CHAPTER

29

Area Bounded by Curves

Exercise

1. The area between the curves $y = \sin x$ and the X-axis from $x = 0$ to $x = 2\pi$ is equal to
   (a) 2 sq. units (b) 4 sq. units
   (c) 1/2 sq. units (d) 1/4 sq. units
2. Area lying in the first quadrant and bounded by the curve $y = x^3$ and the line $y = 4x$ is
   (a) 2 (b) 3
   (c) 4 (d) 5
3. The area bounded by the curve $|x| + y = 1$ and the axis of X is
   (a) 4 (b) 2
   (c) 1 (d) 1/2
4. The area enclosed between the curves $y^2 = x$ and $y = |x|$ is
   (a) $\frac{2}{3}$ (b) 1
   (c) $\frac{1}{6}$ (d) $\frac{1}{3}$
5. The area cut off the parabola $4y = 3x^2$ by the straight line $2y = 3x + 12$ in sq. unit is
   (a) 16 (b) 21
   (c) 27 (d) 36
6. The area of region bounded by $y = |x - 1|$ and $y = 1$ is
   (a) 1 (b) 2
   (c) $\frac{1}{2}$ (d) None of these
7. The area inside the parabola $5x^2 - y = 0$ but outside the parabola $2x^2 - y + 9 = 0$ is
   (a) $12\sqrt{3}$ (b) $6\sqrt{3}$
   (c) $8\sqrt{3}$ (d) $4\sqrt{3}$
8. Area bounded by the curves $x = 1$, $x = 3$, $xy = 1$ and X-axis is
   (a) log 2 (b) log 3
   (c) log 4 (d) None of these
9. The area of the figure bounded by the curve $|y| = 1 - x^2$ is **[NDA-II 2016]**
   (a) 2/3 (b) 4/3
   (c) 8/3 (d) None of these
10. Area lying between the parabola $y^2 = 4ax$ and its latus rectum is
    (a) $\frac{8}{3}a^2$ (b) $\frac{8}{3}a$
    (c) $\frac{4}{3}a$ (d) $\frac{4}{3}a^2$
11. The area bounded by $y = \cos x$ and $x = -\frac{\pi}{2}$ and $x = 2\pi$ and the axis of X in square units is
    (a) 4 (b) 5
    (c) 6 (d) 7
12. The area common to the circle $x^2 + y^2 = 16a^2$ and the parabola $y^2 = 6ax$ is
    (a) $\frac{4a^2}{3}(4\pi - \sqrt{3})$ (b) $\frac{4a^2}{3}(8\pi - 3)$
    (c) $\frac{4a^2}{3}(4\pi + \sqrt{3})$ (d) None of these
13. Area enclosed between $y = ax^2$ and $x = ay^2$ $(a > 0)$ is 1, then a is
    (a) $1/\sqrt{3}$ (b) 1/2
    (c) 1 (d) 1/3
14. Smaller area enclosed by the circle $x^2 + y^2 = 4$ and the line $x + y = 2$ is
    (a) $2(\pi - 2)$ (b) $\pi - 2$
    (c) $2\pi - 1$ (d) None of these
15. Area common to the parabolas $y = 2x^2$ and $y = x^2 + 4$ is
    (a) 16/3 (b) 8/3
    (c) 32/3 (d) None of these

16. If the ordinate $x = a$ divides the area bounded by the curve $y = \left(1 + \frac{8}{x^2}\right)$ and the ordinates $x = 2$, $x = 4$ into two equal parts, then a is
(a) 3 (b) $2\sqrt{2}$
(c) $2\sqrt{3}$ (d) $3\sqrt{2}$

17. Area bounded by the curves $y = \sqrt{x}$, $x = 2y + 3$ in first quadrant and X-axis is
(a) $2\sqrt{3}$ (b) 18
(c) 9 (d) 34/3

18. The area bounded by the curve $y = x\,|\,x\,|$, X-axis and the ordinates $x = 1, -1$ is given by
(a) 0 (b) 1/3
(c) 2/3 (d) None of these

19. If A is the area lying between the curve $y = \sin x$ and X-axis between $x = 0$ and $\pi/2$, area of the region between the curve $y = \sin 2x$ and X-axis in the same interval is given by
(a) $A/2$ (b) A
(c) $2A$ (d) None of these

20. Area of the region bounded by the curve $y^2 = 4x$, Y-axis and the line $y = 3$ is
(a) 2 sq. units (b) 9/4 sq. units
(c) $6\sqrt{3}$ sq. units (d) None of these

21. The area of the region bounded by the curves $y^2 = 2x + 1$ and $x - y - 1 = 0$ is
(a) 4/3 (b) 8/3
(c) 14/3 (d) 16/3

22. The area bounded by the parabola $y = 2 - x^2$ and the straight line $y + x = 0$ is
(a) $\frac{17}{6}$ (b) $\frac{34}{7}$
(c) $\frac{9}{2}$ (d) $\frac{7}{2}$

23. If A is the area between the curve $y = \sin x$ and the X-axis in the interval $\left[0, \frac{\pi}{4}\right]$, then the area between the curve $y = \cos x$ and X-axis, in the same interval is
(a) A (b) $1 - A$
(c) $\frac{\pi}{2} - A$ (d) $\frac{\pi}{2} + A$

24. If A_1, A_2 be the areas of the curves $x^2 + y^2 + 18x + 24y = 0$ and $\frac{x^2}{14} + \frac{y^2}{13} = 1$, then
(a) $A_1 > A_2$ (b) $A_1 < A_2$
(c) $A_1 = A_2$ (d) None of these

