CHAPTER

3

Function

Exercise

1. If A = {1, 2, 3, 4}, then which of the following functions is from A to itself ?
(a) $f_1 = \{(x, y) : y = x + 1)\}$
(b) $f_2 = \{(x, y) : x + y > 4\}$
(c) $f_3 = \{(x, y) : y < x\}$
(d) $f_4 = \{(x, y) : x + y = 5\}$

2. Set A has 3 elements and set B has 4 elements. The number of injections that can be defined from A to B is
(a) 144 (b) 12
(c) 24 (d) 64

3. If $f(x) = \frac{3x+2}{5x-3}$, then
(a) $f^{-1}(x) = f(x)$ (b) $f^{-1}(x) = -f(x)$
(c) $fof(x) = -x$ (d) $f^{-1}(x) = \frac{-1}{19} f(x)$

4. Let $f(x) = \frac{1}{\sqrt{2x-1}} - \sqrt{1-x^2}$, then Dom ($f$) is equal to
(a) $]\frac{1}{2}, 1]$ (b) $[-1, \infty[$
(c) $[1, \infty[$ (d) None of these

5. The domain of the function $f(x) = \sqrt{\log(2x - x^2)}$ is
(a)]0, 2[(b) [0, 2]
(c) $]-\infty, 1]$ (d) None of these

6. Let C and R denote the sets of all complex numbers and all real numbers respectively.
Let $f : C \to R : f(z) = |z|$. Then, f is
(a) one-one, into
(b) one-one, onto
(c) many-one, onto
(d) many-one, into

7. Let $A = \{x : -1 \le x \le 1, x \in I\}$ and $f : A \to A$ such that $f(x) = x|x|$, then f is
(a) a bijection
(b) injective but not surjective
(c) surjective but not injective
(d) neither injective nor surjective

8. $f\left(x + \frac{1}{x}\right) = x^2 + \frac{1}{x^2}, x \ne 0$, then $f(x) =$
(a) x^2 (b) $x^2 - 1$
(c) $x^2 - 2$ (d) None of these

9. Let A = {0, 1} and N the set of all natural numbers. Then the mapping $f : N \to A$ defined by $f(2n - 1) = 0$, $f(2n) = 1 \ \forall \ n \in N$ is
(a) one-one onto
(b) one-one into
(c) many one onto
(d) many one into

10. Given $f(x) = \log\left(\frac{1+x}{1-x}\right)$
$g(x) = \frac{3x + x^3}{1 + 3x^2}$, then $fog(x)$ equals to
(a) $-f(x)$ (b) $3f(x)$
(c) $[f(x)]^3$ (d) None of these

11. Let $f : R \to R$, $g : R \to R$ be two functions given by $f(x) = 2x - 3$, $g(x) = x^3 + 5$. Then $(fog)^{-1}(x)$ is equal to
(a) $\left(\frac{x+7}{2}\right)^{1/3}$ (b) $\left(x - \frac{7}{2}\right)^{1/3}$
(c) $\left(\frac{x-2}{7}\right)^{1/3}$ (d) $\left(\frac{x-7}{2}\right)^{1/3}$

12. Which one of the following functions $f : R \to R$ is injective ?
(a) $f(x) = |x| \ \forall \ x \in R$
(b) $f(x) = x^2 \ \forall \ x \in R$
(c) $f(x) = 11, \forall \ x \in R$
(d) $f(x) = -x \ \forall \ x \in R$

13. The function $f : N \to N$ given by $f(n) = n - (-1)^n$ is
(a) one-one and onto (b) many-one and onto
(c) one-one and into (d) many-one and into

14. Let $f(x) = \dfrac{1}{\log_{10}(1-x)} + \sqrt{x+2}$. Then,
Dom (f) = ?
(a) $(]-\infty, 1[- \{0\}) \cap ([-2, \infty[)$
(b) $[-2, \infty[- \{0\}$
(c) $[-2, 1[- \{0\}$
(d) None of the above

15. Let $f(x) = \dfrac{\sin^{-1} x}{x}$. Then, Dom (f) = ?
(a) $]-1, 1[$ (b) $]-1, 1[- \{0\}$
(c) $[-1, 1] - \{0\}$ (d) None of these

16. If $F(x) = \dfrac{x-1}{x+1}$, then $F(2x)$ is
(a) $\dfrac{F(x)+1}{F(x)+3}$ (b) $\dfrac{3F(x)+1}{F(x)+3}$
(c) $\dfrac{F(x)+3}{F(x)+1}$ (d) $\dfrac{F(x)+3}{3F(x)+1}$

