CHAPTER 6

Complex Numbers

Exercise

1. Let $\Delta = \begin{vmatrix} 1 & \omega & 2\omega^2 \\ 2 & 2\omega^2 & 4\omega^3 \\ 3 & 3\omega^3 & 6\omega^4 \end{vmatrix}$, where ω is the cube root of unity, then
   (a) $\Delta = 0$ (b) $\Delta = 1$
   (c) $\Delta = 2$ (d) $\Delta = 3$
2. Let z_1, z_2 be two complex numbers such that $z_1 + z_2$ and $z_1 z_2$ both are real, then
   (a) $z_1 = -z_2$ (b) $z_1 = \overline{z}_2$
   (c) $z_1 = -\overline{z}_2$ (d) $z_1 = z_2$
3. Let z be a purely imaginary number such that $\text{Im}(z) > 0$. Then arg (z) is equal to
   (a) π (b) $\frac{\pi}{2}$
   (c) 0 (d) $-\frac{\pi}{2}$
4. Let z be any non-zero complex number. Then, arg (z) + arg $(\overline{z})$ is equal to
   (a) π (b) $-\pi$
   (c) 0 (d) $\frac{\pi}{2}$
5. If the complex numbers z_1, z_2, z_3 are in AP, then they lie on a
   (a) circle (b) parabola
   (c) line (d) ellipse
6. The roots of the equation $(x - 1)^3 + 8 = 0$ are
   (a) $-1, 1 + 2\omega, 1 + 2\omega^2$
   (b) $-1, 1 - 2\omega, 1 - 2\omega^2$
   (c) $2, 2\omega, 2\omega^2$
   (d) $2, 1 + 2\omega, 1 + 2\omega^2$
7. Let z be a complex number. Then the angle between z and iz is
   (a) π (b) 0
   (c) $\frac{\pi}{2}$ (d) None of the above
8. The locus of the point z satisfying the condition $\arg\left(\frac{z-1}{z+1}\right) = \frac{\pi}{3}$ is a
   (a) parabola (b) circle
   (c) pair of straight lines (d) None of these
9. The locus of the point z satisfying the condition $|z - 3i| = 2$ is
   (a) circle (b) Y-axis
   (c) parabola (d) ellipse
10. The complex number $z = x + iy$ which satisfy the equation $|z + 1| = 1$ lie on
    (a) X-axis
    (b) Y-axis
    (c) a circle with centre $(-1, 0)$ and radius 1
    (d) None of these
11. Geometrically Re $(z^2 - i) = 2$, where $i = \sqrt{-1}$ and Re is the real part, represents
    (a) circle (b) ellipse
    (c) rectangular hyperbola (d) parabola
12. If $z = \left(\frac{\sqrt{3}}{2} + \frac{i}{2}\right)^5 + \left(\frac{\sqrt{3}}{2} - \frac{i}{2}\right)^5$, then
    (a) $\text{Re}(z) = 0$
    (b) $\text{Im}(z) = 0$
    (c) $\text{Re}(z) > 0, \text{Im}(z) > 0$
    (d) $\text{Re}(z) > 0, \text{Im}(z) < 0$
13. If z_1, z_2 be any two non-zero complex numbers such that $|z_1 + z_2| = |z_1| + |z_2|$, then arg (z_1) – arg (z_2) is equal to
    (a) $-\pi$ (b) $-\frac{\pi}{2}$

(c) 0 (d) $\frac{\pi}{2}$

14. If $2+i\sqrt{3}$ is a root of the quadratic equation $x^2 + ax + b = 0$, where $a, b \in R$, then the values of a and b are respectively
(a) 4, 7 (b) – 4, – 7
(c) – 4, 7 (d) 4, – 7

15. If the complex numbers $\sin x + i \cos 2x$ and $\cos x - i \sin 2x$ are conjugate to each other, then x is equal to
(a) $n\pi$ (b) $\left(\frac{n+1}{2}\right)\pi$
(c) 0 (d) None of these

16. The origin and the roots of the equation $z^2 + pz + q = 0$ form an equilateral triangle if
(a) $p^2 = q$ (b) $p^2 = 3q$
(c) $q^2 = 3p$ (d) $q^2 = p$

17. If z is a complex number such that $\left|\frac{z-5i}{z+5i}\right| = 1$, then the locus of z is
(a) X-axis
(b) straight line $y = 5$
(c) a circle passing through the origin
(d) None of these

18. If $\frac{1}{x}+x = 2\cos\theta$, then $x^n + \frac{1}{x^n}$ is equal to
(a) $2 \cos n\theta$ (b) $2 \sin n\theta$
(c) $\cos n\theta$ (d) $\sin n\theta$

19. If ω is a complex cube root of unity, then the value of
$\frac{a+b\omega+c\omega^2}{c+a\omega+b\omega^2}+\frac{a+b\omega+c\omega^2}{b+c\omega+a\omega^2}$ is
(a) 1 (b) 0
(c) 2 (d) – 1

20. The expression $\frac{(1+i)^n}{(1-i)^{n-2}}$ is equal to
(a) i^{n+1} (b) i^{n-1}
(c) $2i^{n-1}$ (d) None of these

21. If $c^2 + s^2 = 1$, then $\frac{1+c+is}{1+c-is}$ is equal to
(a) $c + is$ (b) $c - is$
(c) $s + ic$ (d) $s - ic$

22. If $z = 1 + i\sqrt{3}$, then $|\arg(z)| + |\arg(\bar{z})|$ is equal to
(a) $\frac{\pi}{3}$ (b) $\frac{2\pi}{3}$
(c) 0 (d) $\frac{\pi}{2}$

23. $(1 - 2i)^{-3}$ is equal to
(a) $-\frac{11}{125}-\frac{2}{125}i$ (b) $-\frac{11}{125}+\frac{2}{125}i$
(b) $\frac{11}{125}-\frac{2}{125}i$ (d) None of these

24. If $z = \frac{\sqrt{3}+i}{2}$, then z^{69} equals to
(a) $-i$ (b) i
(c) 1 (d) – 1

25. If $|z_1| = |z_2|$ and $\arg(z_1) + \arg(z_2) = 0$, then
(a) $z_1 = z_2$ (b) $z_1 = \bar{z}_2$
(c) $z_1 z_2 = 1$ (d) None of these

26. $(1 + i)^8 + (1 - i)^8$ is equal to
(a) 2^8 (b) 2^5
(c) $2^4 \cos\frac{\pi}{4}$ (d) $2^8 \cos\frac{\pi}{8}$

27. The amplitude of $\frac{1+i\sqrt{3}}{\sqrt{3}+1}$ is
(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{4}$
(c) $\frac{\pi}{3}$ (d) None of these