25. The area bounded by the parabola $y^2 = 4x$ and $x + y = 3$ is :
(a) $\frac{16}{3}$ (b) $\frac{32}{3}$
(c) $\frac{64}{3}$ (d) $\frac{166}{3}$

26. The area of the region lying between the line $x - y + 2 = 0$ and the curve $x = \sqrt{y}$ is
(a) 9 (b) 9/2
(c) 10/3 (d) None of these

Directions (Q. Nos. 27 and 28) :

Consider the curves $f(x) = x\,|\,x\,| - 1$

and $g(x) = \begin{cases} \frac{3x}{2}, & x > 0 \\ 2x, & x \le 0 \end{cases}$

27. Where do the curves intersect? **[NDA-I-2016]**
(a) Only at (2, 3)
(b) Only at (– 1, – 2)
(c) At (2, 3) and (– 1, – 2)
(d) Neither at (2, 3) nor at (– 1, – 2)

28. What is the area bounded by the curves? **[NDA-I-2016]**
(a) $\frac{17}{6}$ sq. units (b) $\frac{8}{3}$ sq. units
(c) 2 sq. units (d) $\frac{1}{3}$ sq. units

Directions (Q. Nos. 29 and 30) :

Consider the curves $y = |\,x - 1|$ and $|\,x\,| = 2$

29. What is/are the points of intersection of the curves? **[NDA-I-2016]**
(a) Only (– 2, 3) (b) Only (2, 1)
(c) (– 2, 3) and (2, 1) (d) Neither (– 2, 3) nor (2, 1)

30. What is the area of the region bounded by the curves and X-axis? **[NDA-I-2016]**
(a) 3 sq. units (b) 4 sq. units
(c) 5 sq. units (d) 6 sq. units

31. What is the area of the region bounded by X-axis, the curve $f(x) = |\,x - 1\,| + x^2$, where $x \in R$ and the two ordinates $x = \frac{1}{2}$ and $x = 1$? **[NDA-I-2016]**
(a) $\frac{5}{12}$ sq. units (b) $\frac{5}{6}$ sq. units
(c) $\frac{7}{6}$ sq. units (d) 2 sq. units

32. What is the area of the region bounded by X-axis, the curve $f(x) = |\,x - 1\,| + x^2$, where $x \in R$ and the two ordinates $x = 1$ and $x = \frac{3}{2}$ **[NDA-I-2016]**

(a) $\frac{5}{12}$ sq. units (b) $\frac{7}{12}$ sq. units

(c) $\frac{2}{3}$ sq. units (d) $\frac{11}{12}$ sq. units

33. The area of the figure bounded by the curve $|y| = 1 - x^2$ is **[NDA-II-2016]**

(a) 2/3 (b) 4/3

(c) 8/3 (d) None of these

34. Area enclosed by $|x| + |y| = 1$ is equal to **[NDA-II-2017]**

(a) $2\sqrt{2}$ sq. units (b) 2 sq. units

(c) 1 sq. units (d) $2\sqrt{3}$ sq. units

35. What is the area of the region bounded by the parabolas $y^2 = 6(x - 1)$ and $y^2 = 3x$? **[NDA-I-2018]**

(a) $\frac{\sqrt{6}}{3}$ (b) $\frac{2\sqrt{6}}{3}$

(c) $\frac{4\sqrt{6}}{3}$ (d) $\frac{5\sqrt{6}}{3}$

36. The area of the loop between the curve $y = c \sin x$ and the X-axis is **[NDA-I-2019]**

(a) c (b) $2c$

(c) $3c$ (d) $4c$

37. What is the area of the region by bounded by $|x| < 5$, $y = 0$ and $y = 8$? **[NDA-II-2019]**

(a) 40 sq. units (b) 80 sq. units

(c) 120 sq. units (d) 160 sq. units

38. What is the area bounded by curve $y^2 = 2x$ and the straight line $y = x$? **[NDA-II-2019]**

(a) $\frac{2}{3}$ sq. units (b) $\frac{4}{3}$ sq. units

(c) $\frac{1}{3}$ sq. units (d) 1 sq. units

39. What is the area bounded by $y = \sqrt{16 - x^2}$, $y \geq 0$ and the X-axis? **[NDA-I-2021]**

(a) 16π sq. units (b) 8π sq. units

(c) 4π sq. units (d) 2π sq. units

40. What is the area bounded by $y = [x]$, where $[\cdot]$ is the greatest integer function, the x-axis and the lines $x = -1.5$ and $x = -1.8$? **[NDA-II-2021]**

(a) 0.3 square unit (b) 0.4 square unit

(c) 0.6 square unit (d) 0.8 square unit

ANSWERS

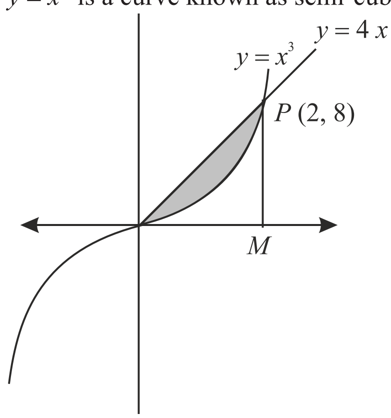
1.	(b)	**2.**	(c)	**3.**	(c)	**4.**	(c)	**5.**	(c)	**6.**	(a)	**7.**	(a)	**8.**	(b)	**9.**	(c)	**10.**	(a)
11.	(b)	**12.**	(c)	**13.**	(a)	**14.**	(b)	**15.**	(c)	**16.**	(b)	**17.**	(c)	**18.**	(c)	**19.**	(b)	**20.**	(b)
21.	(d)	**22.**	(c)	**23.**	(b)	**24.**	(a)	**25.**	(c)	**26.**	(b)	**27.**	(c)	**28.**	(b)	**29.**	(c)	**30.**	(c)
31.	(a)	**32.**	(d)	**33.**	(c)	**34.**	(b)	**35.**	(c)	**36.**	(b)	**37.**	(b)	**38.**	(a)	**39.**	(b)	**40.**	(b)

Explanations

1. (b) $A = \int_0^{2\pi} \sin x\,dx = \int_0^{\pi} \sin x\,dx + \int_{\pi}^{2\pi} (-\sin x)\,dx$

$= [-\cos x]_0^{\pi} + [\cos x]_{\pi}^{2\pi} = 4$ sq. units

2. (c) $y = x^3$ is a curve known as semi-cubical parabola.



