CHAPTER 8

Sequence and Series

Exercise

1. If second term of a GP is 2 and the sum of infinite terms is 8, then its first term is
   (a) $\frac{1}{4}$ (b) $\frac{1}{2}$
   (c) 2 (d) 4
2. If 7th and 13th terms of an AP be 34 and 64 respectively, then its 18th term is
   (a) 87 (b) 88
   (c) 89 (d) 90
3. If a, b, c are in GP and $a^{1/x} = b^{1/y} = c^{1/z}$, then x, y, z are in
   (a) AP (b) GP
   (c) HP (d) None of these
4. If x, y, z are in AP, then the value of $(x + y - z)(y + z - x)$ is
   (a) $8yz - 3y^2 - 4z^2$ (b) $4xz - 3y^2$
   (c) $8xy - 4x^2 - 3y^2$ (d) All are correct
5. If $\log(x + z) + \log(x - 2y + z) = 2\log(x - z)$ then x, y, z are in
   (a) HP (b) GP
   (c) AP (d) None of these
6. If the sum of the series $1+\frac{3}{x}+\frac{9}{x^2}+\frac{27}{x^3}+\infty$ is a finite number, then
   (a) $x < 3$ (b) $x > \frac{1}{3}$
   (c) $x < \frac{1}{3}$ (d) $x > 3$
7. If a, b, c are in HP then the value of $\frac{b+a}{b-a}+\frac{b+c}{b-c}$ is
   (a) 1 (b) 2
   (c) 3 (d) None of these
8. If $x = \sum_{n=0}^{\infty} a^n, y = \sum_{n=0}^{\infty} b^n, z = \sum_{n=0}^{\infty} (ab)^n$, where $a, b < 1$, then
   (a) $xyz = x + y + z$ (b) $xz + yz = xy + z$
   (c) $xy + yz = xz + y$ (d) $xy + xz = yz + x$
9. $(\underbrace{666.....6}_{n\text{-digits}})^2 + (\underbrace{888.....8}_{n\text{-digits}})$ is equal to
   (a) $\frac{4}{9}(10^n - 1)$ (b) $\frac{4}{9}(10^{2n} - 1)$
   (c) $\frac{4}{9}(10^n - 1)^2$ (d) None of these
10. If a, b, c are in GP then $\log_a x, \log_b x, \log_c x$ are in
    (a) AP (b) GP
    (c) HP (d) None of these
11. The sum of the first four terms of an AP is 56. The sum of the last four terms is 112. If its first term is 11, then the number of terms is
    (a) 10 (b) 11
    (c) 12 (d) None of these
12. If pth, qth and rth terms of a GP are x, y, z respectively, then $x^{q-r} y^{r-p} z^{p-q}$ is equal to
    (a) 0 (b) 1
    (c) – 1 (d) none of these
13. The sum of all two digit numbers which when divided by 4, yields unity as a remainder is
    (a) 1012 (b) 1201
    (c) 1212 (d) 1210
14. If $x^{18} = y^{21} = z^{28}$, then $3, 3\log_y x, 3\log_z y, 7\log_x z$ are in
    (a) AP (b) GP
    (c) HP (d) None of these
15. If $\frac{a^n + b^n}{a^{n-1} + b^{n-1}}$ is the AM between a and b, then the value of n is
    (a) 0 (b) 1
    (c) – 1 (d) none of these
16. If x, y, z are in GP and $a^x = b^y = c^z$ then
    (a) $\log_b a = \log_a c$ (b) $\log_c b = \log_a c$
    (c) $\log_b a = \log_c b$ (d) None of these

17. Sum of three numbers in GP be 14. If one is added to first and second and 1 is subtracted from the third, the new numbers are in AP. The smallest of them is
(a) 2 (b) 4
(c) 6 (d) 8

18. If the sum of an infinite GP be 3 and the sum of the squares of its terms is also 3, then its first term and common ratio are
(a) $\frac{3}{2}, \frac{1}{2}$ (b) $\frac{1}{2}, \frac{3}{2}$
(c) $1, \frac{1}{2}$ (d) None of these

19. If the p^{th}, q^{th} and r^{th} terms of an A.P. be a, b, c respectively, then
$a(q-r) + b(r-p) + c(p-q)$ is equal to
(a) 0 (b) 2
(c) pqr (d) $p+q+r$

20. If the roots of the equation $x^3 - 12x^2 + 39x - 28 = 0$ are in A.P., then their common difference will be
(a) ± 1 (b) ± 2
(c) ± 3 (d) ± 4

21. The least value of n such that $1 + 3 + 5 + 7 + \ldots\ldots\ldots n$ terms ≥ 500 is
(a) 18 (b) 19
(c) 22 (d) 23

22. The maximum value of the sum of the series
$20 + 19\frac{1}{3} + 18\frac{2}{3} + \ldots.$ is
(a) 300 (b) 310
(c) 320 (d) None of these