17. If $f(x) = \cos(\log_e x)$, then
$f(x).f(y) - \dfrac{1}{2}\,[f(x/y) + f(xy)]$ is equal to
(a) 0 (b) $\dfrac{1}{2} f(x).f(y)$
(c) $f(x+y)$ (d) None of these

18. Let $g(x) = \sin x + \cos x$. Then Range (g) is equal to
(a) $\left[-\dfrac{1}{\sqrt{2}}, \dfrac{1}{\sqrt{2}}\right]$ (b) $\left]-\dfrac{1}{\sqrt{2}}, \dfrac{1}{\sqrt{2}}\right[$
(c) $[-\sqrt{2}, \sqrt{2}]$ (d) $]-\sqrt{2}, \sqrt{2}[$

19. Which of the following function from Z to itself is bijection ?
(a) $f(x) = x^3$ (b) $f(x) = x + 2$
(c) $f(x) = 2x + 1$ (d) $f(x) = x^2 + x$

20. Let $f : N \to N$ is defined by $f(n) = n^2 + n + 1$. Then f is
(a) one-one, onto (b) one-one, into
(c) many-one onto (d) many-one, into

21. Let A and B be two sets such that A be the empty set and B has m elements, then the total number of mappings from A to B is
(a) m (b) 0
(c) 1 (d) None of these

22. Let $A = \{x \in R : x \le 1\}$ and $f : A \to A$ be defined as $f(x) = x(2-x)$, then $f^{-1}(x)$ is
(a) $1+\sqrt{1-x}$ (b) $1-\sqrt{1-x}$
(c) $\sqrt{1-x}$ (d) $1 \pm \sqrt{1-x}$

23. If $f(x_1) - f(x_2) = f\left(\dfrac{x_1 - x_2}{1 - x_1 x_2}\right)$ for $x_1, x_2 \in (-1, 1)$, then what is $f(x)$ equal to? **[NDA-I 2016]**
(a) $\ln\left(\dfrac{1-x}{1+x}\right)$ (b) $\ln\left(\dfrac{2+x}{1-x}\right)$
(c) $\tan^{-1}\left(\dfrac{1-x}{1+x}\right)$ (d) $\tan^{-1}\left(\dfrac{1+x}{1-x}\right)$

24. Let $f(x)$ be the greatest integer function and $g(x)$ be the modulus function. What is $(fof)\left(-\dfrac{9}{5}\right) + (gog)(-2)$ equal to? **[NDA-I 2016]**
(a) – 1 (b) 0
(c) 1 (d) 2

25. What is the domain of the function $f(x) = \dfrac{1}{\sqrt{|x| - x}}$? **[NDA-II 2016]**
(a) $(-\infty, 0)$ (b) $(0, \infty)$
(c) $0 < x < 1$ (d) $x > 1$

26. What is the range of the function $y = \dfrac{x^2}{1+x^2}$? **[NDA-I 2017]**
(a) [0 1) (b) [0 1]
(c) (0 1) (d) (0 1]

27. If $f(x) = \dfrac{x}{x-1}$, then what is $\dfrac{f(a)}{f(a+1)}$ equal to ? **[NDA-I 2017]**
(a) $f\left(\dfrac{-a}{a+1}\right)$ (b) $f(a^2)$
(c) $f\left(\dfrac{1}{a}\right)$ (d) $f(-a)$

28. The function $f : X \to Y$ defined by $f(x) = \cos x$, where $x \in X$, is one-one and onto then X and Y are respectively equal to **[NDA-I 2017]**
(a) $[0, \pi]$ and $[-1, 1]$ (b) $\left[-\dfrac{\pi}{2}, \dfrac{\pi}{2}\right]$ and $[-1, 1]$
(c) $[0, \pi]$ and $(-1, 1)$ (d) $[0, \pi]$ and $[0, 1]$

29. Let $f : [-6, 6] \to R$ be defined by $f(x) = x^2 - 3$. Consider the following
1. $(fofof)(-1) = (fofof)(1)$
2. $(fofof)(-1) = 4\,(fofof)(1) = (fof)(0)$
Which of the above is/are correct? **[NDA-I 2017]**
(a) Only 1 (b) Only 2
(c) Both 1 and 2 (d) Neither 1 nor 2

30. Let $f(x) = \begin{cases} x, & x \text{ is rational} \\ 0, & x \text{ is irrational} \end{cases}$
and $g(x) = \begin{cases} 0, & x \text{ is rational} \\ x, & x \text{ is irrational} \end{cases}$
If $f : R \to R$ and $g : R \to R$ then $f - g$ is **[NDA-I 2017]**
(a) one-one and into
(b) neither one-one nor onto
(c) many-one and onto
(d) one-one and onto