If $x \to -x$ and $y \to -y$ the equation does not change. It is symmetrical in Ist and 3rd quadrants.

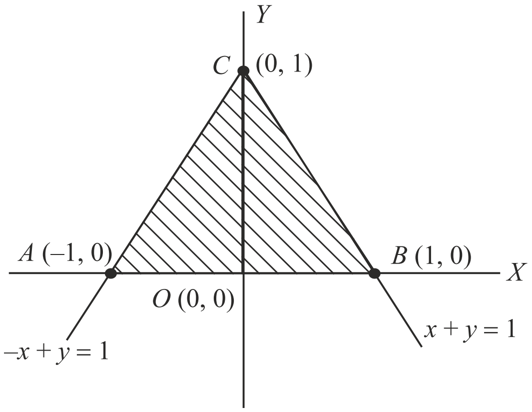
The line $y = 4x$ meets $y = x^3$ at $x = 0, 2, -2$ where $y = 0, 8, -8$.

Area in Ist quadrant $= \int_0^2 (y_1 - y_2)\,dx$

$= \int_0^2 (4x - x^3)\,dx = 4$

3. (c) $|x| + y = 1$ can be written as $x + y = 1; x \geq 0$ and $-x + y = 1; x < 0$.

These are the two straight lines,

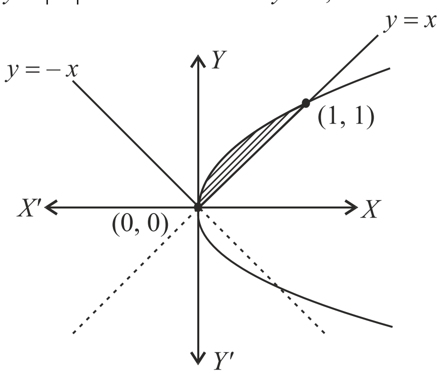


So, area bounded by these lines and X-axis is

A = area of $\Delta ABC = 2$ (area of ΔOBC)

$$= 2\left\{\frac{1}{2}\times 1\times 1\right\} = 1$$

4. (c) $y = |x|$ can be written as $y = x$,



when $x \geq 0$ and $y = -x$, when $x < 0$

Area bounded by $y^2 = x$ and $y = |x|$

$$A = \int_0^1 (y_1 - y_2)\, dx = \int_0^1 (\sqrt{x} - x)\, dx$$

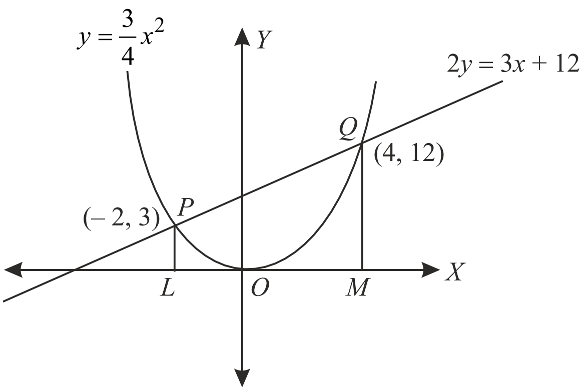
$$= \left[\frac{x^{3/2}}{3/2} - \frac{x^2}{2}\right]_0^1 = \left[\frac{2}{3} - \frac{1}{2}\right] = \frac{1}{6}$$

5. (c) Eliminating y, we get $2\,(3x + 12) = 3x^2$

or $(x - 4)\,(3x + 6) = 0$

or $x = -2, x = 4$

$\Rightarrow$ $y = 3, y = 12$



i.e., the points of intersection are $(-2, 3)$ and $(4, 12)$

$$A = \int_{-2}^{4} (y_1 - y_2)\, dx$$

$$= \int_{-2}^{4} \left(\frac{3x + 12}{2} - \frac{3}{4}x^2\right) dx$$

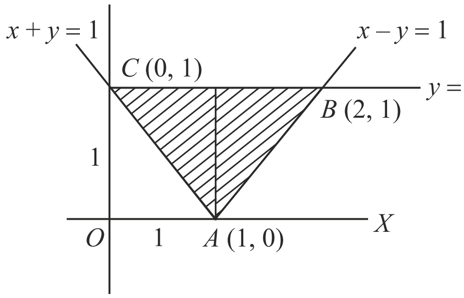
$= 27$ sq. units.