23. Let S_n denote the sum of first n terms of an AP if $S_{2n} = 3S_n$, then the ratio $3_{3n}/S_n$ is equal to
(a) 4 (b) 6
(c) 8 (d) 10

24. The ratio between the sum of n terms of two AP's is $3n + 8 : 7n + 15$. Then the ratio between their 12^{th} terms is
(a) 5 : 7 (b) 7 : 16
(c) 12 : 11 (d) None of these

25. If $a_1, a_2, a_3, \ldots.$ is an AP such that $a_1 + a_5 + a_{10} + a_{15} + a_{20} + a_{24} = 225$, then $a_1 + a_2 + a_3 + \ldots. + a_{23} + a_{24}$ is equal to
(a) 909 (b) 75
(c) 750 (d) 900

26. If S_1, S_2, S_3 be the sum of n, $2n$, $3n$ terms respectively of an AP, then
(a) $S_3 = S_1 + S_2$ (b) $S_3 = 2(S_1 + S_2)$
(c) $S_3 = 3(S_2 - S_1)$ (d) None of these

27. In a GP, $T_2 + T_5 = 216$ and $T_4 : T_6 = 1 : 4$ and all terms are integers, then its first term is
(a) 16 (b) 14
(c) 12 (d) None of these

28. In a geometric progression consisting of positive terms, each term equals to the sum of the next two terms. Then the common ratio of this progression equals to
(a) $\frac{1}{2}(1-\sqrt{5})$ (b) $\frac{1}{2}\sqrt{5}$
(c) $\sqrt{5}$ (d) $\frac{1}{2}(\sqrt{5}-1)$

29. If a, b, c be three successive terms of a G.P. with common ratio r and $a > 0$ satisfying the relation $c > 4b - 3a$, then
(a) $1 < r < 3$ (b) $-3 < r < -1$
(c) $r > 3$ or $r < 1$ (d) None of these

30. If $(1-k)(1 + 2x + 4x^2 + 8x^3 + 16x^4 + 32x^5) = 1 - k^6$, where $k \neq 1$ then the value of $\frac{k}{x}$ is
(a) 2 (b) 4
(c) $\frac{1}{2}$ (d) $\frac{1}{4}$

31. A GP consists of $2n$ terms. If the sum of the terms occupying the odd places is S_1 and that of the terms in the even place is S_2 then the common ratio of the GP is
(a) $\frac{S_2}{S_1}$ (b) $\frac{S_1}{S_2}$
(c) $S_1 + S_2$ (d) $S_1 - S_2$

32. Three distinct real numbers a, b, c are in GP such that $a + b + c = xb$, then
(a) $0 < x < 1$ (b) $-1 < x < 3$
(c) $x < -1$ or $x > 3$ (d) $-1 < x < 2$

33. If the AM and GM between two numbers are in the ratio $m : n$, then the numbers are in the ratio
(a) $m + \sqrt{n^2 - m^2} : m - \sqrt{n^2 + m^2}$
(b) $m + \sqrt{n^2 + m^2} : m - \sqrt{n^2 + m^2}$
(c) $m + \sqrt{m^2 - m^2} : m - \sqrt{m^2 - n^2}$
(d) None of these

34. If $x = \sum_{n=0}^{\infty} a^n, y = \sum_{n=0}^{\infty} b^n, z = \sum_{n=0}^{\infty} c^n$, where a, b, c are in A.P. such that $|a| < 1$, $|b| < 1$ and $|c| < 1$ then x, y, z are in
(a) AP (b) GP
(c) HP (d) None of these

35. Sum to infinity of the series
$\frac{3}{4} - \frac{5}{4^2} + \frac{3}{4^3} - \frac{5}{4^4} + \frac{3}{4^5} - \frac{5}{4^6} + \ldots\ldots$ is
(a) $\frac{1}{3}$ (b) $\frac{1}{5}$
(c) $\frac{7}{15}$ (d) None of these

36. If $S_1, S_2, ..., S_\lambda$ are the sums of infinite GP's whose first terms are respectively 1, 2, 3, ..., λ and common ratios are $\frac{1}{2}, \frac{1}{3}, ..., \frac{1}{\lambda+1}$ respectively, then $S_1 + S_2 + S_3 + ... + S_\lambda$ is equal to
    (a) $\frac{\lambda(\lambda+1)}{2}$ (b) $\frac{\lambda(\lambda+2)}{2}$
    (c) $\frac{\lambda(\lambda+3)}{2}$ (d) None of these

37. If a, b, c are in AP, then $2^{ax+1}, 2^{bx+1}, 2^{cx+1}, x \neq 0$ are in
    (a) AP (b) GP only when $x > 0$
    (c) GP if $x < 0$ (d) GP $\forall\, x \neq 0$

