

Quadratic Equation

Exercise

- The number of real roots of $3^{2x^2-7x+7} = 9$ is
 (a) 0 (b) 2
 (c) 1 (d) 4
- If $\cos \alpha$ is a root of $25x^2 - 5x - 12 = 0$, $-1 < x < 0$. Then the value of $\sin 2\alpha$ is
 (a) $\frac{12}{25}$ (b) $\frac{-12}{25}$
 (c) $\frac{-24}{25}$ (d) $\frac{20}{25}$
- If $ax^2 + bx + c = 0$ is satisfied by every value of x , then
 (a) $b, c = 0$ (b) $c = 0$
 (c) $a = 0$ (d) $a = b = c = 0$
- If both the roots of the equation $ax^2 + bx + c = 0$ are zero, then
 (a) $b = c = 0$ (b) $b = 0, c \neq 0$
 (c) $b \neq 0, c = 0$ (d) $b \neq 0, c \neq 0$
- If p and q are roots of the quadratic equation $x^2 + mx + m^2 + a = 0$, then the value of $p^2 + q^2 + pq$ is
 (a) 0 (b) a
 (c) $-a$ (d) $\pm m^2$
- The roots of the equation $\log_2(x^2 - 4x + 5) = (x - 2)$ are
 (a) 4, 5 (b) 2, -3
 (c) 2, 3 (d) 3, 5
- Let x_1, x_2 be the roots of the equation $x^2 - 3x + p = 0$ and let x_3, x_4 be the roots of the equation $x^2 - 12x + q = 0$. If the numbers x_1, x_2, x_3, x_4 (in order) form an increasing GP, then
 (a) $p = 2, q = 16$ (b) $p = 2, q = 32$
 (c) $p = 4, q = 16$ (d) $p = 4, q = 32$
- The quadratic equation whose roots are reciprocal of the roots of the equation $ax^2 + bx + c = 0$ is
 (a) $cx^2 + bx + a = 0$ (b) $bx^2 + cx + a = 0$
 (c) $cx^2 + ax + b = 0$ (d) $bx^2 + ax + c = 0$
- The condition that one root of the equation $ax^2 + bx + c = 0$ may be double of the other, is
 (a) $b^2 = 9ac$ (b) $2b^2 = 9ac$
 (c) $2b^2 = ac$ (d) $b^2 = ac$
- If α, β are the roots of $x^2 + bx + c = 0$, then the equation whose roots are b and c is
 (a) $x^2 + \alpha x - \beta = 0$
 (b) $x^2 - x(\alpha + \beta + \alpha\beta) - \alpha\beta(\alpha + \beta) = 0$
 (c) $x^2 + (\alpha + \beta - \alpha\beta)x - \alpha\beta(\alpha + \beta) = 0$
 (d) $x^2 + x(\alpha + \beta + \alpha\beta) - \alpha\beta(\alpha + \beta) = 0$
- Real roots of the equation $x^2 + 5|x| + 4 = 0$ are
 (a) -1, -4 (b) 1, 4
 (c) -4, 4 (d) None of these
- If $2 + i\sqrt{3}$ is a root of $x^2 + px + q = 0$ where $p, q \in \mathbb{R}$, then
 (a) $p = -4, q = 7$ (b) $p = 4, q = 7$
 (c) $p = 4, q = -7$ (d) $p = -4, q = -7$
- The values of x satisfying $x = \sqrt{6 + \sqrt{6 + \sqrt{6 + \dots}}}$ are
 (a) 3, -2 (b) -2
 (c) 3 (d) None of these
- The set of values of p for which the roots of the equation $3x^2 + 2x + p(p - 1) = 0$ are of opposite signs is
 (a) $(-\infty, 0)$ (b) $(0, 1)$
 (c) $(1, \infty)$ (d) $(0, \infty)$
- If p and q are the roots of the equation $x^2 + px + q = 0$, then
 (a) $p = 1$ (b) $p = 1$ or 0
 (c) $p = -2$ (d) $p = -2$ or 0
- The value of p for which the difference between the roots of the equation $x^2 + px + 8 = 0$ is 2 are
 (a) ± 2 (b) ± 4
 (c) ± 6 (d) ± 8