6. (a) The given curves are

(i) $y = x - 1, x > 1$

(ii) $y = -(x - 1), x < 1$

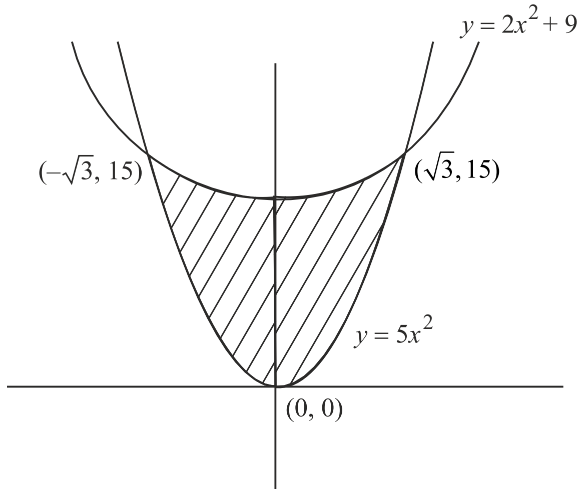
(iii) $y = 1$



These three lines enclose a triangle whose area is

$$A = \frac{1}{2}\times \text{base}\times \text{height} = \frac{1}{2}\times 2\times 1 = 1$$

7. (a) Solving $5x^2 - y = 0$ and $2x^2 - y + 9 = 0$



We get $x = -\sqrt{3},\ \sqrt{3}$

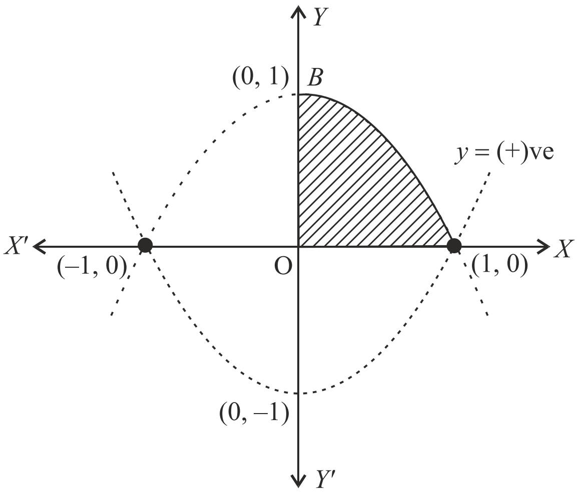
So, required area $= 2\int_0^{\sqrt{3}} \{(2x^2 + 9) - 5x^2\}\, dx$

$$= 2\int_0^{\sqrt{3}} (9 - 3x^2)\, dx = 2[9x - x^3]_0^{\sqrt{3}}$$

$$= 2[9\sqrt{3} - 3\sqrt{3}] = 12\sqrt{3}$$

8. (b) Bounded area $= \int_1^3 \frac{1}{x}\, dx = \log 3$

9. (c) $y = 1 - x^2, y > 0$



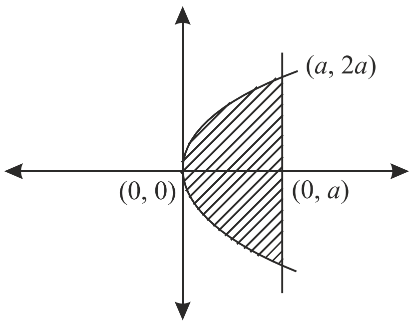
or $\quad x^2 = -(y-1)$ ...(i)

and $\quad -y = 1 - x^2, y < 0$

or $\quad x^2 = y + 1$ ...(ii)

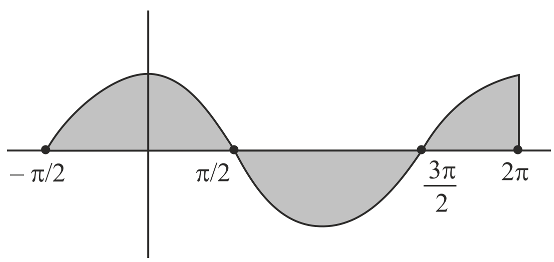
Bounded area $= 4\int_0^1 (1-x^2)\,dx = 4\left[x - \frac{x^3}{3}\right]_0^1 = \frac{8}{3}$

10. (a) Area $= 2\int_0^a 2\sqrt{ax}\,dx$



$= 4\sqrt{a}\left[\frac{x^{3/2}}{3/2}\right]_0^a = \frac{8\sqrt{a}}{3}[a^{3/2}] = \frac{8}{3}a^2$

11. (b) Bounded area



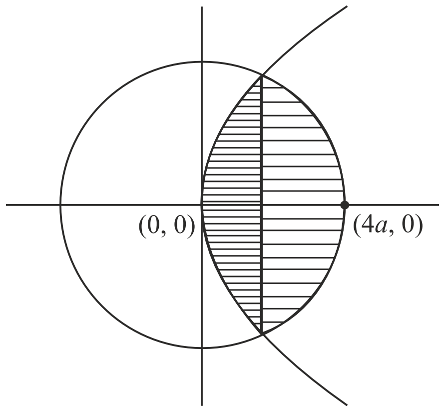
$= \int_{-\pi/2}^{\pi/2} \cos x\,dx + \int_{\pi/2}^{3/\pi} (-\cos x)\,dx + \int_{3\pi/2}^{2\pi} \cos x\,dx$

$= [\sin x]_{-\pi/2}^{\pi/2} - [\sin x]_{\pi/2}^{3\pi/2} + [\sin x]_{3\pi/2}^{2\pi}$

$= (1 + 1) - (-1 - 1) + (0 + 1)$

$= 5$

12. (c) $x^2 + y^2 = 16a^2$ and $y^2 = 6ax$ intersect at $x = 2a$



Area $= 2\left[\int_0^{2a} y_{\text{parabola}}\,dx + \int_{2a}^{4a} y_{\text{circle}}\,dx\right]$

$= 2\left[\int_0^{2a} \sqrt{6ax}\,dx + \int_{2a}^{4a} \sqrt{(4a)^2 - x^2}\,dx\right]$