38. If a, b, c are in HP then the value of $\left(\frac{1}{b}+\frac{1}{c}-\frac{1}{a}\right)\left(\frac{1}{c}+\frac{1}{a}-\frac{1}{b}\right)$ is
    (a) $\frac{2}{bc}-\frac{1}{b^2}$ (b) $\frac{1}{4}\left(\frac{3}{c^2}+\frac{2}{ca}-\frac{1}{a^2}\right)$
    (c) $\frac{3}{b^2}-\frac{2}{ab}$ (d) All of these

39. 1.2.3 + 2.3.4 + 3.4.5 + ... n terms is $\frac{n(n+1)(n+2)(n+3)}{P}$ where P is equal to
    (a) 4 (b) 6
    (c) 8 (d) 12

40. If $a_1, a_2, a_3, ...\ a_n$ be an A.P. of non-zero terms, then $\frac{1}{a_1a_2}+\frac{1}{a_2a_3}+......+\frac{1}{a_{n-1}\,a_n}$ is equal to
    (a) $\frac{1}{a_1a_n}$ (b) $\frac{n}{a_1a_n}$
    (c) $\frac{n-1}{a_1a_n}$ (d) None of these

41. If the m^{th} term of an HP is n and n^{th} term be m, then $(m + n)^{th}$ term is
    (a) $\frac{mn}{m+n}$ (b) $\frac{m}{m+n}$
    (c) $\frac{n}{m+n}$ (d) $\frac{m-n}{m+n}$

42. The sum of 0.2 + 0.004 + 0.00006 + ... ∞ is :
    (a) $\frac{200}{891}$ (b) $\frac{2000}{9801}$
    (c) $\frac{1000}{9801}$ (d) None of these

43. The value of $2^{1/4} \cdot 4^{1/8} \cdot 8^{1/16} \cdot 16^{1/32} \cdot$ ..... is
    (a) 1 (b) 2
    (c) $\frac{3}{2}$ (d) $\frac{5}{2}$

Directions (Q. Nos. 44-45):
Given that $\log_x y, \log_z x, \log_y z$ are in GP, $xyz = 64$ and x^3 y^3, z^3 are in AP.

44. Which one of the following is correct?
    x, y and z are **[NDA-I 2016]**
    (a) in AP only
    (b) in GP only
    (c) in both AP and GP
    (d) Neither in AP nor in GP

45. Which one of the following is correct?
    xy, yz and zx are **[NDA-I 2016]**
    (a) in AP only
    (b) in GP only
    (c) in both AP and GP
    (d) Neither in AP nor in GP

46. What is the greatest value of the positive integer n satisfying the condition **[NDA-II 2016]**
    $$1+\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+...+\frac{1}{2^{n-1}} < 2-\frac{1}{1000}\ ?$$
    (a) 8 (b) 9
    (c) 10 (d) 11

47. How many geometric progressions is/are possible containing 27, 8 and 12 as three of its/their terms? **[NDA-II 2016]**
    (a) One (b) Two
    (c) Four (d) Infinitely many

Directions (Q. Nos. 48-50):
Let a, x, y, z, b be in AP, where $x + y + z = 15$. Let a, p, q, r, b be in HP, where $p^{-1} + q^{-1} + r^{-1} = 5/3$.

48. What is the value of ab? **[NDA-II 2016]**
    (a) 10 (b) 9
    (c) 8 (d) 6

49. What is the value of xyz? **[NDA-II 2016]**
    (a) 120 (b) 105
    (c) 90 (d) Cannot be determined

50. What is the value of pqr? **[NDA-II 2016]**
    (a) 35/243 (b) 81/35
    (c) 243/35 (d) Cannot be determined

Directions (Q. Nos. 51-52):
The sixth term of an AP is 2 and its common difference is greater than 1.

51. What is the common difference of the AP so that the product of the first, fourth and fifth terms is greatest? **[NDA-II 2016]**
    (a) 8/5 (b) 9/5
    (c) 2 (d) 11/5

52. What is the first term of the AP, so that the product of the first, fourth and fifth terms is greatest? **[NDA-II 2016]**
    (a) – 4 (b) – 6
    (c) – 8 (d) – 10

Directions (Q. Nos. 53–54):

The interior angles of a polygon of n sides are in AP. The smallest angle is 120° and the common difference is 5°.

53. How many possible values can n have? **[NDA-II 2016I]**
(a) One (b) Two
(c) Three (d) Infinitely many

54. What is the largest interior angle of the polygon? **[NDA-II 2016]**
(a) 160° only
(b) 195° only
(c) Either 160° or 195°
(d) Neither 160° nor 195°

55. What is the sum of the series
$0.3 + 0.33 + 0.333 + \dots n$ terms? **[NDA-I 2017]**
(a) $\frac{1}{3}\left[n-\frac{1}{9}\left(1-\frac{1}{10^n}\right)\right]$ (b) $\frac{1}{3}\left[n-\frac{2}{9}\left(1-\frac{1}{10^n}\right)\right]$
(c) $\frac{1}{3}\left[n-\frac{1}{3}\left(1-\frac{1}{10^n}\right)\right]$ (d) $\frac{1}{3}\left[n-\frac{1}{9}\left(1+\frac{1}{10^n}\right)\right]$