28. $(\sin\theta + i\cos\theta)^4$ is equal to
(a) $\sin 4\theta + i\cos 4\theta$ (b) $\sin 4\theta - i\cos 4\theta$
(c) $\cos 4\theta + i\sin 4\theta$ (d) $\cos 4\theta - i\sin 4\theta$

29. The smallest positive integer n for which $(1 + i)^{2n} = (1 - i)^{2n}$ is
(a) 4 (b) 8
(c) 2 (d) 12

30. The complex number $\left(\frac{1+2i}{1-i}\right)$ lies in the
(a) I quadrant (b) II quadrant
(c) III quadrant (d) IV quadrant

31. $\left(\frac{1+\cos\frac{\pi}{8}+i\sin\frac{\pi}{8}}{1+\cos\frac{\pi}{8}-i\sin\frac{\pi}{8}}\right)^8$ is equal to
(a) $1 + i$ (b) $1 - i$
(c) 1 (d) – 1

32. Area of the triangle formed by 3 complex numbers $1 + i$, $i - 1$, $2i$ in the Argand plane is
(a) $\frac{1}{2}$ (b) 1
(c) $\sqrt{2}$ (d) 2

33. If ω is a complex cube root of unity, then $(1 - \omega + \omega^2)^6 + (1 - \omega^2 + \omega)^6$ is equal to
(a) 0 (b) 6
(c) 64 (d) 128

34. The locus represented by the equation $|z - 1| = |z - i|$ is
(a) a circle of radius 1
(b) an ellipse with foci at 1 and $-i$
(c) a line passing through the origin
(d) a circle on the line joining 1 and $-i$ as diameter

35. The value of $\left(\frac{1+i\sqrt{3}}{1-i\sqrt{3}}\right)^6+\left(\frac{1-i\sqrt{3}}{1+i\sqrt{3}}\right)^6$ is
(a) 2 (b) – 2
(c) 1 (d) 0

36. The value of $\frac{i^{592}+i^{590}+i^{588}+i^{586}+i^{584}}{i^{582}+i^{580}+i^{578}+i^{576}+i^{574}}$ is
(a) – 1 (b) – 2
(c) – 3 (d) – 4

37. Which of the following is correct ?
(a) $2+3i>1+4i$ (b) $6+2i>3+3i$
(c) $5+8i>5+7i$ (d) None of these

38. If $x+iy=\sqrt{\frac{a+ib}{c+id}}$, then $(x^2+y^2)^2$ is equal to
(a) $\frac{a^2+b^2}{c^2+d^2}$ (b) $\frac{c^2+d^2}{a^2+b^2}$
(c) $\frac{a^2-b^2}{c^2-d^2}$ (d) None of these

39. The conjugate of complex number is $\frac{1}{i-1}$. Then the complex number is
(a) $\frac{-1}{i-1}$ (b) $\frac{1}{i+1}$
(c) $\frac{-1}{i+1}$ (d) $\frac{1}{i-1}$

40. For any integer n, arg of $z=\frac{(\sqrt{3}+i)^{4n+1}}{(1-i\sqrt{3})^{4n}}$ is
(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{3}$
(c) $\frac{\pi}{2}$ (d) $\frac{2\pi}{3}$

41. If z_1 and z_2 be two complex numbers such that $|z_1-z_2|=|z_1|-|z_2|$, then arg $\frac{z_1}{z_2}$ is
(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{2}$
(c) 0 (d) None of these

42. If arg $z=\pi/4$, then
(a) Re z^2 = Im z^2 (b) Im $z^2=0$
(c) Re $z^2=0$ (d) None of these

43. If z_1, z_2 are conjugate complex numbers, and z_3, z_4 are also conjugate, then arg $\frac{z_3}{z_2}$ is equal to
(a) $\arg\frac{z_1}{z_4}$ (b) $\arg\frac{z_4}{z_1}$
(c) $\arg\frac{z_2}{z_4}$ (d) $\arg\frac{z_1}{z_3}$

44. (z_1, z_2) and (z_3, z_4) are two pairs of conjugate complex numbers then $\arg\frac{z_1}{z_3}+\arg\frac{z_2}{z_4}$ is equal to
(a) 0 (b) $\frac{\pi}{2}$
(c) π (d) $-\frac{\pi}{2}$

45. If z_1 and z_2 are two complex numbers satisfying the equation $\left|\frac{z_1+z_2}{z_1-z_2}\right|=1$, then $\frac{z_1}{z_2}$ is a number which is
(a) positive real (b) negative real
(c) zero (d) purely imaginary

46. If $z=x+iy$ and $\omega=\frac{1-iz}{z-i}$, then $|\omega|=1$ implies that in the complex plane
(a) z lies on the imaginary axis
(b) z lies on the real axis
(c) z lies on the unit axis
(d) None of the above

47. If $|z+\bar{z}|+|z-\bar{z}|=2$, then z lies on
(a) a straight line (b) a square
(c) a circle (d) None of these

48. If the number $\frac{z-1}{z+1}$ is purely imaginary, then
(a) $|z|>1$ (b) $|z|<1$
(c) $|z|=1$ (d) $|z|>2$

49. If $\omega^3=1$ and $\omega\neq1$, then $(1+\omega)(1+\omega^2)(1+\omega^4)(1+\omega^8)$ is equal to
(a) 3 (b) – 3
(c) 9 (d) 1

50. If ω is a cube root of unity, then the value of $(1+\omega-\omega^2)(1-\omega+\omega^2)$ is
(a) 1 (b) 0
(c) 2 (d) 4

51. $1+i^2+i^4+\dots\dots\dots+i^{2n}$ is
(a) positive (b) negative
(c) 0 (d) cannot be determined

52. $i^2+i^4+i^6+\dots\dots$ $(2n+1)$ terms is
(a) i (b) $-i$
(c) 1 (d) –1

53. If $z=x+iy$, $z^{1/3}=a-ib$ and $\frac{x}{a}-\frac{y}{b}=\lambda(a^2-b^2)$, then λ is equal to
(a) 3 (b) 4
(c) 2 (d) None of these

54. If $x=a+b$, $y=a\omega+b\omega^2$, $z=a\omega^2+b\omega$, then $x^3+y^3+z^3$ is equal to
(a) $3(a^3+b^3)$
(b) $3(a^3-b^3)$
(c) 0
(d) $a^3+b^3+c^3-3abc$

55. If $z=\frac{\sqrt{3}+i}{2}$ then $(z^{101}+i^{103})^{105}$ equals to

(a) z (b) z^2
(c) z^3 (d) None of these

56. If $(\sqrt{3}+i)^{100}=2^{99}(a+ib)$, then a^2+b^2 is equal to

(a) $\sqrt{2}$ (b) 4
(c) $\sqrt{3}$ (d) None of these

57. If $(\sqrt{3}+i)^{10}=a+ib,$ then a and b are respectively

(a) 128 and $128\sqrt{3}$ (b) 64 and $-64\sqrt{3}$
(c) 512 and $-512\sqrt{3}$ (d) None of these

58. If $(x+iy)^5=p+iq$, then $(y+ix)^5$ is equal to

(a) $q+ip$ (b) $p-iq$
(c) $q-ip$ (d) $-p-iq$

Directions (Q. Nos. 59-60):
Let z_1, z_2 and z_3 be non-zero complex numbers satisfying $z^2=i\bar{z}$, where $i=\sqrt{-1}$.