Quadratic Equation

17. If $f(x) = 2x^2 + mx^2 - 13x + n$ and 2, 3 are roots of the equation $f(x) = 0$, then the values of m and n are
 (a) $-5, -30$ (b) $-5, 30$
 (c) $5, 30$ (d) None of these
18. If $7^{\log_7(x^2 - 4x + 5)} = x - 1$ then x may have values
 (a) 2, 3 (b) 7
 (c) $-2, -3$ (d) 2, -3
19. If α, β are the roots of $ax^2 + bx + c = 0$ then the equation whose roots are $2 + \alpha, 2 + \beta$ is
 (a) $ax^2 + x(4a - b) + 4a - 2b + c = 0$
 (b) $ax^2 + x(4a - b) + 4a + 2b + c = 0$
 (c) $ax^2 + x(b - 4a) + 4a + 2b + c = 0$
 (d) $ax^2 + x(b - 4a) + 4a - 2b + c = 0$
20. If 8, 2 are the roots of $x^2 + ax + \beta = 0$, and 3, 3 are the roots of $x^2 + \alpha x + b = 0$, then the roots of $x^2 + ax + b = 0$ are
 (a) 8, -1 (b) $-9, 2$
 (c) $-8, -2$ (d) 9, 1
21. If $x = 1 + i$ is a root of the equation $x^3 - ix + 1 - i = 0$, then the other real root is
 (a) 1 (b) -1
 (c) 0 (d) None of these
22. The number of roots of the quadratic equation $8 \sec^2 \theta - 6 \sec \theta + 1 = 0$ is
 (a) infinite (b) 1
 (c) 2 (d) 0
23. If the roots of the equation $(a^2 + b^2)x^2 - 2b(a + c)x + (b^2 + c^2) = 0$ are equal then a, b, c are in
 (a) GP (b) AP
 (c) HP (d) None of these
24. If $x^2 - ax - 21 = 0$ and $x^2 - 3ax + 35 = 0, a > 0$ have a common root, then a is equal to
 (a) -4 (b) 4
 (c) 2 (d) None of these
25. If the equations $x^2 + bx + c = 0, x^2 + cx + b = 0 (b \neq c)$ have a common root, then
 (a) $b + c = 0$ (b) $b + c - 1 = 0$
 (c) $b + c + 1 = 0$ (d) None of the above
26. If α, β are the roots of $ax^2 - 26x + c = 0$ then $\alpha^3\beta^3 + \alpha^2\beta^3 + \alpha^3\beta^2$ is equal to
 (a) $\frac{c^2}{a^3}(c + 26)$ (b) $\frac{c^2}{a^3}(c - 26)$
 (c) $\frac{bc^3}{a^3}$ (d) None of these
27. $\sqrt{2 + \sqrt{2 + \sqrt{2 + \dots}}}$ to ∞ is equal to
 (a) -1 (b) $\sqrt{2}$
 (c) 2 (d) $\frac{1}{2}$
28. If the quadratic equations $ax^2 + 2cx + b = 0$ and $ax^2 + 2bx + c = 0 (b \neq c)$ have a common root, then $a + 4b + 4c$ is equal to
 (a) 0 (b) $-\frac{1}{2}$
 (c) 2 (d) 1
29. If α, β are the roots of the equation $ax^2 + bx + c = 0$, then $\frac{1}{a\alpha + b} + \frac{1}{a\beta + b}$ is equal to
 (a) $\frac{c}{ab}$ (b) $\frac{a}{bc}$
 (c) $\frac{b}{ac}$ (d) None of the above
30. The positive value of m for which the roots of the equation $12x^2 + mx + 5 = 0$ are in the ratio 3 : 2 is
 (a) $5\sqrt{10}$ (b) $\frac{5\sqrt{10}}{2}$
 (c) $\frac{5}{12}$ (d) $\frac{12}{5}$
31. If α and β are the roots of $ax^2 + bx + c = 0$, then the value of $(a\alpha + b)^{-2} + (a\beta + b)^{-2}$ is equal to
 (a) $\frac{b^2 - 2ac}{a^2c^2}$ (b) $\frac{c^2 - 2ab}{a^2b^2}$
 (c) $\frac{a^2 - 2bc}{b^2c^2}$ (d) None of these
32. If α, β are the roots of the equation $8x^2 - 3x + 27 = 0$ then the value of $\left(\frac{\alpha^2}{\beta}\right)^{1/3} + \left(\frac{\beta^2}{\alpha}\right)^{1/3}$ is
 (a) $1/4$ (b) $1/3$
 (c) $7/2$ (d) 4
33. If α, β are the roots of the equation $x^2 - (1 + n^2)x + \frac{1}{2}(1 + n^2 + n^4) = 0$, then $\alpha^2 + \beta^2$ is equal to
 (a) n^2 (b) $2n^2$
 (c) $n^2 + 2$ (d) $-n^2$
34. If $(1 - p)$ is a root of the quadratic equation $x^2 + px + (1 - p) = 0$, then its roots are
 (a) 0, 1 (b) $-1, 1$
 (c) 0, -1 (d) $-1, 2$
35. In a quadratic equation with leading coefficient 1, a student reads the coefficient 16 of x wrongly as 19 and obtain the roots as -15 and -4 . The correct roots are
 (a) 6, 10 (b) $-6, -10$
 (c) $-7, -9$ (d) None of these
36. Ramesh and Mahesh solve an equation. In solving Ramesh commits a mistake in constant term and finds the roots 8 and 2. Mahesh commits a mistake in the

- coefficient of x and finds the roots -9 and -1 . The correct roots are
- (a) $-8, 2$ (b) $9, 1$
 (c) $9, -1$ (d) $-8, -2$
37. Two candidates attempt to solve a quadratic equation of the form $x^2 + px + q = 0$. One starts with a wrong value of p and finds the roots to be 2 and 6 . The other starts with a wrong value of q and finds the roots to be $2, -9$. The correct roots are
- (a) $3, 4$ (b) $5, 3$
 (c) $-3, -4$ (d) None of these
38. If one root of the equation $(a^2 - 5a + 3)x^2 + (3a - 1)x + 2 = 0$ be double of the other, then the value of a is
- (a) $\frac{2}{3}$ (b) $-\frac{2}{3}$
 (c) $\frac{1}{3}$ (d) $-\frac{1}{3}$
39. If the equation $\frac{x^2 - bx}{ax - c} = \frac{m - 1}{m + 1}$ has roots equal in magnitude but opposite in signs, then m is equal to
- (a) $\frac{a + b}{a - b}$ (b) $\frac{a - b}{a + b}$
 (c) 0 (d) 1
40. If the equation $\frac{a}{x - a} + \frac{b}{x - b} = 1$ has two roots equal in magnitude and opposite in signs then the value of $a + b$ is
- (a) 0 (b) 1
 (c) -1 (d) None of these
41. If α and β are the roots of $x^2 - p(x + 1) - c = 0$, then the value of $\frac{\alpha^2 + 2\alpha + 1}{\alpha^2 + 2\alpha + c} + \frac{\beta^2 + 2\beta + 1}{\beta^2 + 2\beta + c}$ is
- (a) 2 (b) 1
 (c) -1 (d) 0
42. If α, β be the roots of $ax^2 + 2bx + c = 0$ and $\alpha + \delta, \beta + \delta$ be those of $Ax^2 + 2Bx + C = 0$, then the value of $(b^2 - ac)/(B^2 - AC)$ is
- (a) $\left(\frac{a}{A}\right)^2$ (b) $\left(\frac{A}{a}\right)^2$
 (c) 0 (d) 1
43. The number of values of λ for which $(\lambda^2 - 3\lambda + 2)x^2 + (\lambda^2 - 5\lambda + 6)x + \lambda^2 - 4 = 0$ is an identity in x is
- (a) 1 (b) 2
 (c) -2 (d) 0
44. The equation $(b - c)x^2 + (c - a)x + (a - b) = 0$ has
- (a) equal roots (b) irrational roots
 (c) rational roots (d) None of these
45. If $a + b + c = 0$, then the roots of the equation $(c^2 - ab)x^2 - 2(a^2 - bc)x + (b^2 - ac) = 0$ are
- (a) imaginary (b) real and equal
 (c) real and unequal (d) None of these
46. The value of 'a' for which the quadratic equation $3x^2 + 2(a^2 + 1)x + (a^2 - 3a + 2) = 0$ possesses roots of opposite signs lies in
- (a) $(-\infty, 1)$ (b) $(-\infty, 0)$
 (c) $(1, 2)$ (d) $\left(\frac{3}{2}, 2\right)$
47. If $x^2 + 2ax + 10 - 3a > 0$ for all $x \in \mathbb{R}$, then
- (a) $a < -5$ (b) $-5 < a < 2$
 (c) $a > 5$ (d) $2 < a < 5$
48. If $(x + a)$ is a factor of both the quadratic polynomial $x^2 + px + q$ and $x^2 + lx + m$, where, p, q, l and m are constants, then which one of the following is correct ?
- (a) $a = \frac{m - q}{l - p} (l \neq p)$ (b) $a = \frac{m + q}{l + p} (l \neq -p)$
 (c) $l = \frac{m - q}{a - p} (a \neq p)$ (d) $p = \frac{m - q}{a - l} (a \neq l)$
49. If the difference between the roots of the equation $x^2 + ax + 1 = 0$ is less than $\sqrt{5}$, then the set of possible values of a is
- (a) $(-3, 3)$ (b) $(-3, \infty)$
 (c) $(3, \infty)$ (d) $(-\infty, -3)$
50. If the sum of the roots of $ax^2 + bx + c = 0$ be equal to sum of the squares, then
- (a) $2ac = ab + b^2$ (b) $2ab = bc + c^2$
 (c) $2bc = ac + c^2$ (d) None of these
51. If α, β are the roots of the equation $x^2 + px + 1 = 0$ and γ, δ are the roots of the equation $x^2 + qx + 1 = 0$, then $(\alpha - \gamma)(\alpha + \delta)(\beta - \gamma)(\beta + \delta)$ is equal to
- (a) $q^2 - p^2$ (b) $p^2 - q^2$
 (c) $p^2 + q^2$ (d) None of these
52. If a, b are the roots of the equation $x^2 + x + 1 = 0$, then $a^2 + b^2$ is equal to
- (a) 1 (b) 2
 (c) -1 (d) 3
53. If α, β are the roots of the equation $x^2 + x + 1 = 0$ and $\frac{\alpha}{\beta}, \frac{\beta}{\alpha}$, are roots of the equation $x^2 + px + q = 0$, then p equals to
- (a) -1 (b) 1
 (c) -2 (d) 2
54. The value of x satisfying $\log_3(x^2 + 4x + 12) = 2$ are
- (a) $2, -4$ (b) $1, -3$
 (c) $-1, 3$ (d) $-1, -3$
55. The value of $x^2 - 2bx + c$ is positive if
- (a) $b^2 - 4c > 0$ (b) $b^2 - 4c < 0$
 (c) $c^2 < b$ (d) $b^2 < c$
56. If $x^2 + x + 1$ is a factor of $ax^3 + bx^2 + cx + d$ then the real root of $ax^3 + bx^2 + cx + d = 0$ is
- (a) $-\frac{d}{a}$ (b) $\frac{d}{a}$
 (c) $\frac{a}{d}$ (d) None of these