56. If the sum of m terms of an AP is n and the sum of n terms is m, then the sum of $(m + n)$ terms is **[NDA-I 2017]**
(a) mn (b) $m + n$
(c) $2(m + n)$ (d) $-(m + n)$

57. The fifth term of an AP of n terms, whose sum is $n^2 - 2n$, is **[NDA-I 2017]**
(a) 5 (b) 7
(c) 8 (d) 15

58. The sum of the first n terms of the series $\frac{1}{2}+\frac{3}{4}+\frac{7}{8}+\frac{15}{16}+\dots$ is equal to **[NDA-I 2017]**
(a) $2^n - n - 1$ (b) $1 - 2^{-n}$
(c) $2^{-n} + n - 1$ (d) $2^n - 1$

59. The sum of the roots of the equation $x^2 + bx + c = 0$ (where b and c are non-zero) is equal to the sum of the reciprocals of their squares. Then $\frac{1}{c}, b, \frac{c}{b}$ are in **[NDA-I 2017]**
(a) AP (b) GP
(c) HP (d) None of these

60. The sum of the roots of the equation $ax^2 + x + c = 0$ (where a and c are non-zero) is equal to the sum of the reciprocals of their squares. Then a, ca^2, c^2 are in **[NDA-I 2017]**
(a) AP (b) GP
(c) HP (d) None of these

61. If $S_n = nP + \frac{n(n-1)Q}{2}$, where S_n denotes the sum of the first n terms of an AP, then the common difference is **[NDA-II 2017]**
(a) P + Q (b) 2P + 3Q
(c) 2Q (d) Q

62. The value of the product $6^{\frac{1}{2}} \times 6^{\frac{1}{4}} \times 6^{\frac{1}{8}} \times 6^{\frac{1}{16}} \times \dots$ up to infinite terms is **[NDA-II 2017]**
(a) 6 (b) 36
(c) 216 (d) 512

63. A person is to count 4500 notes. Let a_n denote the number of notes he counts in the n^{th} minute. If $a_1 = a_2$ $a_3 = \dots = a_{10} = 150$, and $a_{10}, a_{11}, a_{12}, \dots$ are in AP with the common difference –2, then the time taken by him to count all the notes is **[NDA-II 2017]**
(a) 24 minutes (b) 34 minutes
(c) 125 minutes (d) 135 minutes

64. If $y = x + x^2 + x^3 + \dots$ up to infinite terms where $x < 1$, then which one of the following is correct? **[NDA-II 2017]**
(a) $x = \frac{y}{1+y}$ (b) $x = \frac{y}{1-y}$
(c) $x = \frac{1+y}{y}$ (d) $x = \frac{1-y}{y}$

65. If x_1 and x_2 are positive quantities, then the condition for the difference between the arithmetic mean and the geometric mean is to be greater than 1 is **[NDA-II 2017]**
(a) $x_1 + x_2 > 2\sqrt{x_1 x_2}$ (b) $\sqrt{x_1} + \sqrt{x_2} > \sqrt{2}$
(c) $|\sqrt{x_1} - \sqrt{x_2}| > \sqrt{2}$ (d) $x_1 + x_2 < 2(\sqrt{x_1 x_2} + 1)$

66. If $1.3 + 2.3^2 + 3.3^3 + \dots + n.3^n = \frac{(2n-1)3^a + b}{4}$ then a and b are respectively **[NDA-II 2017]**
(a) n, 2 (b) n, 3
(c) $n + 1$, 2 (d) $n + 1$, 3

67. If $n \in N$, then $121^n - 25^n + 1900^n - (-4)^n$ is divisible by which one of the following ? **[NDA-I 2018]**
(a) 1904 (b) 2000
(c) 2002 (d) 2006

68. If the ratio of AM to GM of two positive numbers a and b is 5 : 3, then $a : b$ is equal to **[NDA-I 2018]**
(a) 3 : 5 (b) 2 : 9
(c) 9 : 1 (d) 5 : 3

69. If $x = 1 - y + y^2 - y^3 + \dots$ up to infinite terms, where $|y| < 1$, then which one of the following is correct ? **[NDA-I 2018]**
(a) $x = \frac{1}{1+y}$ (b) $x = \frac{1}{1-y}$
(c) $x = \frac{y}{1+y}$ (d) $x = \frac{y}{1-y}$

70. The third term of a GP is 3. What is the product of the first five terms ? **[NDA-I 2018]**
(a) 216
(b) 226
(c) 243
(d) Cannot be determined due to insufficient data