59. What is $z_1+z_2+z_3$ equal to? **[NDA-I 2016]**

(a) i (b) $-i$
(c) 0 (d) 1

60. Consider the following statements: **[NDA-I 2016]**

1. $z_1z_2z_3$ is purely imaginary.
2. $z_1z_2+z_2z_3+z_3z_1$ is purely real.

Which of the above statements is/are correct?

(a) Only 1 (b) Only 2
(c) Both 1 and 2 (d) Neither 1 nor 2

Directions (Q. Nos. 61-62):
Let z be a complex number satisfying

$$\left|\frac{z-4}{z-8}\right|=1 \text{ and } \left|\frac{z}{z-2}\right|=\frac{3}{2}$$

61. What is $|z|$ equal to? **[NDA-I 2016]**

(a) 6 (b) 12
(c) 18 (d) 36

62. What is $\left|\frac{z-6}{z+6}\right|$ equal to ? **[NDA-I 2016]**

(a) 3 (b) 2
(c) 1 (d) 0

63. Suppose, ω is a cube root of unity with $\omega \neq 1$. Suppose, P and Q are the points on the complex plane defined by ω and ω^2. If O is the origin, then what is the angle between OP and OQ? **[NDA-I 2016]**

(a) 60° (b) 90°
(c) 120° (d) 150°

64. If $z=x+iy=\left(\frac{1}{\sqrt{2}}-\frac{i}{\sqrt{2}}\right)^{-25}$, where $i=\sqrt{-1}$, then what is the fundamental amplitude of $\frac{z-\sqrt{2}}{z-i\sqrt{2}}$? **[NDA-I 2016]**

(a) π (b) $\frac{\pi}{2}$
(c) $\frac{\pi}{3}$ (d) $\frac{\pi}{4}$

65. Suppose ω_1 and ω_2 are two distinct cube roots of unity different from 1. Then, what is $(\omega_1-\omega_2)^2$ equal to? **[NDA-I 2016]**

(a) 3 (b) 1
(c) – 1 (d) – 3

66. What is $\sqrt{\frac{1+\omega^2}{1+\omega}}$ equal to, where ω is the cube root of unity? **[NDA-II 2016]**

(a) 1 (b) ω
(c) ω^2 (d) $i\omega$, where $i=\sqrt{-1}$

67. What is $\omega^{100}+\omega^{200}+\omega^{300}$ equal to, where ω is the cube root of unity? **[NDA-II 2016]**

(a) 1 (b) 3ω
(c) $3\omega^2$ (d) 0

68. If $\text{Re}\left(\frac{z-1}{z+1}\right)=0$, where $z=x+iy$ is a complex number, then which one of the following is correct? **[NDA-II 2016]**

(a) $z=1+i$ (b) $|z|=2$
(c) $z=1-i$ (d) $|z|=1$

69. If $z=\left(\frac{\sqrt{3}}{2}+\frac{i}{2}\right)^{107}+\left(\frac{\sqrt{3}}{2}-\frac{i}{2}\right)^{107}$, then what is the imaginary part of z equal to? **[NDA-II 2016]**

(a) 0 (b) $\frac{1}{2}$
(c) $\frac{\sqrt{3}}{2}$ (d) 1

70. What is the number of distinct solutions of the equation $z^2+|z|=0$ (where z is a complex number)? **[NDA-II 2016]**

(a) One (b) Two
(c) Three (d) Five

ANSWERS																			
1.	(a)	**2.**	(b)	**3.**	(c)	**4.**	(c)	**5.**	(c)	**6.**	(b)	**7.**	(c)	**8.**	(b)	**9.**	(a)	**10.**	(c)
11.	(c)	**12.**	(b)	**13.**	(c)	**14.**	(c)	**15.**	(d)	**16.**	(b)	**17.**	(a)	**18.**	(a)	**19.**	(d)	**20.**	(c)
21.	(a)	**22.**	(b)	**23.**	(a)	**24.**	(a)	**25.**	(b)	**26.**	(b)	**27.**	(c)	**28.**	(d)	**29.**	(c)	**30.**	(b)
31.	(d)	**32.**	(b)	**33.**	(d)	**34.**	(c)	**35.**	(a)	**36.**	(a)	**37.**	(d)	**38.**	(a)	**39.**	(c)	**40.**	(a)
41.	(c)	**42.**	(c)	**43.**	(a)	**44.**	(a)	**45.**	(d)	**46.**	(b)	**47.**	(b)	**48.**	(c)	**49.**	(d)	**50.**	(d)
51.	(d)	**52.**	(d)	**53.**	(b)	**54.**	(a)	**55.**	(c)	**56.**	(b)	**57.**	(c)	**58.**	(a)	**59.**	(c)	**60.**	(c)
61.	(a)	**62.**	(d)	**63.**	(c)	**64.**	(a)	**65.**	(d)	**66.**	(b)	**67.**	(d)	**68.**	(d)	**69.**	(a)	**70.**	(c)

Explanations

1. (a) $\Delta = \begin{vmatrix} 1 & \omega & 2\omega^2 \\ 2 & 2\omega^2 & 4\omega^3 \\ 3 & 3\omega^3 & 6\omega^4 \end{vmatrix}$

$$\Delta = \omega \times 2\omega^2 \begin{vmatrix} 1 & 1 & 1 \\ 2 & 2\omega & 2\omega \\ 3 & 3\omega^2 & 3\omega^2 \end{vmatrix}$$

$= 0$ $\{\because C_2$ and C_3 are identical$\}$

2. (b) Im $(z_1 + z_2) = 0$ and Im $(z_1 z_2) = 0$.
It is possible only when z_1 and z_2 are conjugate of each other.
i.e., $z_1 = \bar{z}_2$ or $z_2 = \bar{z}_1$

3. (c) Im $(z) > 0$ and Re $(z) = 0$

$\arg(z)\ \tan^{-1}\left\{\dfrac{\text{Im}(z)}{\text{Re}(z)}\right\} = \tan^{-1}\infty = \dfrac{\pi}{2}$

4. (c) $\arg(z) + \arg(\bar{z}) = \arg(z\bar{z}) = \arg(|z|^2)$

$= \tan^{-1} 0 = 0$ $\{\because \text{Im}(|z|^2) = 0\}$

5. (c) z_1, z_2, z_3 are in A.P.
$\Rightarrow 2z_2 = z_1 + z_3 \Rightarrow 2x_2 + 2iy_2 = (x_1 + x_3) + i(y_1 + y_3)$

$\Rightarrow x_2 = \dfrac{x_1 + x_3}{2}$ and $y_2 = \dfrac{y_1 + y_3}{2}$

Hence, z_1, z_2, z_3 lies on a straight line.