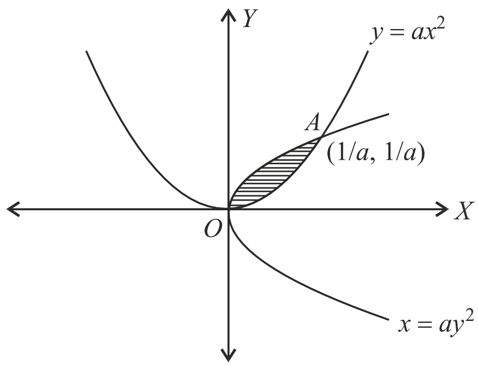
$= 2\left[\sqrt{6a}\left(\frac{x^{3/2}}{3/2}\right)_0^{2a} + \left(\frac{x}{2}\sqrt{(4a)^2 - x^2} + \frac{1}{2}(4a)^2 \sin^{-1}\frac{x}{4a}\right)_{2a}^{4a}\right]$

$= \frac{16}{3}\sqrt{3}a^2 + 2\left[-2\sqrt{3}a^2 + 8a^2\left(\frac{\pi}{2} - \frac{\pi}{6}\right)\right]$

$= \frac{4\sqrt{3}a^2}{3} + \frac{16\pi a^2}{3} = \frac{4a^2}{3}(4\pi + \sqrt{3})$

13. (a) The two curves meet at O (0, 0)

and $A\left(\frac{1}{a}, \frac{1}{a}\right)$.



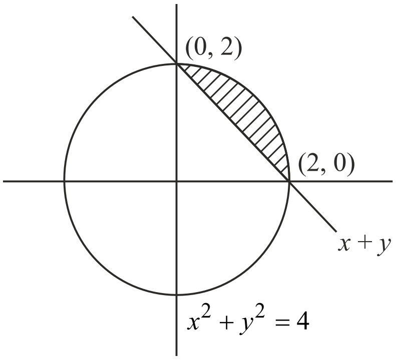
Bounded area $A = \int_0^{1/a}\left(\sqrt{\frac{x}{a}} - ax^2\right)dx$

$1 = \left[\frac{2}{3}\frac{x^{3/2}}{\sqrt{a}} - a\frac{x^3}{3}\right]_0^{1/a}$

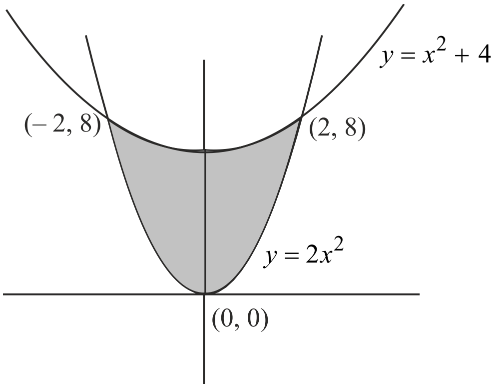
$1 = \left(\frac{2}{3} - \frac{1}{3}\right)\frac{1}{a^2} \Rightarrow a = \frac{1}{\sqrt{3}}$

14. (b) Bounded area $= \int_0^2 \{\sqrt{4-x^2} - (2-x)\}\, dx$

$= \pi - 2$



15. (c) $y = 2x^2$ cuts $y = x^2 + 4$ at $x = 2$ and $x = -2$



Bounded Area $= 2\int_0^2 (x^2 + 4 - 2x^2)\, dx$

$= 2\int_0^2 (4 - x^2)\, dx$

$= 2\left[4x - \frac{x^3}{3}\right]_0^2 = \frac{32}{3}$

16. (b) $A = \int_2^4 \left(1 + \frac{8}{x^2}\right) dx = \left[x - \frac{8}{x}\right]_2^4 = 4$ sq. units

$A_1 = \int_2^a \left(1 + \frac{8}{x^2}\right) dx = \left[x - \frac{8}{x}\right]_2^a$

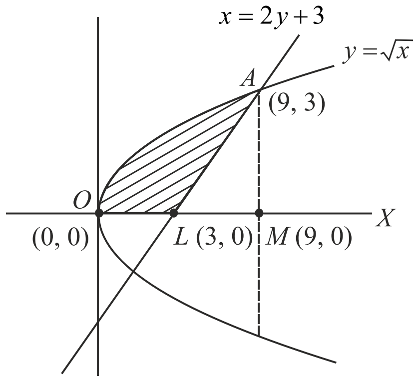
$= \left(a - \frac{8}{a} + 2\right)$ sq. units

Given, $A_1 = \frac{1}{2} A$

$\Rightarrow a - \frac{8}{a} + 2 = 2$

$\Rightarrow a^2 - 8 = 0 \Rightarrow a = 2\sqrt{2}$

17. (c) $y = \sqrt{x}$ and $x = 2y + 3$ intersects at (9, 3) and (1, – 1).



Area bounded by $y = \sqrt{x}$, $x = 2y + 3$ and X-axis in first quadrant $= \int_0^9 y\, dx -$ area of ΔALM

$= \int_0^9 \sqrt{x}\, dx - \frac{1}{2} \times 6 \times 3 = 9$

18. (c) $y = x^2, x \geq 0$ and $y = -x^2, x < 0$

$A = 2\int_0^1 y\, dx = 2\int_0^1 x^2\, dx = \frac{2}{3}$

or $\left| \int_{-1}^0 (-x^2)\, dx + \int_0^1 (x^2)\, dx \right| = \frac{2}{3}$

19. (b) $A = \int_0^{\pi/2} \sin x\, dx = [-\cos x]_0^{\pi/2} = 1$

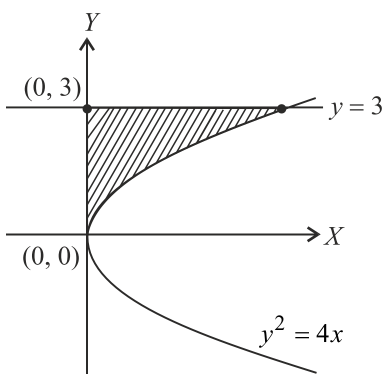
Then, area between $\sin 2x$ and X-axis from $x = 0$ to $\frac{\pi}{2}$