ANSWERS																			
1.	(d)	**2.**	(c)	**3.**	(a)	**4.**	(d)	**5.**	(a)	**6.**	(d)	**7.**	(b)	**8.**	(b)	**9.**	(b)	**10.**	(c)
11.	(b)	**12.**	(b)	**13.**	(d)	**14.**	(a)	**15.**	(b)	**16.**	(c)	**17.**	(a)	**18.**	(a)	**19.**	(a)	**20.**	(c)
21.	(d)	**22.**	(b)	**23.**	(b)	**24.**	(b)	**25.**	(d)	**26.**	(c)	**27.**	(c)	**28.**	(d)	**29.**	(c)	**30.**	(a)
31.	(a)	**32.**	(c)	**33.**	(c)	**34.**	(c)	**35.**	(c)	**36.**	(c)	**37.**	(d)	**38.**	(d)	**39.**	(a)	**40.**	(c)
41.	(a)	**42.**	(b)	**43.**	(b)	**44.**	(c)	**45.**	(c)	**46.**	(c)	**47.**	(d)	**48.**	(b)	**49.**	(b)	**50.**	(c)
51.	(a)	**52.**	(b)	**53.**	(a)	**54.**	(a)	**55.**	(a)	**56.**	(d)	**57.**	(b)	**58.**	(c)	**59.**	(c)	**60.**	(a)
61.	(d)	**62.**	(a)	**63.**	(b)	**64.**	(b)	**65.**	(c)	**66.**	(d)	**67.**	(b)	**68.**	(c)	**69.**	(a)	**70.**	(c)

Explanations

1. (d) $ar = 2$ ...(i)

 and $\frac{a}{1-r} = 8$ ...(ii)

 From eqs. (i) and (ii),

 $a = 4$ and $r = \frac{1}{2}$

2. (c) $T_7 = a + 6d = 34$

 $T_{13} = a + 12d = 64$

 $\Rightarrow a = 4$ and $d = 5$

 So, $T_{18} = a + 17d = 89$

3. (a) $\because$ a, b, c are in GP.

 $\Rightarrow b^2 = ac$ ...(i)

 Let $a^{1/x} = b^{1/y} = c^{1/z} = k$

 $\Rightarrow a = k^x, b = k^y, c = k^z$

 From eq (i), $k^{2y} = k^x . k^z$

 $\Rightarrow 2y = x + z$

 $\Rightarrow x, y, z$ are in AP.

4. (d) x, y, z are in AP.

 $\Rightarrow 2y = x + z$ ...(i)

 Now, $(x + y - z)(y + z - x)$

 $= (2y - z + y - z)(y + z - 2y + z)$

 {From eq. (i) $x = 2y - z$}

 $= 8yz - 3y^2 - 4z^2$

 Similarly, from eq (i), put $z = 2y - x$

 So, we get $(x + y - z)$

 $(y + z - x) = 8xy - 4x^2 - 4y^2$

 Hence, all are correct.

5. (a) $\log(x + z) + \log(x - 2y + z) = 2\log(x - z)$

 $\Rightarrow (x + z)(x - 2y + z) = (x - z)^2$

 $\Rightarrow 2xy + 2yz = 4xz$

 $\Rightarrow \frac{1}{x} + \frac{1}{z} = \frac{2}{y}$

 $\Rightarrow \frac{1}{x}, \frac{1}{y}, \frac{1}{z}$ are in AP.

 $\Rightarrow x, y, z$ are in HP.

6. (d) For the sum to be finite of an infinite GP common ratio should be lesser than 1.

 $\Rightarrow \frac{3}{x} < 1 \Rightarrow x > 3$

7. (b) a, b, c are in HP.

 $\Rightarrow b = \frac{2ac}{a+c} \Rightarrow \frac{b}{a} = \frac{2c}{a+c}$ and $\frac{b}{c} = \frac{2a}{a+c}$

 Applying componendo dividendo theorem

 $\Rightarrow \frac{b+a}{b-a} = \frac{3c+a}{c-a}$ and $\frac{b+c}{b-c} = \frac{3a+c}{a-c}$

 Adding both terms

 $\Rightarrow \frac{b+a}{b-a} + \frac{b+c}{b-c} = \frac{3c+a}{c-a} + \frac{3a+c}{a-c}$

 $= 2\frac{(c-a)}{(c-a)} = 2$

8. (b) $x = \sum_{n=1}^{\infty} a^n = 1 + a + a^2 + ...\infty$

 $\Rightarrow x = \frac{1}{1-a} \Rightarrow a = \frac{x-1}{x}$

 Similarly, $b = \frac{y-1}{y}$ and $z = \frac{1}{1-ab}$

 $z = \frac{xy}{y+x-1}$

 $\Rightarrow xz + yz = xy + z$

9. (b) $[666 .....6]^2 + [888 .... 8]$

 $= 36\,[111.... 1]^2 + 8\,[111.... 1]$

 $= 36\left\{\frac{1(10^n - 1)}{9}\right\}^2 + 8\left\{1\frac{(10^n - 1)}{9}\right\}$

 $= \frac{36}{81}(10^n - 1)^2 + \frac{8}{9}(10^n - 1)$

$= \frac{4}{9}[10^{2n} + 1 - 2.10^n + 2.10^n - 2]$

$= \frac{4}{9}\{10^{2n} - 1\}$

10. (c) $\because$ a, b, c are in GP.