6. (b) $(x - 1)^3 + 8 = 0$
$\because x = -1,\ 1 - 2\omega,\ 1 - 2\omega^2$ satisfies the given equation, so these are the roots of given equation.

7. (c) Angle between z and $iz = \dfrac{\pi}{2}$

8. (b) $\arg\left(\dfrac{z-1}{z+1}\right) = \dfrac{\pi}{3}$

$\arg\left[\dfrac{(x-1)+iy}{(x+1)+iy}\right] = \dfrac{\pi}{3}$ $\left\{\because \arg\dfrac{z_1}{z_2} = \arg z_1 - \arg z_2\right\}$

$\Rightarrow \tan^{-1}\dfrac{y}{x-1} - \tan^{-1}\dfrac{y}{x+1} = \dfrac{\pi}{3}$

$\Rightarrow \tan^{-1}\left[\dfrac{\dfrac{y}{x-1} - \dfrac{y}{x+1}}{1 + \dfrac{y}{x-1}\cdot\dfrac{y}{x+1}}\right] = \dfrac{\pi}{3}$

$\Rightarrow \dfrac{xy + y - xy + y}{x^2 - 1 + y^2} = \sqrt{3}$ or $x^2 + y^2 - 1 = \dfrac{2y}{\sqrt{3}}$

This is the equation of circle.
Hence, the locus is circle.

9. (a) $|z - 3i| = 2$
$\Rightarrow |x + i(y - 3)| = 2$
$\Rightarrow x^2 + (y - 3)^2 = 2^2$ $\{\because (x - h)^2 + (y - k)^2 = r^2\}$
Hence, the locus of the point satisfying the given condition is circle.

10. (c) $|z + 1| = 1$
$\Rightarrow |(x + 1) + iy| = 1$
$\Rightarrow (x + 1)^2 + y^2 = 1$ $\{\because (x - h)^2 + (y - k)^2 = r^2\}$
Hence, the locus is the circle with centre $(-1, 0)$ and radius 1.

11. (c) Re $(z^2 - i) = 2$
Let $z = x + iy$
$\Rightarrow$ Re $\{(x + iy)^2 - 1\} = 2$
$\Rightarrow$ Re $\{(x^2 - y^2) + i(2xy - 1)\} = 2$
$\Rightarrow x^2 - y^2 = 2$
It is the equation of rectangular hyperbola.

12. (b) $\dfrac{\sqrt{3}}{2} + \dfrac{i}{2} = r(\cos\theta + i\sin\theta)$

$\Rightarrow r = 1$ and $\theta = \dfrac{\pi}{6}$

So, $\dfrac{\sqrt{3}}{2} \pm \dfrac{i}{2} = \dfrac{\cos\pi}{6} \pm i\sin\dfrac{\pi}{6}$

$z = \left(\dfrac{\sqrt{3}}{2} + \dfrac{i}{2}\right)^5 + \left(\dfrac{\sqrt{3}}{2} - \dfrac{i}{2}\right)^5$

$$= \left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right)^5 + \left(\cos\frac{\pi}{6} - i\sin\frac{\pi}{6}\right)^5$$

$$= \cos\frac{5\pi}{6} + i\sin\frac{5\pi}{6} + \cos\frac{5\pi}{6} - i\sin\frac{5\pi}{6}$$

$$\Rightarrow z = 2\cos\frac{5\pi}{6} \Rightarrow \text{Im}(z) = 0$$

13. (c) Given, $|z_1 + z_2| = |z_1| + |z_2|$
It is possible only when $z_1 = z_2$
So, arg (z_1) – arg (z_2) = 0

14. (c) Given one root = $2 + i\sqrt{3}$
$\Rightarrow$ Other root = $2 - i\sqrt{3}$
So, equation is
x^2 – (sum of the roots) x + product of the roots = 0
$\Rightarrow x^2 - 4x + 7 = 0$
On comparing with $x^2 + ax + b = 0$
$a = -4$ and $b = 7$

15. (d) Since, Re (z) = Re $(\bar{z})$ and Im (z) = – Im $(\bar{z})$
So, $\sin x = \cos x \Rightarrow x = \dfrac{\pi}{4}$
and $\cos 2x = -(-\sin 2x)$
$\Rightarrow 2x = \dfrac{\pi}{4} \Rightarrow x = \dfrac{\pi}{8}$
Hence, no value of x satisfies the condition of being the conjugate numbers.

16. (b) Given origin (0, 0) and the roots z_1, z_2 of equation $z^2 + pz + q = 0$ forms an equilateral triangle.
$z_1 + z_2 = -p$ and $z_1 z_2 = q$
Let $z_3 = (0, 0)$
For equilateral triangle
$$z_1^2 + z_2^2 + z_3^2 = z_1z_2 + z_2z_3 + z_3z_1$$
$\Rightarrow z_1^2 + z_2^2 = z_1z_2$
$\Rightarrow (z_1 + z_2)^2 - 2z_1z_2 = z_1z_2$
$\Rightarrow (z_1 + z_2)^2 = 3z_1z_2$
$\Rightarrow p^2 = 3q$

17. (a) $\left|\dfrac{z-5i}{z+5i}\right| = 1$
$\Rightarrow |z - 5i| = |z + 5i|$
$\Rightarrow |x - iy - 5i|^2 = |x + iy + 5i|^2$
$\Rightarrow x + (y - 5)^2 = x^2 + (y + 5)^2$
$\Rightarrow y = 0$
Hence, the locus is X-axis.