Quadratic Equation

57. If the product of the roots of the equation $x^2 - 3kx + 2e^{2 \log k} - 1 = 0$ is 7, then the value of k
- (a) 1 (b) 2
(c) 3 (d) 4

Directions (Q. Nos. 58–59):

Let α and β ($\alpha < \beta$) be the roots of the equation $x^2 + bx + c = 0$, where $b > 0$ and $c < 0$.

58. Consider the following:

1. $\beta < -\alpha$
2. $\beta < |\alpha|$

Which of the above is/are correct? [NDA-I 2016]

- (a) Only 1 (b) Only 2
(c) Both 1 and 2 (d) Neither 1 nor 2

59. Consider the following:

1. $\alpha + \beta + \alpha\beta > 0$
2. $\alpha^2\beta + \beta^2\alpha > 0$

Which of the above is/are correct? [NDA-I 2016]

- (a) Only 1
(b) Only 2
(c) Both 1 and 2
(d) Neither 1 nor 2

60. If $x^2 - px + 4 > 0$ for all real values of x , then which one of the following is correct? [NDA-I 2016]

- (a) $|p| < 4$ (b) $|p| \leq 4$
(c) $|p| > 4$ (d) $|p| \geq 4$

61. If one root of the equation $(l - m)x^2 + lx + 1 = 0$ is double the other and l is real, then what is the greatest value of m ?

- (a) $-\frac{9}{8}$ (b) $\frac{9}{8}$
(c) $-\frac{8}{9}$ (d) $\frac{8}{9}$

Directions (Q. Nos. 62–63):

Let α and β be the roots of the equation $x^2 - (1 - 2a^2)x + (1 - 2a^2) = 0$.

62. Under what condition does the above equation have real roots? [NDA-II 2016]

- (a) $a^2 < \frac{1}{2}$ (b) $a^2 > \frac{1}{2}$
(c) $a^2 \leq \frac{1}{2}$ (d) $a^2 \geq \frac{1}{2}$

63. Under what condition is $\frac{1}{\alpha^2} + \frac{1}{\beta^2} < 1$? [NDA-II 2016]

- (a) $a^2 < \frac{1}{2}$ (b) $a^2 > \frac{1}{2}$

(c) $a^2 > 1$

(d) $a^2 \in \left(\frac{1}{3}, \frac{1}{2}\right)$ only

Directions (Q. Nos. 64–65):

$2x^2 + 3x - \alpha = 0$ has roots -2 and β while the equation $x^2 - 3mx + 2m^2 = 0$ has both roots positive, where $\alpha > 0$ and $\beta > 0$.

64. What is the value of α ? [NDA-II 2016]

- (a) $\frac{1}{2}$ (b) 1
(c) 2 (d) 4

65. If $\beta, 2, 2m$ are in GP, then what is the value of $\beta\sqrt{m}$? [NDA-II 2016]

- (a) 1 (b) 2
(c) 4 (d) 6

66. If $c > 0$ and $4a + c < 2b$, then $ax^2 - bx + c = 0$ has a root in which one of the following intervals? [NDA-II 2016]

- (a) (0, 2) (b) (2, 3)
(c) (3, 4) (d) (-2, 0)

67. If both the roots of the equation $x^2 - 2kx + k^2 - 4 = 0$ lie between -3 and 5 , then which one of the following is correct? [NDA-II 2016]

- (a) $-2 < k < 2$ (b) $-5 < k < 3$
(c) $-3 < k < 5$ (d) $-1 < k < 3$

68. If the difference between the roots of the equation $x^2 + kx + 1 = 0$ is strictly less than 5, where $|k| \geq 2$, then k can be any element of the interval [NDA-I 2017]

- (a) $(-3, -2] \cup [2, 3)$
(b) $(-3, 3)$
(c) $[-3, -2] \cup [2, 3]$
(d) None of these

69. If the roots of the equation $x^2 + px + q = 0$ are in the same ratio as those of the equation $x^2 + lx + m = 0$, then which one of the following is correct? [NDA-I 2017]

- (a) $p^2m = l^2q$
(b) $m^2p = l^2q$
(c) $m^2p = q^2l$
(d) $m^2p^2 = l^2q$

70. If the graph of a quadratic polynomial lies entirely above X-axis, then which one of the following is correct? [NDA-I 2017]

- (a) Both the roots are real
(b) One root is real and the other is complex
(c) Both the roots are complex
(d) Cannot say