$A_2 = \int_0^{\pi/2} \sin 2x\, dx$

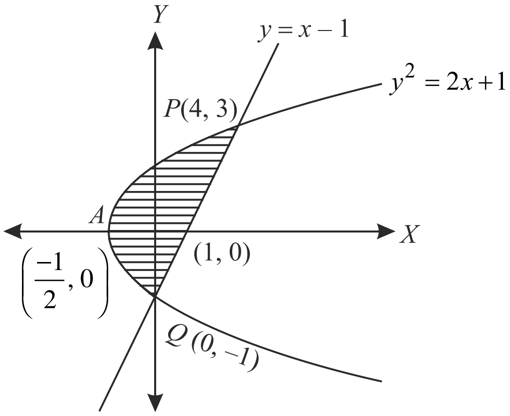
$= \left[\frac{-\cos 2x}{2}\right]_0^{\pi/2} = 1 = A$

20. (b) Bounded area

$= \int_0^3 x\, dy = \int_0^3 \frac{y^2}{4}\, dy = \frac{1}{4}\left[\frac{y^3}{3}\right]_0^3 = \frac{9}{4}$ sq. units



21. (d)



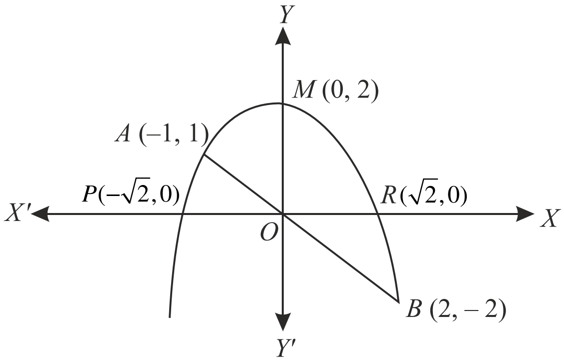
The line $y = x - 1$ meets the parabola $y^2 = 2x + 1$ at (0, – 1) and (4, 3).

So, Area $A = \int_{-1}^{3}(y+1)\,dy - \int_{-1}^{3}\left(\frac{y^2-1}{2}\right)dy$

$$= \left[\frac{y^2}{2}+y\right]_{-1}^{3} - \frac{1}{2}\left[\frac{y^3}{3}-y\right]_{-1}^{3} = \frac{16}{3}$$

22. (c) $x^2 = -(y-2)$ and the line $y + x = 0$, cuts it in the points $x^2 - x - 2 = 0$ or $(x-2)(x+1) = 0$

$\Rightarrow \quad x = 2, -1$



Points are (2, – 2) and (– 1, 1).

So, area $a = \left|\int_{-1}^{2}(2-x^2)dx\right| - \left|\int_{-1}^{2}-x\,dx\right|$

$$= \left[2x - \frac{x^3}{3} + \frac{x^2}{2}\right]_{-1}^{2}$$

$$= \left(4 - \frac{8}{3} + 2\right) - \left(-2 + \frac{1}{3} + \frac{1}{2}\right) = \frac{9}{2}$$

23. (d) $A = \int_0^{\pi/4}\sin x\,dx = 1 - \frac{1}{\sqrt{2}}$

$A_1 = \int_0^{\pi/4}\cos x\,dx = \frac{1}{\sqrt{2}}$

$\Rightarrow \quad A = 1 - A_1 \Rightarrow A_1 = 1 - A.$

24. (a) $x^2 + y^2 + 18x + 24y = 0$ is a circle whose centre is (– 9, – 12) and radius $= \sqrt{81+144} = 15$ and area $A_1 = \pi^2 r = 225\pi$

$\frac{x^2}{14} + \frac{y^2}{13} = 1$ is an ellipse,

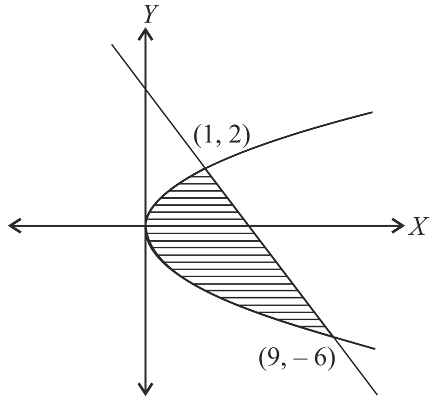
where $a = \sqrt{14}$ and $b = \sqrt{13}$

and area $A_2 = \pi ab = \sqrt{182}\pi \Rightarrow A_1 > A_2.$

25. (c) The line meets the parabola at (9, – 6) and (1, 2).

$$A = \int_{-6}^{2}\left[(3-y) - \left(\frac{y^2}{4}\right)\right]dx$$

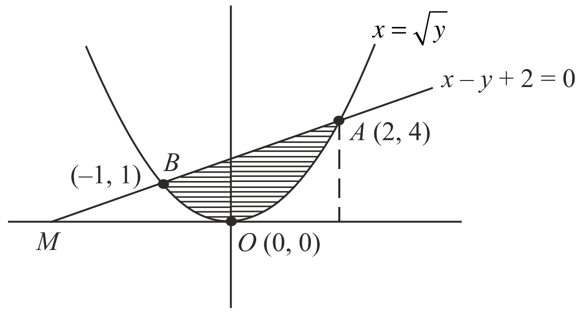
$$= \left[3y - \frac{y^2}{2} - \frac{y^3}{12}\right]_{-6}^{2}$$



$$= 6 - 2 - \frac{2}{3} - \{-18 - 18 + 18\}$$

$$= 4 - \frac{2}{3} + 18 = \frac{64}{3}$$

26. (b) Curve $x = \sqrt{y}$ and line $x - y + 2 = 0$ meets at (– 1, 1) and (2, 4).