18. (a) $x + \dfrac{1}{x} = 2\cos\theta$
$\Rightarrow x^2 - 2\cos\theta x + 1 = 0$
$\Rightarrow x = \cos\theta \pm i\sin\theta$
Let $x = \cos\theta + i\sin\theta$ and $\dfrac{1}{x} = \cos\theta - i\sin\theta$
$$x^n + \frac{1}{x^n} = (\cos\theta + i\sin\theta)^n + (\cos\theta - i\sin\theta)^n$$
$= \cos n\theta + i\sin n\theta + \cos n\theta - i\sin n\theta = 2\cos n\theta$

19. (d) $\dfrac{a+b\omega+c\omega^2}{c+a\omega+b\omega^2} + \dfrac{a+b\omega+c\omega^2}{b+c\omega+a\omega^2}$
$$= \frac{\omega^2(a+b\omega+c\omega^2)}{\omega^2(c+a\omega+b\omega^2)} + \frac{\omega(a+b\omega+c\omega^2)}{\omega(b+c\omega+a\omega^2)}$$
$$= \frac{\omega^2(a+b\omega+c\omega^2)}{a+b\omega+c\omega^2} + \frac{\omega(a+b\omega+c\omega^2)}{(a+b\omega+c\omega^2)}$$
$= \omega^2 + \omega = -1 \qquad (\because 1 + \omega + \omega^2 = 0)$

20. (c) $\dfrac{(1+i)^n}{(1-i)^{n-2}} = \left(\dfrac{1+i}{1-i}\right)^n (1-i)^2$
$$= \left[\frac{(1+i)^2}{(1-i^2)}\right]^n (1 + i^2 - 2i)$$
$$= i^n(-2i) = -2i^{n+1} = \frac{2i^{n+1}}{i^2} = 2i^{n-1}$$

21. (a) $\dfrac{1+c+is}{1+c-is} = \dfrac{[(1+c)+is]^2}{(1+c)^2 - i^2s^2}$
$$= \frac{1+c^2+2c-s^2+2i(1+c)s}{1+c^2+2c+s^2} \qquad \{\because c^2 + s^2 = 1\}$$
$$= \frac{c^2+s^2+c^2+2c-s^2+2i(1+c)s}{2+2c}$$
$$= \frac{2c(1+c)+2i(1+c)s}{2(1+c)} = c + is$$

22. (b) $z = 1 + i\sqrt{3}$
$$\arg z = \tan^{-1}\frac{\sqrt{3}}{1} = \frac{\pi}{3}$$
$\bar{z} = 1 - i\sqrt{3}$
$$\arg \bar{z} = \tan^{-1}\left(\frac{-\sqrt{3}}{1}\right) = -\frac{\pi}{3}$$
$$|\arg(z)| + |\arg(\bar{z})| = \frac{\pi}{3} + \frac{\pi}{3} = \frac{2\pi}{3}$$

23. (a) $(1-2i)^{-3} = \dfrac{1}{(1-2i)^3} = \dfrac{1}{1-8i^3-6i(1-2i)}$
$$= \frac{1}{-11+2i} \times \frac{-11-2i}{-11-2i} = \frac{-11-2i}{125} = \frac{-11}{125} - \frac{2i}{125}$$

24. (a) $z = \dfrac{\sqrt{3}}{2} + \dfrac{i}{2} = \cos\dfrac{\pi}{6} + i\sin\dfrac{\pi}{6}$
$$z^{69} = \left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right)^{69}$$
$$= \cos\frac{69\pi}{6} + i\sin\frac{69\pi}{6}$$

$$= \cos\frac{23\pi}{2} + i\sin\frac{23\pi}{2}$$

$$= \cos\left(11\pi + \frac{\pi}{2}\right) + i\sin\left(11\pi + \frac{\pi}{2}\right)$$

$$= -\cos\frac{\pi}{2} - i\sin\frac{\pi}{2} = -i$$

25. (b) Given $|z_1| = |z_2|$ ...(i)
and $\arg(z_1) + \arg(z_2) = 0$
$\Rightarrow \arg(z_1 z_2) = 0$
$\Rightarrow \text{Im}(z_1 z_2) = 0$ ...(ii)
From eqs. (i) and (ii),
It is clear that z_1, z_2 are conjugate of each other.
i.e., $z_1 = \overline{z_2}$

26. (b) $(1+i)^8 + (1-i)^8 = \{(1+i)^2\}^4 + \{(1-i)^2\}^4$
$= \{1 + i^2 + 2i\}^4 + \{1 + i^2 - 2i\}^4$
$= 16i^4 + 16i^4 = 32$

27. (c) $z = \frac{1+i\sqrt{3}}{\sqrt{3}+1}$

$$\arg(z) = \tan^{-1}\frac{\sqrt{3}/(\sqrt{3}+1)}{1/(\sqrt{3}+1)} = \tan^{-1}\sqrt{3} = \frac{\pi}{3}$$

28. (d) $(\sin\theta + i\cos\theta)^4$
$= \{\cos(\pi/2 - \theta) + i\sin(\pi/2 - \theta)\}^4$

$$= \cos 4\left(\frac{\pi}{2} - \theta\right) + i\sin 4\left(\frac{\pi}{2} - \theta\right)$$

$= \cos(2\pi - 4\theta) + i\sin(2\pi - 4\theta)$
$= \cos 4\theta - i\sin 4\theta$

29. (c) $(1+i)^{2n} = (1-i)^{2n}$

$$\Rightarrow \left(\frac{1+i}{1-i}\right)^{2n} = 1$$

$$\Rightarrow \left(\frac{1+i}{1-i} \times \frac{(1+i)}{1+i}\right)^{2n} = 1 \Rightarrow \left(\frac{2i}{2}\right)^{2n} = 1$$

$\Rightarrow i^{2n} = 1 = i^4 \Rightarrow n = 2$
So, smallest value of $n = 2$

30. (b) $\frac{1+2i}{1-i} \times \frac{1+i}{1+i} = \frac{-1}{2} + \frac{3i}{2} = \left(\frac{-1}{2}, \frac{3}{2}\right)$
So, the complex number lies in II quadrant.

31. (d) $\left\{\dfrac{1+\cos\frac{\pi}{8} + i\sin\frac{\pi}{8}}{1+\cos\frac{\pi}{8} - i\sin\frac{\pi}{8}}\right\}^8$

$$= \left\{\frac{\cos\frac{\pi}{16} + i\sin\frac{\pi}{16}}{\cos\frac{\pi}{16} - i\sin\frac{\pi}{16}}\right\}^8 = \frac{\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}}{\cos\frac{\pi}{2} - i\sin\frac{\pi}{2}} = -1$$

32. (b) $1 + i = (1, 1) = (x_1, y_1)$, $-1 + i = (-1, 1) = (x_2, y_2)$
and $2i = (0, 2) = (x_3, y_3)$

$$\text{So, area} = \frac{1}{2}\begin{vmatrix} 1 & x_1 & y_1 \\ 1 & x_2 & y_2 \\ 1 & x_3 & y_3 \end{vmatrix}$$

$$\Delta = \frac{1}{2}\begin{vmatrix} 1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 0 & 2 \end{vmatrix} = -1$$

Neglecting (–)ve sign because area is never (–)ve.

33. (d) $(1 - \omega + \omega^2)^6 + (1 - \omega^2 + \omega)^6$
$= (-2\omega)^6 + (-2\omega^2)^6 \quad \{\because 1 + \omega + \omega^2 = 0\}$
$= 2^6\omega^6 + 2^6\omega^{12} \quad \{\because \omega^3 = 1\}$
$= 64 + 64 = 128$

34. (c) $|z - 1| = |z - i|$
$\Rightarrow |(x-1) + iy| = |x + i(y-1)|$
$\Rightarrow (x-1)^2 + y^2 = x^2 + (y-1)^2 \Rightarrow x = y$
i.e., a line passing through the origin.