ANSWERS

1.	(b)	2.	(c)	3.	(d)	4.	(a)	5.	(c)	6.	(c)	7.	(b)	8.	(a)	9.	(b)	10.	(c)
11.	(d)	12.	(a)	13.	(c)	14.	(b)	15.	(b)	16.	(c)	17.	(d)	18.	(a)	19.	(d)	20.	(d)
21.	(b)	22.	(a)	23.	(a)	24.	(b)	25.	(c)	26.	(a)	27.	(c)	28.	(a)	29.	(c)	30.	(a)
31.	(a)	32.	(a)	33.	(a)	34.	(c)	35.	(b)	36.	(b)	37.	(c)	38.	(a)	39.	(b)	40.	(a)
41.	(b)	42.	(a)	43.	(a)	44.	(c)	45.	(b)	46.	(c)	47.	(b)	48.	(a)	49.	(a)	50.	(a)
51.	(a)	52.	(c)	53.	(b)	54.	(d)	55.	(d)	56.	(a)	57.	(b)	58.	(c)	59.	(b)	60.	(a)
61.	(b)	62.	(d)	63.	(a)	64.	(c)	65.	(a)	66.	(a)	67.	(d)	68.	(a)	69.	(a)	70.	(c)

Explanations

1. (b) $3^{2x^2-7x+7} = 9 = 3^2$
 $\Rightarrow 2x^2 - 7x + 7 = 2$
 $\Rightarrow 2x^2 - 7x + 5 = 0$
 $b^2 - 4ac = 49 - 4(2)(5) > 0$
 So, there are 2 real roots.
2. (c) $25x^2 - 5x - 12 = 0$
 $\Rightarrow (5x - 4)(5x + 3) = 0$
 $x = \frac{4}{5}$ or $-\frac{3}{5}$
 Given $-1 < x < 0$
 So, $x = -\frac{3}{5}$
 $\Rightarrow \cos \alpha = -\frac{3}{5}$ and $\sin \alpha = \frac{4}{5}$
 $\sin 2\alpha = 2 \sin \alpha \cos \alpha = -\frac{24}{25}$
3. (d) $ax^2 + bx + c = 0$ is satisfied by every value of x if $a = b = c = 0$
4. (a) Both roots of the equation $ax^2 + bx + c = 0$ are zero.
 Hence, $b = c = 0$
5. (c) $x^2 + mx + m^2 + a = 0$
 Sum of the roots $p + q = -m$
 Product of the roots $pq = m^2 + a$
 $p^2 + q^2 + pq = (p + q)^2 - 2pq + pq$
 $= (p + q)^2 - pq = m^2 - (m^2 + a) = -a$
6. (c) $\log_2(x^2 - 4x + 5) = x - 2$
 $x^2 - 4x + 5 = 2^{x-2}$
 By Hit and Trial method,
 $x = 2, 3$ satisfies the above equation.
7. (b) $x^2 - 3x + p = 0$ and $x^2 - 12x + q = 0$
 On putting $p = 2$ and $q = 32$ equation becomes $x^2 - 3x + 2 = 0$ and $x^2 - 12x + 32 = 0$ and thus the roots are 1, 2, 4, 8, which forms an increasing G.P. So, $p = 2$ and $q = 32$.
8. (a) $ax^2 + bx + c = 0$
 For reciprocal roots,
 Replace x by $1/x$
 $a\left(\frac{1}{x}\right)^2 + b\left(\frac{1}{x}\right) + c = 0$
 $\Rightarrow cx^2 + bx + a = 0$
9. (b) Let the roots of the equation $ax^2 + bx + c = 0$ be α and 2α .
 Sum of the roots $3\alpha = -\frac{b}{a}$
 Product of the roots $2\alpha^2 = c/a$
 $\Rightarrow 2\left(\frac{-b}{3a}\right)^2 = \frac{c}{a} \Rightarrow 2b^2 = 9ac$
10. (c) $\because \alpha$ and β are the roots of $x^2 + bx + c = 0$
 So, $\alpha + \beta = -b$ and $\alpha\beta = c$
 New roots are b and c .
 Sum of roots $= b + c = -\alpha - \beta + \alpha\beta$
 Product of roots $= bc = -\alpha\beta(\alpha + \beta)$
 Hence, equation is $x^2 - (b + c)x + bc = 0$
 $x^2 + (\alpha + \beta - \alpha\beta)x - \alpha\beta(\alpha + \beta) = 0$
11. (d) $x^2 \geq 0$ and $|x| \geq 0$
 So, $x^2 + 5|x| + 4 \neq 0$
 Hence, number of real roots = 0
12. (a) One root = $2 + i\sqrt{3}$
 \Rightarrow Other root = $2 - i\sqrt{3}$
 Sum of roots = 4
 Product of roots = $4 + 3 = 7$
 So, equation is $x^2 - 4x + 7 = 0$
 Comparing with $x^2 + px + q = 0$
 we get $p = -4$ and $q = 7$
13. (c) $x = \sqrt{6 + \sqrt{6 + \sqrt{6 + \sqrt{6}}}} \dots \infty$
 Consecutive factors of $6 = 2 \times 3$

Quadratic Equation

\therefore There is (+)ve sign.

So, $x = 3$ (greater number)

14. (b) Roots of the equation $3x^2 + 2x + p(p-1) = 0$ are of opposite signs.

So, product of the roots < 0

$$\frac{p(p-1)}{3} < 0$$

$$\Rightarrow p(p-1) < 0$$

$$\Rightarrow 0 < p < 1$$

$$\Rightarrow p \in (0, 1)$$

15. (b) Roots of the equation $x^2 + px + q = 0$ are p and q .

$$\therefore p + q = -p$$

$$\Rightarrow 2p + q = 0 \quad \dots(i)$$

$$\text{and } pq = q \Rightarrow q(p-1) = 0 \quad \dots(ii)$$

$$\Rightarrow q = 0 \text{ or } p = 1$$

$$\text{If } q = 0 \Rightarrow p = 0 \quad [\text{From (i)}]$$

Hence, $p = 0$ or 1 .

16. (c) Let α, β are the roots of the equation.

$$x^2 + px + 8 = 0$$

$$\alpha + \beta = -p \text{ and } \alpha\beta = 8$$

$$\text{Given } \alpha - \beta = 2$$

$$\Rightarrow (\alpha + \beta)^2 - 4\alpha\beta = 4$$

$$\Rightarrow p^2 - 32 = 4 \Rightarrow p^2 = 36$$

$$\Rightarrow p = \pm 6$$

17. (d) Given 2 and 3 are the roots of the equation.

$$2x^2 + mx^2 - 13x + n = 0$$

$$\text{So } 4m + n = 18 \quad \dots(i)$$

$$\text{and } 9m + n = 21 \quad \dots(ii)$$

On solving eqs. (i) and (ii),

$$m = \frac{3}{5} \text{ and } n = \frac{78}{5}$$

18. (a) $7^{\log_7(x^2 - 4x + 5)} = x - 1$

$$\Rightarrow x^2 - 4x + 5 = x - 1$$

$$\Rightarrow x^2 - 5x + 6 = 0$$

$$\Rightarrow (x-2)(x-3) = 0$$

$$\Rightarrow x = 2, 3$$

19. (d) $\alpha + \beta = -\frac{b}{a}$ and $\alpha\beta = \frac{c}{a}$

New roots are $2 + \alpha$ and $2 + \beta$.

$$\text{So, sum of roots} = 4 + \alpha + \beta = 4 - \frac{b}{a}$$

$$\text{Product of roots} = (2 + \alpha)(2 + \beta) = 4 + \frac{c}{a} - \frac{2b}{a}$$

So, equation is

$$x^2 - \left(4 - \frac{b}{a}\right)x + \left(4 + \frac{c}{a} - \frac{2b}{a}\right) = 0$$

$$\Rightarrow ax^2 + (b - 4a)x + (4a + c - 2b) = 0$$

20. (d) Roots of equation $x^2 + ax + \beta = 0$ are 8 and 2

while roots of $x^2 + \alpha x + b = 0$ are 3 and 3.

