1-t^2}{t}dt=\frac{1}{2}\int\left(\frac{1}{t}-t\right)dt$

$=\frac{1}{2}\log(\cos 2x)-\frac{(\cos 2x)^2}{4}+c$