35. (a) $\left(\frac{1+i\sqrt{3}}{1-i\sqrt{3}}\right)^6 + \left(\frac{1-i\sqrt{3}}{1+i\sqrt{3}}\right)^6$

$$= \left[\frac{\frac{-1-i\sqrt{3}}{2}}{\frac{-1+i\sqrt{3}}{2}}\right]^6 + \left[\frac{\frac{-1+i\sqrt{3}}{2}}{\frac{-1-i\sqrt{3}}{2}}\right]^6$$

$$\because \omega = \frac{-1+i\sqrt{3}}{2} \text{ and } \omega^2 = \frac{-1-i\sqrt{3}}{2}$$

$$= \left(\frac{\omega^2}{\omega}\right)^6 + \left(\frac{\omega}{\omega^2}\right)^6 = \omega^6 + \frac{1}{\omega^6} = 1 + 1 = 2$$

36. (a) $\frac{i^{592} + i^{590} + i^{588} + i^{586} + i^{584}}{i^{582} + i^{580} + i^{578} + i^{576} + i^{574}}$

$$= \frac{i^{584}[i^8 + i^6 + i^4 + i^2 + 1]}{i^{574}[i^8 + i^6 + i^4 + i^2 + 1]}$$

$= i^{584-574} = i^{10} = i^2 = -1$

37. (d) Two complex numbers cannot be compared.

38. (a) $x + iy = \sqrt{\frac{a+ib}{c+id}}$ ...(i)

$\Rightarrow x - iy = \sqrt{\frac{a-ib}{c-id}}$ ...(ii)

Multiplying both equations,

$$x^2 + y^2 = \sqrt{\frac{a^2+b^2}{c^2+d^2}} \Rightarrow (x^2+y^2)^2 = \frac{a^2+b^2}{c^2+d^2}$$

39. (c) Let $z = \frac{1}{i-1} = \frac{1}{-1+i} \times \frac{-1-i}{-1-i}$

$$z = \frac{-1-i}{2}$$

So, $\bar{z} = \dfrac{-1+i}{2} \times \dfrac{-1-i}{-1-i} = \dfrac{1}{-1-i}$ or $\bar{z} = \dfrac{-1}{i+1}$

40. (a) $\dfrac{(\sqrt{3}+i)^{4n+1}}{(1-i\sqrt{3})^{4n}} = \dfrac{2^{4n+1}\left(\dfrac{\sqrt{3}}{2}+\dfrac{i}{2}\right)^{4n+1}}{2^{4n}\left(\dfrac{1}{2}-\dfrac{i\sqrt{3}}{2}\right)^{4n}}$

$= \dfrac{2\left\{\cos\dfrac{\pi}{6}+i\sin\dfrac{\pi}{6}\right\}^{4n+1}}{\left\{\cos\dfrac{\pi}{3}-i\sin\dfrac{\pi}{3}\right\}^{4n}}$

$= 2\dfrac{e^{i(4n+1)\pi/6}}{e^{-i(4n)\pi/3}} = 2e^{i\left\{\frac{(4n+1)\pi}{6}+\frac{4n\pi}{3}\right\}}$

$= 2e^{i(12n+1)\pi/6} = 2e^{2n\pi i}.e^{i\pi/6} = 2e^{i\pi/6}$

So, arg $z = \pi/6$

41. (c) Given $|z_1 - z_2| = |z_1| - |z_2|$

It is possible only when $z_1 = z_2$

So, $\arg\left(\dfrac{z_1}{z_2}\right) = \arg z_1 - \arg z_2 = 0$

42. (c) Let $z = x + iy$

Given arg $z = \pi/4$

$\Rightarrow \tan^{-1}\left(\dfrac{y}{x}\right) = \dfrac{\pi}{4}$

or $x = y$ …(i)

then $z^2 = (x + iy)^2 = (x^2 - y^2) + (2xy)i$

From eq. (*i*), Re $(z^2) = 0$

43. (a) $z_1 = \bar{z}_2$ and $z_3 = \bar{z}_4$

Let $z_1 = re^{i\theta} \Rightarrow z_2 = re^{-i\theta}$

and $z_3 = \text{R}e^{i\phi} \Rightarrow z_4 = \text{R}e^{-i\phi}$

then $\dfrac{z_3}{z_2} = \dfrac{\text{R}e^{i\phi}}{re^{-i\phi}} = \dfrac{R}{r}e^{i(\phi+\theta)}$

$\Rightarrow \arg\left(\dfrac{z_3}{z_2}\right) = \phi+\theta$ and $\dfrac{z_1}{z_4} = \dfrac{re^{i\theta}}{\text{R}e^{-i\phi}} = \dfrac{r}{R}e^{i(\theta+\phi)}$

$\Rightarrow \arg\left(\dfrac{z_1}{z_4}\right) = \theta+\phi$

Hence, $\arg\left(\dfrac{z_3}{z_2}\right) = \arg\left(\dfrac{z_1}{z_4}\right)$

44. (a) Let $z_1 = re^{i\theta} \Rightarrow z_2 = re^{i\theta}$

and $z_3 = \text{R}e^{i\theta} \Rightarrow z_4 = \text{R}e^{-i\phi}$

$\arg\dfrac{z_1}{z_3} + \arg\dfrac{z_2}{z_4} = \arg\dfrac{z_1 z_2}{z_3 z_4}$

$= \arg\left(\dfrac{r^2}{R^2}\right) = 0$ (purely real number)

45. (d) $\left|\dfrac{z_1+z_2}{z_1-z_2}\right| = 1$

$\Rightarrow |z_1 + z_2| = |z_1 - z_2|$

or $|z_1 + z_2|^2 = |z_1 - z_2|^2$

$\Rightarrow |z_1|^2 + |z_2|^2 + 2\,\text{Re}(z_1\bar{z}_2) = |z_1|^2 + |z_2|^2 - 2\,\text{Re}(z_1\bar{z}_2)$

$\Rightarrow \text{Re}(z_1\bar{z}_2) = 0$ …(*i*)

Now, $\dfrac{z_1}{z_2} = \dfrac{z_1\bar{z}_2}{z_2\bar{z}_2} = \dfrac{z_1\bar{z}_2}{|z_2|^2}$

From Eq. (*i*), $\dfrac{z_1}{z_2}$ is purely imaginary number.

46. (b) $|\omega| = 1 \Rightarrow \left|\dfrac{1-iz}{z-1}\right| = 1$

or $|1 - iz|^2 = |z - i|^2$

or $|1 + y - ix|^2 = |x + i(y - 1)|^2$

or $(1 + y)^2 + x^2 = x^2 + (y - 1)^2$

or $y = 0$, i.e., X-axis or real axis.