So, $a = -10$ and $b = 9$

$$\text{Now, } x^2 + ax + b = 0 \Rightarrow x^2 - 10x + 9 = 0$$

$$\Rightarrow (x-1)(x-9) = 0 \Rightarrow x = 1, 9$$

21. (b) By hit and trial method,

$x = -1$ satisfies the given equation.

So, it is the real root.

22. (a) $8 \sec^2 \theta - 6 \sec \theta + 1 = 0$

$$\Rightarrow \cos^2 \theta - 6 \cos \theta + 8 = 0$$

$$\Rightarrow (\cos \theta - 4)(\cos \theta - 2) = 0$$

$$\Rightarrow \cos \theta = 4 \text{ or } 2$$

$$\text{But } -1 \leq \cos \theta \leq 1$$

So, roots are not possible.

23. (a) $(a^2 + b^2)x^2 - 2b(a+c)x + (b^2 + c^2) = 0$

\therefore Root are equal so $B^2 - 4AC = 0$

$$4b^2(a+c)^2 = 4(a^2 + b^2)(b^2 + c^2)$$

$$\Rightarrow b^4 + a^2c^2 - 2ab^2c = 0$$

$$\Rightarrow (b^2 - ac)^2 = 0$$

$$\Rightarrow b^2 = ac$$

i.e., a, b, c are in G.P.

24. (b) $x^2 - ax - 21 = 0$ and $x^2 - 3ax + 35 = 0$ have a common root.

$$\text{So, } \frac{x^2}{-35a - 63a} = \frac{-x}{35 + 21} = \frac{1}{-3a + a}$$

$$= \frac{x^2}{-98a} = \frac{-x}{56} = \frac{1}{-2a}$$

$$\text{I} \quad \text{II} \quad \text{III}$$

From I and III, $x = 7$

$$\text{From II and III, } x = \frac{28}{a}$$

$$\text{Hence, } \frac{28}{a} = 7 \Rightarrow a = 4$$

25. (c) $x^2 + bx + c = 0$ and $x^2 + cx + b = 0$ have a common root.

So, put $x = 1$

$$1 + b + c = 0 \Rightarrow b + c = -1$$

26. (a) α, β are the roots of equation $ax^2 - 26x + c = 0$

$$\alpha + \beta = \frac{26}{a}; \alpha\beta = \frac{c}{a}$$

$$\alpha^3\beta^3 + \alpha^2\beta^3 + \alpha^3\beta^2 = \alpha^3\beta^3 + \alpha^2\beta^2(\alpha + \beta)$$

$$= \frac{c^3}{a^3} + \frac{c^2}{a^2} \left(\frac{26}{a} \right) = \frac{c^2}{a^3} (c + 26)$$

27. (c) $x = \sqrt{2 + \sqrt{2 + \sqrt{2 + \dots \infty}}}$

Consecutive factors of $2 = 1$ and 2

So, $x = 2$

28. (a) $ax^2 + 2cx + b = 0$ and $ax^2 + 2bx + c = 0$ have a common root.

$$\frac{x^2}{2(c^2 - b^2)} = \frac{-x}{ac - ab} = \frac{1}{2ab - 2ac}$$

$$\Rightarrow \frac{x^2}{2(c+b)} = \frac{-x}{a} = \frac{1}{-2a}$$

I II III

From I and III, $x^2 = -\frac{(c+b)}{a}$

From II and III, $x = \frac{1}{2}$

$$\Rightarrow a = -4c - 4b$$

$$\Rightarrow a + 4b + 4c = 0$$

29. (c) $\alpha + \beta = -\frac{b}{a}, \alpha\beta = \frac{c}{a}$

$$\begin{aligned} \frac{1}{a\alpha + b} + \frac{1}{a\beta + b} &= \frac{a(\alpha + \beta) + 2b}{a^2\alpha\beta + ab(\alpha + \beta) + b^2} \\ &= \frac{2b + a(-b/a)}{a^2\left(\frac{c}{a}\right) + ab\left(-\frac{b}{a}\right) + b^2} = \frac{b}{ac} \end{aligned}$$

30. (a) Let α and β be the roots of the equation $12x^2 + mx + 5 = 0$

Then, $\frac{\alpha}{\beta} = \frac{3}{2} \Rightarrow \alpha = 3k$ and $\beta = 2k$

$$\alpha + \beta = 5k = -\frac{m}{12}$$

$$\Rightarrow k = -\frac{m}{60} \quad \dots(i)$$

$$\alpha\beta = 6k^2 = \frac{5}{12}$$

$$\Rightarrow k^2 = \frac{5}{72} \quad \dots(ii)$$

From eqs. (i) and (ii),

$$m = 5\sqrt{10}$$

31. (a) $\alpha + \beta = -b/a; \alpha\beta = c/a$
 $(a\alpha + b)^{-2} + (a\beta + b)^{-2}$

$$= \frac{1}{a^2 \left\{ \alpha + \frac{b}{a} \right\}^2} + \frac{1}{a^2 \left\{ \beta + \frac{b}{a} \right\}^2}$$

$$= \frac{1}{a^2 \{ \alpha - \alpha - \beta \}^2} + \frac{1}{a^2 \{ \beta - \alpha - \beta \}^2}$$

$$= \frac{1}{a^2} \left\{ \frac{1}{\beta^2} + \frac{1}{\alpha^2} \right\} = \frac{1}{a^2} \left\{ \frac{\alpha^2 + \beta^2}{\alpha^2 \beta^2} \right\}$$

$$= \frac{1}{a^2} \left\{ \frac{(\alpha + \beta)^2 - 2\alpha\beta}{\alpha^2 \beta^2} \right\}$$

$$= \frac{1}{a^2} \left\{ \frac{\frac{b^2}{a^2} - \frac{2c}{a}}{\frac{c^2}{a^2}} \right\} = \frac{b^2 - 2ac}{a^2 c^2}$$