ANSWERS																			
1.	(d)	**2.**	(c)	**3.**	(a)	**4.**	(a)	**5.**	(d)	**6.**	(d)	**7.**	(a)	**8.**	(c)	**9.**	(c)	**10.**	(b)
11.	(d)	**12.**	(d)	**13.**	(a)	**14.**	(c)	**15.**	(c)	**16.**	(b)	**17.**	(a)	**18.**	(c)	**19.**	(b)	**20.**	(b)
21.	(c)	**22.**	(b)	**23.**	(a)	**24.**	(b)	**25.**	(a)	**26.**	(a)	**27.**	(b)	**28.**	(a)	**29.**	(a)	**30.**	(d)

Explanations

1. (d) A = {1, 2, 3, 4}
f_1 = {(1, 2), (2, 3) (3, 4), (4, 5)}
f_2 = {(1, 4), (2, 3), (2, 4), (3, 3), (3, 4), (4, 4), (4, 1), (4, 2), (4, 3)}
f_3 = {(2, 1) (3, 1) (3, 2) ...}
and f_4 = {(1, 4) (2, 3) (3, 2) (4, 1)}
Obviously f_4 is a function from A to itself and f_1, f_2, f_3 are not the function from A to itself.

2. (c) $n(A) = 3$ and $n(B) = 4$
Number of injections from A to B = $^4P_3 = 24$

3. (a) Let $f(x) = y$, then
$$\frac{3x+2}{5x-3} = y \Rightarrow x = \frac{3y+2}{5y-3}$$
$$\therefore f^{-1}(y) = \frac{3y+2}{5y-3}$$
$$\text{or } f^{-1}(x) = \frac{3x+2}{5x-3} = f(x) \forall x$$

4. (a) $f(x) = \dfrac{1}{\sqrt{2x-1}} - \sqrt{1-x^2}$
Let $f(x) = g(x) - h(x)$, where $g(x) = \dfrac{1}{\sqrt{2x-1}}$
and $h(x) = \sqrt{1-x^2}$
$g(x)$ is defined when $2x - 1 > 0$, i.e., $x > \dfrac{1}{2}$
$\therefore$ dom $(g) =]\dfrac{1}{2}, \infty[$
$h(x)$ is defined when $1 - x^2 \geq 0$
$\Rightarrow x^2 \leq 1$
$\Rightarrow -1 \leq x \leq 1 \Rightarrow$ Dom $(h) = [-1, 1]$
$\therefore$ Domain of the given function is $]\dfrac{1}{2}, 1]$.

5. (d) Clearly $f(x)$ is defined when $\log(2x - x^2) \geq 0$
i.e., when $(2x - x^2) \geq 1$
i.e., when $1 + x^2 - 2x \leq 0$
$(1 - x)^2 \leq 0$
This happen only when $1 - x = 0$
i.e., $x = 1$
$\therefore$ dom $(f) = \{1\}$

6. (d) $(3 + 4i)$ and $(3 - 4i)$ are two different complex numbers having the same modulus, i.e.,
$f(3 + 4i) = f(3 - 4i)$ so, two different elements have the same image.
$\therefore f$ is many one.
$-1 \in R$ have no pre-image in C.
Hence, f is many one into.

7. (a) $A = \{x : -1 \leq x \leq 1, x \in I\}$
then $A = \{-1, 0, 1\}$
$f(x) = x|x|$
$f(-1) = -1$
$f(0) = 0$
$f(1) = 1$
$\Rightarrow f$ is one one onto, i.e., f is bijection.

8. (c) $f\left(x+\dfrac{1}{x}\right) = x^2 + \dfrac{1}{x^2}$ ...(1)
Let $x + \dfrac{1}{x} = z$
Making square on both sides
$$\left(x+\frac{1}{x}\right)^2 = z^2 \Rightarrow x^2 + \frac{1}{x^2} = z^2 - 2$$
Put in (1) $f(z) = z^2 - 2$
$\Rightarrow f(x) = x^2 - 2$

9. (c) $f : N \rightarrow A$ where $A = \{0, 1\}$
$\because f(2n - 1) = 0$ and $f(2n) = 1 \ \forall \ n \in N$
i.e., All odd numbers are mapped to 0 and all even numbers are mapped to 1, so function is many one onto.