So, bounded area $= \int_{-1}^{2}(x + 2 - x^2)\,dx$

$$= \left[2x + \frac{x^2}{2} - \frac{x^3}{3}\right]_{1}^{2} = \frac{9}{2}$$

27. (c) $f(x) = \begin{cases}\frac{3x}{2}, & x > 0\\ 2x, & x \le 0\end{cases}$ and $f(x) = \begin{cases}x^2 - 1, & x > 0\\ -x^2 - 1, & x \le 0\end{cases}$

When $x > 0$, solving $g(x) = \frac{3x}{2}$ and $f(x) = x^2 - 1$

we get $x = -\frac{1}{2}$ or $x = 2$

$\because x > 0$ so $x = 2 \Rightarrow f(x) = g(x) = 3$

When $x \leq 0$

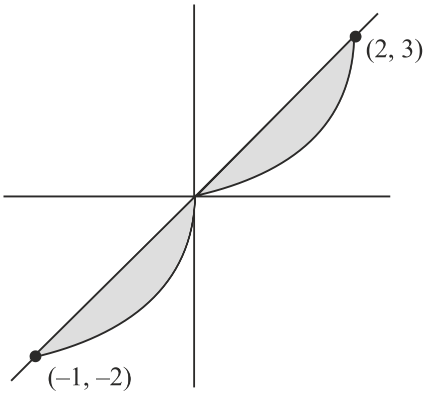
Solving $g(x) = 2x$ and $f(x) = -x^2 - 1$

$\Rightarrow g(x) = f(x) = -2$

So, intersection points of both curves are $(-1, -2)$ and $(2, 3)$.

28. (b) Area bounded by the curve

$$A = \int_{-1}^{2} \{y_1 - y_2\} dx$$



$$= \int_{-1}^{0} \{(-x^2 - 1) - 2x\}\, dx + \int_0^2 \left\{\frac{3x}{2} - (x^2 - 1)\right\} dx$$

$$= \left| -\left[\frac{x^3}{3} + x^2 + x\right]_{-1}^{0} + \left[\frac{3x^2}{4} - \frac{x^3}{3} + x\right]_0^2 \right|$$

$$= -\left\{\frac{1}{3} + 1 - 1\right\} + \left\{\frac{3}{4} \times 4 - \frac{8}{3} + 2\right\} = \frac{8}{3} \text{sq. units}$$

29. (c) $y = |x - 1| = \begin{cases} x - 1, & x > 1 \\ 1 - x, & x \leq 1 \end{cases}$

and $|x| = 2 \Rightarrow x = \pm 2$

So, when $x = 2$, then $y = 2 - 1 = 1$

and when $x = -2$, then $y = 1 - (-2) = 3$

So, points of intersection are $(2, 1)$ and $(-2, 3)$.

30. (c) Area bounded by the curves and X-axis.

$$A = \int_{-2}^{2} |x - 1|\, dx = \int_{-2}^{1} (1 - x)\, dx + \int_1^2 (x - 1)\, dx$$

$$= \left[x - \frac{x^2}{2}\right]_{-2}^{1} + \left[\frac{x^2}{2} - x\right]_1^2$$

$$= \left(1 - \frac{1}{2} + 2 + 2\right) + \left(2 - 2 - \frac{1}{2} + 1\right)$$

= 5 sq. units.

31. (a) $f(x) = |x - 1| + x^2$

Area bounded by X-axis and the ordinates $x = \frac{1}{2}$ and $x = 1$ is

$$A = \int_{1/2}^{1} \{|x - 1| + x^2\}\, dx$$

$$= \int_{1/2}^{1} \{1 - x + x^2\}\, dx$$

$$= \left[x - \frac{x^2}{2} + \frac{x^3}{3}\right]_{1/2}^{1}$$

$$= \frac{5}{12} \text{ sq. units}$$

32. (d) Area bounded by $f(x) = |x - 1| + x^2$ and ordinates $x = 1$ and $x = \frac{3}{2}$ by X-axis.

$$A = \int_1^{3/2} \{|x - 1| + x^2\}\, dx$$

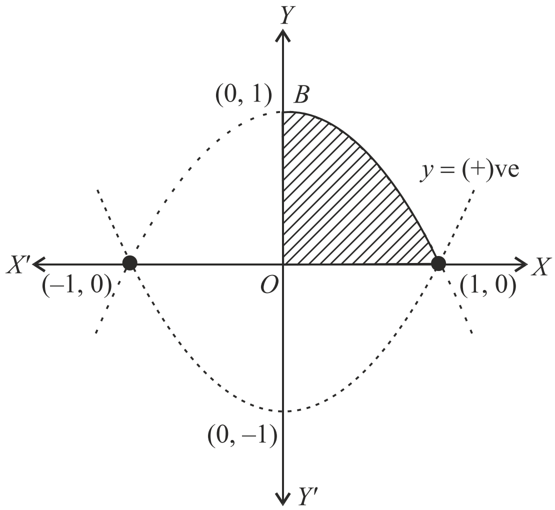
$$= \int_1^{3/2} (x - 1 + x^2)\, dx$$

$$= \left[\frac{x^2}{2} - x + \frac{x^3}{3}\right]_1^{3/2}$$

$$= \frac{9}{8} - \frac{3}{2} + \frac{9}{8} - \frac{1}{2} + 1 - \frac{1}{3}$$

$$= \frac{11}{12} \text{ sq. units}$$

33. (c) $y = 1 - x^2, y > 0$



or $x^2 = -(y - 1)$ …(i)

and $-y = 1 - x^2, y < 0$

or $x^2 = y + 1$ …(ii)