32. (a) $\alpha + \beta = 3/8; \alpha\beta = 27/8$

$$\begin{aligned} \left(\frac{\alpha^2}{\beta} \right)^{1/3} + \left(\frac{\beta^2}{\alpha} \right)^{1/3} \\ = \frac{\alpha^{2/3} \cdot \alpha^{1/3} + \beta^{2/3} \cdot \beta^{1/3}}{(\alpha\beta)^{1/3}} \end{aligned}$$

$$= \frac{\alpha + \beta}{(\alpha\beta)^{1/3}} = \frac{3/8}{3/2} = \frac{1}{4}$$

33. (a) $\alpha + \beta = n^2 + 1; \alpha\beta = \frac{n^4 + n^2 + 1}{2}$

$$\begin{aligned} \alpha^2 + \beta^2 &= (\alpha + \beta)^2 - 2\alpha\beta \\ &= n^4 + 1 + 2n^2 - n^4 - n^2 - 1 = n^2 \end{aligned}$$

34. (c) $1 - p$ is the root of $x^2 + px + (1 - p) = 0$

$$\text{So, } (1 - p)^2 + p(1 - p) + (1 - p) = 0$$

$$\Rightarrow (1 - p)\{(1 - p + p + 1)\} = 0$$

$$\Rightarrow 1 - p = 0 \Rightarrow p = 1$$

If $p = 1$, then equation is $x^2 + x = 0$ and roots are $0, -1$.

35. (b) Let the equation be

$$ax^2 + bx + c = 0$$

Given leading coefficient $a = 1$

So, the equation will become $x^2 + bx + c = 0$

Correct coefficient of $x = 16$

So, correct product of roots

$$= (-15) \times (-4) = 60$$

So, equation will be

$$x^2 + 16x + 60 = 0$$

$$(x + 10)(x + 6) = 0$$

$$x = -6, -10$$

36. (b) Let the equation be $x^2 + ax + b = 0$

Ramesh commits a mistake in constant and find roots 8 and 2.

\Rightarrow He makes mistake in product of roots, sum is correct.

$$\text{So, sum of roots} = 8 + 2 = 10$$

While Mahesh commits mistake in coefficient of x and find roots -9 and -1 .

\Rightarrow He makes mistake in sum of roots, product is correct.

$$\text{So, product of roots} = -9 \times -1 = 9$$

$$\text{Hence, equation is } x^2 - 10x + 9 = 0$$

$$\Rightarrow \text{Correct roots are } 9 \text{ and } 1.$$

37. (c) Same as Q. 36.

Correct roots are -3 and -4 .

38. (a) Let roots are α and 2α . Then

$$\alpha + 2\alpha = \frac{-(3a - 1)}{a^2 - 5a + 3}$$

$$\Rightarrow \alpha = \frac{1 - 3a}{3(a^2 - 5a + 3)}$$

Quadratic Equation

$$\text{and } 2\alpha^2 = \frac{2}{a^2 - 5a + 3}$$

$$\text{or } \frac{(1-3a)^2}{9(a^2 - 5a + 3)^2} = \frac{1}{a^2 - 5a + 3}$$

$$\Rightarrow 1 + 9a^2 - 6a = 9a^2 - 45a + 27$$

$$\Rightarrow 39a = 26$$

$$\Rightarrow a = \frac{26}{39} = \frac{2}{3}$$

$$39. (b) \frac{x^2 - bx}{ax - c} = \frac{m-1}{m+1}$$

$$\Rightarrow (m+1)x^2 - x\{b(m+1) + a(m-1)\} + (m-1)c = 0$$

Roots are equal in magnitude and opposite in sign.

So, sum of roots = 0

$$\text{i.e., } \frac{b(m+1) + a(m-1)}{m+1} = 0$$

$$\Rightarrow m = \frac{a-b}{a+b}$$

$$40. (a) \frac{a}{x-a} + \frac{b}{x-b} = 1$$

$$\Rightarrow x^2 - 2(a+b)x + 3ab = 0$$

Roots are equal in magnitude and opposite in signs.

So, sum of roots = 0

$$\Rightarrow a + b = 0$$

$$41. (b) \alpha + \beta = p, \alpha\beta = -p - c$$

$$(\alpha + 1)(\beta + 1) = \alpha\beta + (\alpha + \beta) + 1$$

$$= p - p - c + 1 = 1 - c$$

$$\text{Then } \frac{\alpha^2 + 2\alpha + 1}{\alpha^2 + 2\alpha + c} + \frac{\beta^2 + 2\beta + 1}{\beta^2 + 2\beta + c}$$

$$= \frac{(\alpha + 1)^2}{(\alpha + 1)^2 - (1 - c)} + \frac{(\beta + 1)^2}{(\beta + 1)^2 - (1 - c)}$$

$$= \frac{(\alpha + 1)^2}{(\alpha + 1)^2 - (\alpha + 1)(\beta + 1)} + \frac{(\beta + 1)^2}{(\beta + 1)^2 - (\beta + 1)(\alpha + 1)}$$

$$= \frac{\alpha + 1}{\alpha - \beta} + \frac{\beta + 1}{\beta - \alpha}$$

$$= \frac{\alpha - \beta}{\alpha - \beta} = 1$$

$$42. (a) \alpha + \beta = \frac{-2b}{a}; \alpha\beta = \frac{c}{a}$$

$$\text{and } (\alpha + \delta)(\beta + \delta) = \frac{C}{A}$$

$$(\alpha + \delta) + (\beta + \delta) = -\frac{2B}{A}$$

$$\text{Now } \{(\alpha + \delta) - (\beta + \delta)\}^2 = (\alpha - \beta)^2$$

$$\{(\alpha + \delta) - (\beta + \delta)\}^2 - 4(\alpha + \delta)(\beta + \delta)$$

$$= (\alpha + \beta)^2 - 4\alpha\beta$$

$$\Rightarrow \frac{4B^2}{A^2} - \frac{4C}{A} = \frac{4b^2}{a^2} - \frac{4c}{a}$$

$$\Rightarrow \frac{B^2 - AC}{A^2} = \frac{b^2 - ac}{a^2}$$

$$\Rightarrow \frac{b^2 - ac}{B^2 - AC} = \left(\frac{a}{A}\right)^2$$

43. (a) To be identity

$$\lambda^3 - 3\lambda + 2 = 0, \lambda^2 - 5\lambda + 6 = 0 \text{ and } \lambda^2 - 4 = 0$$

$$\Rightarrow \lambda = 1 \text{ or } 2 \text{ and } \lambda = 2 \text{ or } 3 \text{ and } \lambda = 2 \text{ or } -2$$

$$\Rightarrow \lambda = 2$$

44. (c) For $(b-c)x^2 + (c-a)x + (a-b) = 0$

$$B^2 - 4AC = (c-a)^2 - 4(b-c)(a-b)$$

$$= c^2 + a^2 + 4b^2 + 2ac - 4ab - 4ac$$

$$= (c+a-2b)^2 = \text{Perfect square}$$

So, roots are rational.