$\Rightarrow b^2 = ac$

$\Rightarrow 2 \log_x b = \log_x a + \log_x c$

$\Rightarrow \frac{2}{\log_b x} = \frac{1}{\log_a x} + \frac{1}{\log_c x}$

$\Rightarrow \log_a x, \log_b x, \log_c x$ are in HP.

11. (b) Let the A.P. be $a, a + d, a + 2d, ...l - d, l$

Sum of first four terms,

$4a + 6d = 56$ ...(i)

Sum of last four terms,

$4l - 6d = 112$ ...(ii)

Adding eqs. (i) and (ii),

$\Rightarrow a + l = 42$

Given $a = 11$, then $l = 31$ and $d = 2$

$\because l = a + (n - 1)d$

$\Rightarrow 31 = 11 + (n - 1)2 \Rightarrow n = 11$

12. (b) In GP, $T_p = AR^{p-1} = x$

$T_q = AR^{q-1} = y$

$T_r = AR^{r-1} = z$

$x^{q-r} . y^{r-p} . z^{p-q}$

$= (AR^{p-1})^{q-r} . (AR^{q-1})^{r-p} . (AR^{r-1})^{p-q}$

$= A^{q-r+r-p+p-q} R^{(p-1)(q-r) + (q-1)(r-p) + (r-1)(p-q)}$

$= A^0R^0 = 1$

13. (d) Two digits numbers when divided by 4 leaves remainder 1 are 13, 17, 21, ..., 97

This is an AP where $l = a + (n - 1)d$

$\Rightarrow 97 = 13 + (n - 1)4 \Rightarrow n = 22$

Sum = S = $\frac{n}{2}[a + l] = \frac{22}{2}[13 + 97] = 1210$

14. (a) $x^{18} = y^{21} = z^{28}$

$\Rightarrow 18 \log x = 21 \log y = 28 \log z$

$\Rightarrow \log_y x = \frac{\log x}{\log y} = \frac{21}{18} = \frac{7}{6}$

$\Rightarrow \log_z y = \frac{\log y}{\log z} = \frac{28}{21} = \frac{4}{3}$

$\Rightarrow \log_x z = \frac{\log z}{\log x} = \frac{18}{28} = \frac{9}{14}$

Now, 3, $3 \log_y x$, $3 \log_z y$, $7 \log_x y$

$= 3, \frac{21}{6}, \frac{12}{3}, \frac{63}{14}$

$\Rightarrow 3, \frac{7}{2}, 4, \frac{9}{2}$ are in A.P.

15. (b) AM of a and b = $\frac{a+b}{2}$

$\Rightarrow \frac{a^n + b^n}{a^{n-1} + b^{n-1}} = \frac{a+b}{2}$

By Hit and Trial Method, $n = 1$

16. (c) x, y, z are in GP $\Rightarrow \frac{y}{x} = \frac{z}{y}$ ...(i)

Given $a^x = b^y = c^z$

$\Rightarrow x \log a = y \log b = z \log c$

$\Rightarrow \frac{y}{x} = \frac{\log a}{\log b} = \log_b a$

and $\frac{z}{y} = \frac{\log b}{\log c} = \log_c b$

From eq. (i),

$\log_b a = \log_c b$

17. (a) Let 3 numbers in G.P. be $\frac{a}{r}, a, ar$.

Then $\frac{a}{r} + a + ar = 14$ ...(i)

Now, $\frac{a}{r} + 1, a + 1, ar - 1$ are in AP.

$2(a+1) = \left(\frac{a}{r} + 1\right) + (ar - 1)$ ...(ii)

Solving eqs. (i) and (ii),

$a = 4$ and $r = 2$ or $\frac{1}{2}$

So, smallest terms = $\frac{4}{2} = 2$

18. (a) Let the GP of infinite terms be $a, ar, ar^2, ar^3, ... \infty$

Sum $= S_\infty = \frac{a}{1-r} = 3$ ...(i)

Given $a^2, a^2r^2, a^2r^4, ... \infty$ is also GP.

Sum $= S_\infty = \frac{a^2}{1-r^2} = 3$ ...(ii)

From eq. (i) and (ii), $r = \frac{1}{2}$ and $a = \frac{3}{2}$

19. (a) Given $A + (p - 1) D = a$;

$A + (q - 1) D = b$; $A + (r - 1) D = c$.

Then $a(q - r) + b(r - p) + c(p - q)$

$= A\{q - r + r - p + p - q\}$

$+ D\{(p - 1)(q - r) + (q - 1)(r - p) + (r - 1)(p - q)\}$

$= 0$

20. (c) Let $a - d, a, a + d$ are the roots of $x^3 - 12x^2 + 39x - 28 = 0$

So, sum $a - d + a + a + d = 12 \Rightarrow a = 4$

$\Rightarrow a = 4$ is one root

then $x^3 - 12x^2 + 39x - 28 = 0$

will become $(x-4)(x^2-8x+7)=0$

$\Rightarrow x = 1, 4, 7$ or $7, 4, 1$

So, common difference is ± 3.