10. (b) $f(x) = \log\left(\dfrac{1+x}{1-x}\right)$ and $g(x) = \dfrac{3x + x^3}{1 + 3x^2}$
$fog(x) = f\{g(x)\}$

$$= f\left\{\frac{3x+x^3}{1+3x^2}\right\} = \log\left\{\frac{1+\frac{3x+x^3}{1+3x^2}}{1-\frac{3x+x^3}{1+3x^2}}\right\}$$

$$= \log\left\{\frac{1+3x+3x^2+x^3}{1-3x+3x^2-x^3}\right\} = \log\left(\frac{1+x}{1-x}\right)^3$$

$$= 3\log\left(\frac{1+x}{1-x}\right) = 3f(x)$$

11. (d) $f(x) = 2x - 3$, $g(x) = x^3 + 5$

$fog(x) = f\{g(x)\} = f\{x^3 + 5\}$

$fog(x) = 2(x^3 + 5) - 3 = 2x^3 + 7$

Let $fog(x) = y$

$\Rightarrow y = 2x^3 + 7$

or $x = \left(\frac{y-7}{2}\right)^{1/3}$

So, $(fog)^{-1}(x) = \left(\frac{x-7}{2}\right)^{1/3}$

12. (d) An injective function means one-one

In option (d),

$f(x) = -x$

For every value of x, we get a different value of f.

Hence, it is injective.

13. (a) $f : N \to N$

s.t. $f(n) = n - (-1)^n$

$f(1) = 1 + 1 = 2, f(2) = 2 - 1 = 1$

$f(3) = 3 + 1 = 4, f(4) = 4 - 1 = 3$

i.e., $f(n) = \begin{cases} n+1; & n \text{ is odd} \\ n-1; & n \text{ is even} \end{cases}$

Clearly, function is one-one onto.

14. (c) Let $f(x) = g(x) + h(x)$, where

$g(x) = \frac{1}{\log_{10}(x-1)}$

and $h(x) = \sqrt{x+2}$

$g(x)$ is defined only when $\log_{10}(1-x) \neq 0$

and $(1 - x) > 0$

i.e., when $x \neq 0$ and $x < 1$

Dom $(g) =]-\infty, 1[- \{0\}$

$h(x)$ is defined only when $x + 2 \geq 0$

i.e., when $x \geq -2$

$\therefore$ dom $(h) = [-2, \infty[$

Dom (f) = Dom $(g) \cap$ Dom (h)

$= (]-\infty, 1[- \{0\} \cap ([-2, \infty[\ ) = [-2, 1[- \{0\}$

15. (c) $\frac{\sin^{-1}x}{x}$ is defined only when $x \neq 0$ and $x \in [-1, 1]$

$\therefore$ dom $(f) = [-1, 1] - \{0\}$

16. (b) $\frac{F(x)}{1} = \frac{x-1}{x+1} \Rightarrow \frac{F(x)+1}{F(x)-1} = \frac{2x}{-2}$

{Applying Componendo Dividendo Theorem}

$$\therefore F(2x) = \frac{2x-1}{2x+1} = \frac{2\left[\frac{F(x)+1}{1-F(x)}\right]-1}{2\left[\frac{F(x)+1}{1-F(x)}\right]+1}$$

$$= \frac{3F(x)+1}{F(x)+3}$$

17. (a) $f(x) = \cos(\log_e x)$

$$f(x).f(y) - \frac{1}{2}\left[f\left(\frac{x}{y}\right) + f(xy)\right]$$

$= \cos(\log x)\cos(\log y)$

$$-\frac{1}{2}\left[\cos\left(\log\frac{x}{y}\right) + \cos(\log xy)\right]$$

$= \cos(\log x)\cos(\log y)$

$$-\frac{1}{2}[\cos(\log x - \log y) + \cos(\log x + \log y)]$$

$= \cos(\log x)\cos(\log y)$

$$-\frac{1}{2}[2\cos(\log x)\cos(\log y)]$$

$= 0$

18. (c) $g(x) = \cos x + \sin x = \sqrt{2}\left[\frac{1}{\sqrt{2}}\cos x + \frac{1}{\sqrt{2}}\sin x\right]$

$$= \sqrt{2}\sin\left(x + \frac{\pi}{4}\right)$$

$$\Rightarrow -\sqrt{2} \leq g(x) \leq \sqrt{2} \quad \left\{\because -1 \leq \sin\left(\frac{\pi}{4}\right) \leq 1\right\}$$

Hence, Range $= [-\sqrt{2}, \sqrt{2}]$

19. (b) The function $f(x) = x^3$ is not a surjection from Z to itself because $2 \in Z$ does not have any pre image in Z.

The function $f(x) = x + 2$ is a bijection from Z to itself.