45. (b) For $(c^2 - ab)x^2 - 2(a^2 - bc)x + (b^2 - ac) = 0$

$$B^2 - 4AC$$

$$= 4(a^2 - bc)^2 - 4(c^2 - ab)(b^2 - ac)$$

$$= 4\{a^4 + ab^3 + ac^3 - 3a^2bc\}$$

$$= 4a(a^3 + b^3 + c^3 - 3abc)$$

$$\{\because a + b + c = 0 \Rightarrow a^3 + b^3 + c^3 = 3abc\}$$

\Rightarrow Roots are real and equal.

46. (c) Roots are of opposite sign.

So, product of roots < 0

$$\Rightarrow \frac{a^2 - 3a + 2}{3} < 0$$

$$\Rightarrow (a-1)(a-2) < 0$$

$$\Rightarrow 1 < a < 2$$

47. (b) $x^2 + 2ax + 10 - 3a > 0$

$$\Rightarrow 4a^2 - 4(10 - 3a) < 0$$

$$\Rightarrow a^2 + 3a - 10 < 0$$

$$\Rightarrow (a+5)(a-2) < 0$$

$$\Rightarrow -5 < a < 2$$

48. (a) Put $x = -a$ in both the given equations.

$$a^2 - pa + q = 0 \Rightarrow a^2 = pa - q$$

$$\text{and } a^2 - la + m = 0 \Rightarrow a^2 = la - m$$

$$\text{So, } pa - q = la - m$$

$$\text{or } a = \frac{q-m}{p-l} (p \neq l)$$

49. (a) Let α, β be the roots of $x^2 + ax + 1 = 0$

$$\therefore \alpha + \beta = -a \text{ and } \alpha\beta = 1$$

$$\text{Given } \alpha - \beta < \sqrt{5}$$

$$\text{or } (\alpha - \beta)^2 < 5 \text{ or } (\alpha + \beta)^2 - 4\alpha\beta < 5$$

$$\Rightarrow a^2 - 4 < 5$$

$$\Rightarrow a^2 - 9 < 0$$

$$\Rightarrow (a+3)(a-3) < 0$$

$$\Rightarrow -3 < a < 3$$

50. (a) Let α, β are the roots of $ax^2 + bx + c = 0$.

$$\therefore \alpha + \beta = -b/a \text{ and } \alpha\beta = c/a$$

$$\begin{aligned} \text{Given } \alpha + \beta &= \alpha^2 + \beta^2 \\ \text{or } \alpha + \beta &= (\alpha + \beta)^2 - 2\alpha\beta \\ \Rightarrow -\frac{b}{a} &= \frac{b^2}{a^2} - \frac{2c}{a} \end{aligned}$$

$$\Rightarrow 2ac = ab + b^2$$

51. (a) α, β are the roots of $x^2 + px + 1 = 0$
 $\Rightarrow \alpha + \beta = -p$ and $\alpha\beta = 1$
 γ, δ are the roots of $x^2 + qx + 1 = 0$
 $\Rightarrow \gamma + \delta = -q$ and $\gamma\delta = 1$
 So, $(\alpha - \gamma)(\alpha + \delta)(\beta - \gamma)(\beta + \delta)$
 $= [\alpha\beta - \gamma(\alpha + \beta) + \gamma^2][\alpha\beta + \delta(\alpha + \beta) + \delta^2]$
 $= [1 + p\gamma + \gamma^2][1 - p\delta + \delta^2]$
 $= [q\gamma + p\gamma][q\delta - p\delta]$
 $\{ \because \gamma^2 + q\gamma + 1 = 0 \text{ and } \delta^2 + q\delta + 1 = 0 \}$
 $= (p + q)\gamma(q - p)\delta$
 $= (q^2 - p^2)\gamma\delta = q^2 - p^2$
52. (c) Roots of $x^2 + x + 1 = 0$ are ω and ω^2 .
 So, $a = \omega$ and $b = \omega^2$
 $a^2 + b^2 = \omega^2 + \omega^4 = \omega^2 + \omega = -1$
53. (b) Roots of the equation $x^2 + x + 1 = 0$ are ω and ω^2 .
 Let $\alpha = \omega$ and $\beta = \omega^2$
 Then, $\frac{\alpha}{\beta} = \frac{1}{\omega}$ and $\frac{\beta}{\alpha} = \omega$
 Sum of the roots $\frac{\alpha}{\beta} + \frac{\beta}{\alpha} = -p$
 $\Rightarrow \frac{1}{\omega} + \omega = -p \Rightarrow \frac{1 + \omega^2}{\omega} = -p$
 $\Rightarrow -\frac{\omega}{\omega} = -p \Rightarrow p = 1$
54. (d) $\log_3(x^2 + 4x + 12) = 2$
 $\Rightarrow x^2 + 4x + 12 = 3^2$
 $\Rightarrow x^2 + 4x + 3 = 0$
 $\Rightarrow (x + 1)(x + 3) = 0$
 $\Rightarrow x = -1$ or $x = -3$
55. (d) $x^2 - 2bx + c > 0$
 If $4b^2 - 4c < 0$ $\{ \because b^2 - 4ac < 0 \}$
 $\Rightarrow b^2 < c$
56. (a) Factors of $x^2 + x + 1$ are ω and ω^2 .
 They are also the factor of
 $ax^3 + bx^2 + cx + d$
 i.e., ω and ω^2 are the roots of $ax^3 + bx^2 + cx + d = 0$
 So, product of roots $\alpha\beta\gamma = -\frac{d}{a}$
 $\omega \cdot \omega^2 \cdot \gamma = \frac{d}{a}$
 So, third root = $-\frac{d}{a}$ $\{ \because \omega^3 = 1 \}$
57. (b) Product of roots = 7
 $2e^{2 \log k} - 1 = 7$
 $e^{2 \log k} = 4$
 $\Rightarrow k^2 = 4 \Rightarrow k = 2$