21. (d) $1 + 3 + 5 + 7 + ...$ n terms ≥ 500

$$\Rightarrow \frac{n}{2}\{2\times 1+(n-1)(2)\} \geq 500$$

$n^2 \geq 500 \Rightarrow n \geq 22 \times 36$

So, least value of $n = 23$

22. (b) Given series $20+19\frac{1}{3}+18\frac{2}{3}+...$ is an AP

having $a = 20$, $d = \frac{58}{3}-20=-\frac{2}{3}$

$\because$ Sum will be maximum when all the numbers are taken (+).

So, $T_n = 20+(n-1)\left(-\frac{2}{3}\right) \geq 0$

$\Rightarrow 31 - n \geq 0$ or $n \leq 31$

i.e., S_{31} is max.

So, $S_{31} = \frac{31}{2}\left\{2\times 20+(30)\left(-\frac{2}{3}\right)\right\} = 310$

23. (b) $S_{2n} = 3S_n$

$$\Rightarrow \frac{2n}{2}[2a+(2n-1)d] = \frac{3n}{2}[2a+(n-1)d]$$

$$\Rightarrow 2a\left(n-\frac{3n}{2}\right) = d\left\{\frac{3n(n-1)}{2}-\frac{2n(n-1)}{2}\right\}$$

$\Rightarrow 2a = (n+1)d$

$$\Rightarrow \frac{S_{3n}}{S_n} = \frac{\frac{3n}{2}\{2a+(3n-1)d\}}{\frac{n}{2}\{2a+(n-1)d\}}$$

$$= \frac{\{d(n+1+3n-1)\}}{\{d(n+1+n-1)\}} = \frac{6}{1}$$

24. (b) $\frac{S_n}{S'_n} = \frac{3n+8}{7n+15}$

$$\Rightarrow \frac{\frac{n}{2}[2a+(n-1)d]}{\frac{n}{2}[2A+(n-1)D]} = \frac{3n+8}{7n+15}$$

$$\Rightarrow \frac{a+\left(\frac{n-1}{2}\right)d}{A+\left(\frac{n-1}{2}\right)D} = \frac{3n+8}{7n+15}$$

Put $\frac{n-1}{2} = 11 \Rightarrow n = 23$

So, $\frac{a+11d}{A+11D} = \frac{3\times 23+8}{7\times 23+15}$

or $\frac{T_{12}}{T'_{12}} = \frac{77}{176} = \frac{7}{16}$

25. (d) $a_1+a_2+a_3+....+a_{24} = \frac{24}{2}(a_1+a_{24})$

$S_{24} = 12\,(a_1 + a_{24})$ ...(i)

$\because$ Sum of terms equidistance from starting and last are same.

So, $a_1 + a_{24} = a_5 + a_{20} = a_{10} + a_{15}$

$\because$ $a_1 + a_5 + a_{10} + a_{15} + a_{20} + a_{24} = 225$

$$\Rightarrow a_1 + a_{24} = \frac{225}{3} = 75$$

From eq. (i), $S_{24} = 12 \times 75 = 900$

26. (c) $S_1 = \frac{n}{2}[2a+(n-1)d]$

$$S_2 = \frac{2n}{2}[2a+(2n-1)d]$$

$$S_3 = \frac{3n}{2}[2a+(3n-1)d]$$

$$S_2 - S_1 = \frac{n}{2}[4a+(4n-2)d-2a-(n-1)d]$$

$$\Rightarrow S_2 - S_1 = \frac{n}{2}[2a+(3n-1)\ d]$$

$$\Rightarrow 3(S_2 - S_1) = \frac{3n}{2}[2a+(3n-1)\ d] = S_3$$

27. (c) In G.P., $T_2 + T_5 = 216$ and $\frac{T_4}{T_6} = \frac{1}{4}$

$ar + ar^4 = 216$ and $\frac{ar^3}{ar^5} = \frac{1}{4}$

$\Rightarrow ar(1 + r^3) = 216$ and $r^2 = 4 \Rightarrow r = \pm 2$

If $r = \pm 2$, then $a = 12$ and $\frac{27}{2}$

$\because$ a is integer.

So, $a = 12$

28. (d) Given $T_n = T_{n+1} + T_{n+2}$

or $ar^{n-1} = ar^n + ar^{n+1}$

$\Rightarrow 1 = r + r^2$

or $r^2 + r - 1 = 0$

$$\Rightarrow r = \frac{-1\pm\sqrt{5}}{2}$$

$\because$ G.P. is of positive terms.