47. (b) Let $z = x + iy$ and $\bar{z} = x - iy$

Now $|z + \bar{z}| + |z - \bar{z}| = 2$

$\Rightarrow |x| + |y| = 1$

$\Rightarrow x + y = 1; x - y = 1; x + y = -1$

and $x - y = -1$

i.e., a set of four lines forming a square.

48. (c) $\dfrac{z-1}{z+1}$ is purely imaginary.

So, $\text{Re}\left(\dfrac{z-1}{z+1}\right) = 0$

$\Rightarrow \text{Re}\left\{\dfrac{(x-1)+iy}{(x+1)+iy}\right\} = 0$

$\Rightarrow \text{Re}\left\{\dfrac{[(x-1)+iy][(x+1)-iy]}{(x+1)^2+y^2}\right\} = 0$

$\Rightarrow \dfrac{(x-1)(x+1)+y^2}{(x+1)^2+y^2} = 0$

or $x^2 + y^2 = 1 \Rightarrow |z| = 1$

49. (d) $(1 + \omega)(1 + \omega^2)(1 + \omega^4)(1 + \omega^8)$

$\{\because 1 + \omega + \omega^2 = 0\}$

$= (-\omega^2)(-\omega)(1 + \omega)(1 + \omega^2)$

$= (\omega^3)(-\omega^2)(-\omega) = \omega^3 . \omega^3 = 1$

50. (d) $(1 + \omega - \omega^2)(1 - \omega + \omega^2)$ $\{\because 1 + \omega + \omega^2 = 0\}$

$= (-\omega^2 - \omega^2)(-\omega - \omega)$

$= (-2\omega^2) \times (-2\omega) = 4$

51. (d) Let $S = 1 + i^2 + i^4 + ... + i^{2n}$

When number of terms are even then S = 0.

But when number of terms are odd then S = 1.

So, value of S cannot be determined.

52. (d) Let S = $i^2 + i^4 + i^6 + \dots (2n + 1)$ terms
$\because$ Number of terms are fixed and odd so S = – 1

53. (b) $z = x + iy = (a - ib)^3$
$z = a^3 + ib^3 - 3a^2bi - 3ab^2$
$\Rightarrow x + iy = a(a^2 - 3b^2) + b(b^2 - 3a^2)i$
$\Rightarrow x = a(a^2 - 3b^2)$ and $y = b(b^2 - 3a^2)$
$$\Rightarrow \frac{x}{a} - \frac{y}{b} = a^2 - 3b^2 - b^2 + 3a^2 = 4(a^2 - b^2)$$
$\Rightarrow \lambda = 4$

54. (a) $x = a + b$, $y = a\omega + b\omega^2$ and $z = a\omega^2 + b\omega$
$x + y + z = a(1 + \omega + \omega^2) + b(1 + \omega + \omega^2)$
$x + y + z = 0$
If $x + y + z = 0$, then $x^3 + y^3 + z^3 = 3xyz$
$\Rightarrow x^3 + y^3 + z^3 = 3(a + b)(a\omega + b\omega^2)(a\omega^2 + b\omega)$
$= 3(a + b)\{a^2\omega^3 + ab(\omega + \omega^2) + b^2\omega^3\}$
$= 3(a + b)\{a^2 - ab + b^2\} = 3(a^3 + b^3)$

55. (c) $$z = \frac{\sqrt{3}+i}{2} = i\left\{-\frac{1}{2} + \frac{i\sqrt{3}}{2}\right\} = -i\omega \quad ...(i)$$
$z^{101} + i^{103} = (-i\omega)^{101} + i^{103}$
$= -i\omega^2 - i = -i(\omega^2 + 1) = i\omega$
$(z^{101} + i^{103})^{105} = (i\omega)^{105}$
$= i\omega^3 = i(iz)^3$
$= z^3$ {From (i)}

56. (b) $(\sqrt{3}+i)^{100} = 2^{99}(a+ib)$
$$2^{100}\left(\frac{\sqrt{3}}{2} + \frac{i}{2}\right)^{100} = 2^{99}(a+ib)$$
$$\Rightarrow a + ib = 2\left(\frac{\sqrt{3}}{2} + \frac{i}{2}\right)^{100} \quad \dots(i)$$
$$\text{and } a - ib = 2\left(\frac{\sqrt{3}}{2} - \frac{i}{2}\right)^{100} \quad \dots(ii)$$
Multiplying both equations
$$a^2 + b^2 = 4\left(\frac{3}{4} + \frac{1}{4}\right)^{100} = 4$$

57. (c) $$a + ib = (\sqrt{3}+i)^{10} = 2^{10}\left\{\frac{\sqrt{3}}{2} + \frac{i}{2}\right\}^{100}$$
$$a + ib = 2^{10}\left\{\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right\}^{10}$$
$$a + ib = 2^{10}\left\{\cos\frac{5\pi}{3} + i\sin\frac{5\pi}{3}\right\}$$
$$\Rightarrow a = 2^{10}\cos\frac{5\pi}{3} \text{ and } b = 2^{10}\sin\frac{5\pi}{3}$$
$$a = 2^{10}\cos\frac{\pi}{3} \text{ and } b = -2^{10}\sin\frac{\pi}{3}$$
$a = 512$ and $b = -512\sqrt{3}$

58. (a) $(y + ix)^5 = i^5(x - iy)^5 = i(x - iy)^5$
or $(y + ix)^5 = i(p - iq) = q + ip$

59. (c) $z^2 = i\bar{z}$
Let $z = x + iy$
$\Rightarrow (x^2 - y^2) + 2ixy = i(x - iy)$
$\Rightarrow (x^2 - y^2) + 2ixy = y + ix$
$\Rightarrow x^2 - y^2 = y$ …(i)
and $2xy = x$ …(ii)
From eq. (ii),
$2xy = x \Rightarrow x(2y - 1) = 0$
$\Rightarrow x = 0$ or $y = \frac{1}{2}$
From eq. (i)
If $x = 0$ then $y = 0$ or $x = -1$
If $y = \frac{1}{2}$ then $x = \pm\frac{\sqrt{3}}{2}$
So, three non-zero complex numbers are
$$z_1 = -i, z_2 = \frac{\sqrt{3}}{2} + \frac{i}{2} \text{ and } z_3 = -\frac{\sqrt{3}}{2} + \frac{i}{2}$$
Hence, $z_1 + z_2 + z_3 = 0$

60. (c) From the solution of Q. 59.
$$z_1 = -i, z_2 = \frac{\sqrt{3}}{2} + \frac{i}{2} \text{ and } z_3 = -\frac{\sqrt{3}}{2} + \frac{i}{2}$$
$$z_1z_2z_3 = i\left(\frac{\sqrt{3}}{2} + \frac{i}{2}\right)\left(-\frac{\sqrt{3}}{2} + \frac{i}{2}\right)$$
$$= i\left(\frac{3}{4} + \frac{1}{4}\right) = i = \text{purely imaginary}$$
Now, $z_1z_2 + z_2z_3 + z_3z_1$
$$= (-i)\left(\frac{\sqrt{3}}{2} + \frac{i}{2}\right) + \left(\frac{\sqrt{3}}{2} + \frac{i}{2}\right)\left(-\frac{\sqrt{3}}{2} + \frac{i}{2}\right) + \left(-\frac{\sqrt{3}}{2} + \frac{i}{2}\right)(-i)$$
$$= -\frac{\sqrt{3}}{2}i + \frac{1}{2} - 1 + \frac{\sqrt{3}}{2}i + \frac{1}{2}$$
= 0 = purely real
Hence, both the statements are correct.