58. (c) $\because \alpha$ and β are the roots of $x^2 + bx + c = 0$
 So, $\alpha + \beta = -b$ and $\alpha\beta = c$
 Given, $b > 0$ and $c < 0$
 $\Rightarrow \alpha + \beta < 0$ and $\alpha\beta < 0$... (i)
 This is possible only when one of α and β is negative and the negative number is such that its magnitude is greater than β .
 Given, $\alpha < \beta$
 $\Rightarrow \alpha < 0$ and $\beta > 0$
 So, $\beta < -\alpha$ and $\beta < |\alpha|$
 So, both the statements are correct.
59. (b) $\alpha + \beta + \alpha\beta < 0$ {From eq. (i) of Q. 58}
 So, statement 1 is not correct.
 And $\alpha^2\beta + \beta^2\alpha = \alpha\beta(\alpha + \beta)$
 $= (-)(-) = (+)$
 i.e., $\alpha^2\beta + \beta^2\alpha > 0$
 So, statement 2 is correct.
60. (a) Given, $x^2 - px + 4 > 0$
 $\Rightarrow (-p)^2 - 4(1)(4) < 0$ $\{ \because b^2 - 4ac < 0 \}$
 $\Rightarrow p^2 - 16 < 0$
 $\Rightarrow p^2 < 16$
 $\Rightarrow |p| < 4$
61. (b) Let the roots of the equation $(l - m)x^2 + lx + 1 = 0$ and α and 2α .
 Then, $\alpha + 2\alpha = \frac{-l}{l - m}$ and $\alpha(2\alpha) = \frac{1}{l - m}$
 $\Rightarrow \alpha = \frac{l}{3(m - l)}$ and $2\alpha^2 = \frac{1}{l - m}$
 $\Rightarrow \frac{2l^2}{9(m - l)^2} = \frac{1}{(l - m)}$
 $\Rightarrow 2l^2 = 9(l - m)$
 $\Rightarrow 2l^2 - 9l + 9m = 0$
 $\because l$ is real (given) $\{ b^2 - 4ac \geq 0 \}$
 $\Rightarrow 81 - 4(2)(9m) \geq 0$
 $\Rightarrow 8 \times 9m \leq 81$
 $\Rightarrow m \leq \frac{9}{8}$
 So, greater value of $m = \frac{9}{8}$
62. (d) Given, α, β are the roots of
 $x^2 - (1 - 2a^2)x + (1 - 2a^2) = 0$
 For real roots
 $(1 - 2a^2)^2 - 4(1)(1 - 2a^2) \geq 0$
 $\Rightarrow (1 - 2a^2)(1 - 2a^2 - 4) \geq 0$
 $\Rightarrow (2a^2 - 1)(2a^2 + 3) \geq 0$
 $\Rightarrow 2a^2 - 1 \geq 0$ $\{ \because 2a^2 + 3 \geq 0 \}$
 $\Rightarrow a^2 \geq \frac{1}{2}$
63. (a) Here, $\alpha + \beta = 1 - 2a^2$ and $\alpha\beta = 1 - 2a^2$
 Given, $\frac{1}{\alpha^2} + \frac{1}{\beta^2} < 1$

Quadratic Equation

$$\begin{aligned} &\Rightarrow \alpha^2 + \beta^2 < \alpha^2 \beta^2 \\ &\Rightarrow (\alpha + \beta)^2 - 2\alpha\beta < (\alpha\beta)^2 \\ &\Rightarrow (1 - 2a^2)^2 - 2(1 - 2a^2) < (1 - 2a^2)^2 \\ &\Rightarrow 1 - 2a^2 > 0 \\ &\Rightarrow a^2 < \frac{1}{2} \end{aligned}$$

64. (c) Given, -2 is the root of the equation

$$\begin{aligned} 2x^2 + 3x - \alpha &= 0 \\ \Rightarrow 8 - 6 - \alpha &= 0 \\ \Rightarrow \alpha &= 2 \\ \text{So, eq. becomes } 2x^2 + 3x - 2 &= 0 \\ \Rightarrow (2x - 1)(x + 2) &= 0 \\ \Rightarrow x = \frac{1}{2}, x = -2 \end{aligned}$$

So, other root $\beta = \frac{1}{2}$

65. (a) From solution of Q. 64, $\beta = \frac{1}{2}$

Given, $\beta, 2, 2m$ are in GP.
 $\Rightarrow (2)^2 = \beta(2m)$
 $\Rightarrow 2 = \beta m$

Putting $\beta = \frac{1}{2}$, we get $m = 4$

Now, $\beta\sqrt{m} = \frac{1}{2} \times 2 = 1$

66. (a) Let $f(x) = ax^2 - bx + c$

Then, $f(0) = c$
 and $f(2) = 4a - 2b + c$
 Given, $c > 0$ and $4a - 2b + c < 0$
 $\Rightarrow f(0) > 0$ and $f(2) < 0$

Hence, $f(x) = 0$ has both roots lying between 0 and 2 i.e., in the interval $(0, 2)$.

67. (d) $x^2 - 2kx + k^2 - 4 = 0$

$$\Rightarrow x = \frac{2k \pm \sqrt{4k^2 - 4(k^2 - 4)}}{2}$$

$$\Rightarrow x = k \pm 2$$

Given, roots lie between -3 and 5 .

$$\Rightarrow -3 \leq k \pm 2 \leq 5$$

$$\Rightarrow -3 < k - 2 < 5 \text{ and } -3 < k + 2 < 5$$

$$\Rightarrow -1 < k < 7 \text{ and } -5 < k < 3$$

Hence, the correct answer is $-1 < k < 3$.

68. (a) Let α and β are the roots of equation $x^2 + kx + 1 = 0$.

Then, $\alpha + \beta = -k$ and $\alpha\beta = 1$

Given, $\alpha - \beta < \sqrt{5}$

$$\Rightarrow (\alpha - \beta)^2 < 5$$

$$\Rightarrow (\alpha + \beta)^2 - 4\alpha\beta < 5$$

$$\Rightarrow k^2 - 4 < 5 \Rightarrow k^2 < 9$$

$$\Rightarrow |k| < 3$$

$$\Rightarrow -3 < k < 3$$

...(i)

Also given $|k| \geq 2$

$$\Rightarrow k \leq -2 \text{ or } k \geq 2$$

...(ii)

From (i) and (ii),

$$-3 < k \leq -2 \text{ or } 2 \leq k < 3$$

$$\Rightarrow k \in (-3, -2] \cup [2, 3)$$

69. (a) Let α, β be the roots of $x^2 + px + q = 0$ and γ, δ be the roots of $x^2 + lx + m = 0$

So, $\alpha + \beta = -p, \gamma + \delta = -l$

$\alpha\beta = q, \gamma\delta = m$

Given, $\frac{\alpha}{\beta} = \frac{\gamma}{\delta}$

By Componendo Dividendo Theorem

$$\frac{\alpha + \beta}{\alpha - \beta} = \frac{\gamma + \delta}{\gamma - \delta}$$

$$\frac{p}{l} = \frac{\alpha - \beta}{\gamma - \delta}$$

$$\frac{p^2}{l^2} = \frac{(\alpha + \beta)^2 - 4\alpha\beta}{(\gamma + \delta)^2 - 4\gamma\delta}$$

$$\frac{p^2}{l^2} = \frac{p^2 - 4q}{l^2 - 4m}$$

$$\Rightarrow p^2 m = l^2 q$$

70. (c) Let the quadratic polynomial be $ax^2 + bx + c$

Given, $ax^2 + bx + c > 0$

$$\Rightarrow b^2 - 4ac < 0, a > 0$$

\Rightarrow Both roots are imaginary, i.e., complex.