Bounded area $= 4\int_0^1 (1 - x^2)\, dx = 4\left[x - \frac{x^3}{3}\right]_0^1 = \frac{8}{3}$

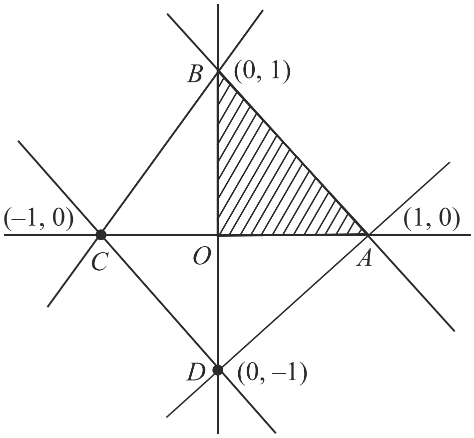
34. (b) $|x| + |y| = 1$

Represents four lines, *i.e.*,

$x + y = 1$

$x - y = 1$

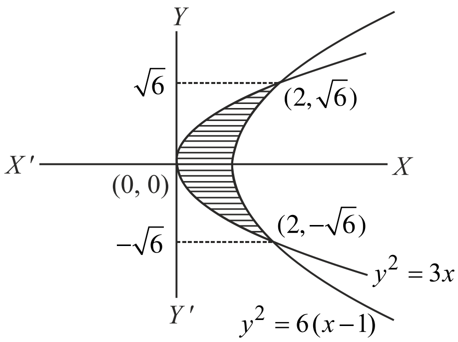
$-x + y = 1$ and $-x - y = 1$



So, bounded area = 4 × area of ΔOAB

$= 4 \times \frac{1}{2} = 2$ sq. units

35. (c) Given curves are $y^2 = 6(x - 1)$ and $y^2 = 3x$
On solving, we get
$3x = 6(x - 1) \Rightarrow 2(x - 1) = x$
$\Rightarrow x = 2$ and $y = \pm\sqrt{6}$



Bounded area is

$$A = 2\int_0^{\sqrt{6}} \left\{\left(\frac{y^2}{6} + 1\right) - \left(\frac{y^2}{3}\right)\right\} dy$$

$$= 2\int_0^{\sqrt{6}} \left(1 - \frac{y^2}{6}\right) dy = 2\left\{y - \frac{y^3}{18}\right\}_0^{\sqrt{6}}$$

$$= 2\left[\sqrt{6} - \frac{6\sqrt{6}}{18}\right] = 2\left[\sqrt{6} - \frac{\sqrt{6}}{3}\right] = \frac{4}{3}\sqrt{6}$$

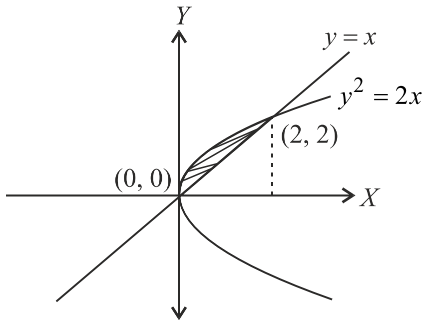
36. (b) When $y = 0$, we have $c \sin x = 0$, hence $x = (0, \pi)$.
So, we have two consecutive values 0 and π of x for which $y = 0$.
Hence, one loop of curve lies between $x = 0$ and $x = \pi$.

$\therefore$ Area of this loop $= \int_0^{\pi} y\, dx$

$$= \int_0^{\pi} c \sin x\, dx = [-a \cos x]_0^{\pi} = 2c$$

37. (b) Area bounded by $|x| < 5$ and $y = 0$ and $y = 8$ is given by rectangle ABCD.
So, bounded area = length × breadth
= 10 × 8 = 80 sq. units.

38. (a) Intersection point of line $y = x$ and parabola $y^2 = 2x$ is (0, 0) and (2, 2).



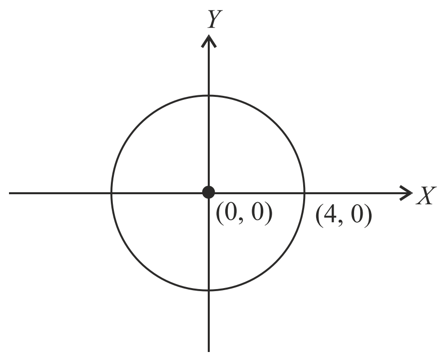
So, bounded area $= \int_0^2 (\sqrt{2x} - x)\, dx$

$$= \left[\sqrt{2}\,\frac{2}{3}x^{3/2} - \frac{x^2}{2}\right]_0^2 = \frac{8}{3} - 2 = \frac{2}{3} \text{sq. units}$$

39. (b) Given curve is

$y = \sqrt{16 - x^2} \Rightarrow x^2 + y^2 = 16$

$A = \int_{-1}^{4} \sqrt{16 - x^2}\, dx$



= area of semicircle $= \frac{1}{2}\pi r^2 = 8\pi$ sq. units

40. (b) $A = \int_{x=-1.5}^{-1.8} [x]\, dx$

As $-1.5 \le x < -1.8 \Rightarrow [x] = -2$

So, Area $A = \int_{-1.5}^{-1.8} -2\, dx$

$= -2\,[x]_{-1.5}^{-1.8}$

$= -2\,[-1.8 + 1.5]$

$= -2\,[-0.3] = 0.6$