61. (a) Let $z = x + iy$
$$\left|\frac{z-4}{z-8}\right| = 1$$
$\Rightarrow |z - 4| = |z - 8|$
$\Rightarrow |(x - 4) + iy| = |(x - 8) - iy|$
$\Rightarrow \sqrt{(x-4)^2 + y^2} = \sqrt{(x-8)^2 + y^2}$
$\Rightarrow x^2 + 16 - 8x + y^2 = x^2 + 64 - 16x + y^2$
$\Rightarrow x = 6$

Now, $\left|\frac{z}{z-2}\right| = \frac{3}{2}$

$\Rightarrow 2|z| = 3|z-2|$

$\Rightarrow 2|x+iy| = 3|(x-2)+iy|$

$\Rightarrow 2\sqrt{x^2+y^2} = 3\sqrt{(x-2)^2+y^2}$

$\Rightarrow 4(x^2+y^2) = 9(x^2+4-4x+y^2)$

$\Rightarrow y = 0$ when $x = 6$

So, $z = 6 + 0i = 6 \Rightarrow |z| = 6$

62. (d) From the solution of Q. 61.

$z = 6$

So, $\left|\frac{z-6}{z+6}\right| = \left|\frac{6-6}{6+6}\right| = 0$

63. (c) Given P and Q are the points representing ω and ω^2.

$\because \arg\omega = \frac{2\pi}{3}$ and $\arg\omega^2 = \frac{4\pi}{3}$

⇒ Angles made by lines OP and OQ at the origin are 120° and 240°.

So, angle between OP and OQ

= 240° – 120° = 120°

64. (a) Given $z = \left(\frac{1}{\sqrt{2}} - \frac{i}{\sqrt{2}}\right)^{-25}$

$\Rightarrow z = \left(\cos\frac{\pi}{4} - i\sin\frac{\pi}{4}\right)^{-25}$ {Polar form}

$= \cos\frac{25\pi}{4} + i\sin\frac{25\pi}{4}$ {Demoiver's theorem}

$= \cos\left(6\pi+\frac{\pi}{4}\right) + i\sin\left(6\pi+\frac{\pi}{4}\right)$

$z = \cos\frac{\pi}{4} + i\sin\frac{\pi}{4} = \frac{1+i}{\sqrt{2}}$

Now, $\frac{z-\sqrt{2}}{z-i\sqrt{2}} = \frac{\frac{1+i}{\sqrt{2}}-\sqrt{2}}{\frac{1+i}{\sqrt{2}}-i\sqrt{2}} = \frac{i-1}{1-i} = -1$

So, fundamental amplitude = π

65. (d) $\because \omega_1$ and ω_2 are two distinct cube roots of unity different from 1.

$\Rightarrow \omega_1 = \omega$ and $\omega_2 = \omega^2$

So, $(\omega_1 - \omega_2)^2 = (\omega - \omega^2)^2$

$= \omega^2 + \omega^4 - 2\omega^3$ $\{\because \omega^3 = 1\}$

$= \omega^2 + \omega - 2$ $\{\because 1 + \omega + \omega^2 = 0\}$

$= -1 - 2 = -3$

66. (b) $\because 1 + \omega + \omega^2 = 0$

$\Rightarrow 1 + \omega^2 = -\omega$ and $1 + \omega = -\omega^2$

So, $\sqrt{\frac{1+\omega^2}{1+\omega}} = \sqrt{\frac{-\omega}{-\omega^2}} = \sqrt{\frac{1}{\omega}}$

$= \sqrt{\frac{\omega^2}{\omega^3}} = \omega$ $\{\because \omega^3 = 1\}$

67. (d) $\omega^{100} + \omega^{200} + \omega^{300}$

$= (\omega^3)^{33}.\omega + (\omega^3)^{66}.\omega^2 + (\omega^3)^{100}$

$= \omega + \omega^2 + 1$ $\{\therefore \omega^3 = 1\}$

$= 0$ $\{\because 1 + \omega + \omega^2 = 0\}$

68. (d) Let $z = x + iy$

$\Rightarrow \frac{z-1}{z+1} = \frac{(x-1)+iy}{(x+1)+iy}$

$= \frac{\{(x-1)+iy\}\{(x-1)-iy\}}{(x+1)^2+y^2}$

$= \frac{\{(x-1)(x+1)+y^2\}+i\{y(x-1)-y(x-1)\}}{(x+1)^2+y^2}$

$\because \text{Re}\left(\frac{z-1}{z+1}\right) = 0$ {Given}

$\Rightarrow (x-1)(x+1) + y^2 = 0$

$\Rightarrow x^2 + y^2 = 1$

$\Rightarrow |z| = 1$

69. (a) $z = \left(\frac{\sqrt{3}}{2}+\frac{i}{2}\right)^{107} + \left(\frac{\sqrt{3}}{2}-\frac{i}{2}\right)^{107}$

$= \left(\cos\frac{\pi}{6}+i\sin\frac{\pi}{6}\right)^{107} + \left(\cos\frac{\pi}{6}-i\sin\frac{\pi}{6}\right)^{107}$ {Polar form}

$z = \cos\frac{107\pi}{6} + i\sin\frac{107\pi}{6} + \cos\frac{107\pi}{6}$

$-i\sin\frac{107\pi}{6}$ {Demoiver theorem}

$= 2\cos\frac{107\pi}{6}$ = real part

Hence, Im $(z) = 0$

70. (c) Given $z^2 + |z| = 0$ …(i)

$\Rightarrow z^2 = -|z|$

$\Rightarrow |z^2| = |-|z||$

$\Rightarrow |z|^2 = |z|$

$\Rightarrow |z|\{|z| - 1\} = 0$

$\Rightarrow |z| = 0$ or $|z| = 1$

If $|z| = 0$, then $z = 0$ {From (i)}

If $|z| = 1$, then $z^2 + 1 = 0$ {From (i)}

Clearly, it has 2 solutions.

Hence, total three solutions are there of the given equation.