The function $f(x) = 2x + 1$ is not a surjection from Z to itself and the function $f(x) = x^2 + x$ is not an injection from Z to itself.

20. (b) $f : N \to N : f(n) = n^2 + n + 1$

$f(n_1) = f(n_2)$

$\Rightarrow n_1^2 + n_1 + 1 = n_2^2 + n_2 + 1$

$\Rightarrow (n_1 - n_2)(n_1 + n_2 + 1) = 0$

$\Rightarrow (n_1 - n_2) = 0 \qquad (\because n_1 + n_2 + 1 \neq 0)$

$\Rightarrow n_1 = n_2$

$\Rightarrow f$ is one-one.

Let $n^2 + n + 1 = 1$ for some $n \in N$

Then, $n(n + 1) = 0$

$\Rightarrow n = -1 \notin N$

$\Rightarrow$ 1 is not the image of any natural number

$\Rightarrow f$ is into.

21. (c) Given |A| = 0 and |B| = m

$f : A \to B$

Total number of mappings from A to B = $m^\circ = 1$

22. (b) Let $f(x) = y$

$\Rightarrow y = 2x - x^2$

or $x^2 - 2x + y = 0$

$$\Rightarrow x = \frac{2 \pm \sqrt{4-4y}}{2}$$

$$f^{-1}(y) = x = 1 \pm \sqrt{1-y}$$

$\because A = \{x \in R : x \le 1\}$

So, $f^{-1}(x) = 1 - \sqrt{1-x}$

23. (a) Let $f(x) = \log\left(\frac{1-x}{1+x}\right)$

$$f\left(\frac{x_1 - x_2}{1 - x_1x_2}\right) = \log\left\{\frac{1 - \frac{x_1 - x_2}{1 - x_1x_2}}{1 + \frac{x_1 - x_2}{1 - x_1x_2}}\right\}$$

$$= \log\left\{\frac{1 - x_1x_2 - x_1 + x_2}{1 - x_1x_2 + x_1 - x_2}\right\}$$

$$= \log\left\{\frac{(1-x_1)(1+x_2)}{(1+x_1)(1-x_2)}\right\}$$

$$= \log\left\{\frac{1-x_1}{1+x_1}\right\} + \log\left\{\frac{1+x_2}{1-x_2}\right\}$$

$$= \log\left\{\frac{1-x_1}{1+x_2}\right\} - \log\left\{\frac{1-x_2}{1+x_2}\right\}$$

$$= f(x_1) - f(x_2)$$

24. (b) $f(x) = [x]$ and $g(x) = |x|$

$$(fof)\left(\frac{-9}{5}\right) + (gog)(-2)$$

$$= f\left\{f\left(-\frac{9}{5}\right)\right\} + g\{g(-2)\}$$

$$= f\left\{\left[-\frac{9}{5}\right]\right\} + g\{|-2|\}$$

$= f(-2) + g(2)$

$= [-2] + |2| = -2 + 2 = 0$

25. (a) $f(x) = \dfrac{1}{\sqrt{|x| - x}}$

For domain $|x| - x > 0$

$\Rightarrow |x| > x$

This is true for all negative values of x.

Hence, domain = $(-\infty, 0)$

26. (a) Let $f(x) = y = \dfrac{x^2}{1+x^2}$

$$\Rightarrow x^2 = \frac{y}{1-y} \Rightarrow x = \sqrt{\frac{y}{1-y}} = f^{-1}(y)$$

For range $\dfrac{y}{1-y} \ge 0 \Rightarrow \dfrac{y}{y-1} \le 0$

$\Rightarrow 0 \le y < 1$

Hence, range = [0 1)

27. (b) $f(x) = \dfrac{x}{x-1}$

$$\frac{f(a)}{f(a+1)} = \frac{\frac{a}{a-1}}{\frac{a+1}{a+1-1}} = \frac{a^2}{a^2-1} = f(a^2)$$

28. (a) $f : X \to Y$ s.t. $f(x) = \cos x$ is one-one onto.

$\cos x$ is one-one for $0 \le x \le \pi$

and onto when $-1 \le \cos x \le 1$

So, X = $[0\ \pi]$ and Y = $[-1\ 1]$

29. (a) $f : [-6\ 6] \to R$ s.t. $f(x) = x^2 - 3$

$\because f(1) = (f - 1)$

so, $(fofof)(-1) = (fofof)(1)$

Hence, only Statement 1 is correct.

30. (d) $f(x) - g(x) = \begin{cases} x, & x \text{ is rational.} \\ -x, & x \text{ is irrational.} \end{cases}$

$\Rightarrow f - g$ is one-one, onto.