So, $r = \frac{\sqrt{5}-1}{2}$

29. (c) Let $a = a$, $b = ar$ and $c = ar^2$

$\because$ $c > 4b - 3a$

$\Rightarrow ar^2 > 4ar - 3a$

or $r^2 - r + 3 > 0$

or $(r-1)(r-3) > 0$ or $r < 1$ or $r > 3$

30. (a) $(1 - k)(1 + 2x + 4x^2 + 8x^3 + 16x^4 + 32x^5) = 1 - k^6$

or $1 + (2x) + (2x)^2 + (2x)^3 + (2x)^4 + (2x)^5 = \frac{1-k^6}{1-k}$

or $\frac{1-(2x)^6}{1-2x} = \frac{1-k^6}{1-k}$

$\Rightarrow k = 2x$ or $\frac{k}{x} = 2$

31. (a) Given, $S_1 = T_1 + T_3 + T_5 + ...$

$= a\{1 + r^2 + r^4 + ... \}$

and $S_2 = T_2 + T_4 + T_6 + ...$

$= a\{r + r^3 + r^5 + ...\}$

or $S_2 = ar\{1 + r^2 + r^4 + ...\}$

$\Rightarrow S_2 = rS_1$

So, $r = \frac{S_2}{S_1}$

32. (c) a, b, c are in GP

and $a = a$, $b = ar$ and $c = ar^2$

then $a + b + c = axr$

$\Rightarrow a\{1 + r + r^2\} = axr$

or $r^2 + r(1 - x) + 1 = 0$

$\because$ r is real.

So, $(1 - x)^2 - 4 > 0$

or $x^2 - 2x - 3 > 0$ or $(x + 1)(x - 3) > 0$

$\Rightarrow x < -1$ or $x > 3$

33. (c) Let the numbers be a and b.

$$\frac{a+b}{2\sqrt{ab}} = \frac{m}{n}$$

Applying Componendo Dividendo theorem,

$$\frac{(\sqrt{a}+\sqrt{b})^2}{(\sqrt{a}-\sqrt{b})^2} = \frac{m+n}{m-n}$$

or $\frac{\sqrt{a}+\sqrt{b}}{\sqrt{a}-\sqrt{b}} = \frac{\sqrt{m+n}}{\sqrt{m-n}}$

Again applying Componendo Dividendo theorem,

$$\frac{2\sqrt{a}}{2\sqrt{b}} = \frac{\sqrt{m+n}+\sqrt{m-n}}{\sqrt{m+n}-\sqrt{m-n}}$$

or $\frac{a}{b} = \frac{(\sqrt{m+n}+\sqrt{m-n})^2}{(\sqrt{m+n}-\sqrt{m-n})^2}$

$$= \frac{m+\sqrt{m^2-n^2}}{m-\sqrt{m^2-n^2}}$$

34. (c) $x = \sum_{n=0}^{\infty} a^n = 1 + a + a^2 + ...\infty = \frac{1}{1-a}$ ... (i)

$y = \sum_{n=0}^{\infty} b^n = 1 + b + b^2 + ...\infty = \frac{1}{1-b}$ ... (ii)

$z = \sum_{n=0}^{\infty} c^n = 1 + c + c^2 + ...\infty = \frac{1}{1-c}$ ... (iii)

$\because$ a, b, c are in A.P.

So, $2b = a + c$ $\left\{\begin{array}{l}\text{From eqs. (i), (ii), (iii)} \\ a = \frac{x-1}{x}, b = \frac{y-1}{y} \text{ and } c = \frac{z-1}{z}\end{array}\right\}$

$$\Rightarrow 2\left(1-\frac{1}{y}\right) = \left(1-\frac{1}{x}\right) + \left(1-\frac{1}{z}\right)$$

$$\Rightarrow \frac{2}{y} = \frac{1}{x} + \frac{1}{z}$$

$\Rightarrow x, y, z$ are in HP.

35. (c) $\frac{3}{4} - \frac{5}{4^2} + \frac{3}{4^3} - \frac{5}{4^4} + \frac{3}{4^5} - \frac{5}{4^6} + ...\infty$

$$= \frac{3}{4}\left\{1 + \frac{1}{4^2} + \frac{1}{4^4} + ...\infty\right\} - \frac{5}{4^2}\left\{1 + \frac{1}{4^2} + \frac{1}{4^4} + ...\infty\right\}$$

$$= \left(\frac{3}{4} - \frac{5}{16}\right)\left(1 + \frac{1}{4^2} + \frac{1}{4^4} + ...\infty\right)$$

$$= \frac{7}{16}\left(\frac{1}{1-\frac{1}{16}}\right) = \frac{7}{15}$$

36. (c) $S_1 + S_2 + ... + S_\lambda$

$$= \frac{1}{1-\frac{1}{2}} + \frac{2}{1-\frac{1}{3}} + ... + \frac{\lambda}{1-\frac{1}{\lambda+1}}$$

$= 2 + 3 + 4 + ... + (\lambda + 1)$

$= \{1 + 2 + 3 + 4 + ... + (\lambda + 1)\} - 1$

$$= \frac{(\lambda+1)(\lambda+2)}{2} - 1 = \frac{\lambda(\lambda+3)}{2}$$

37. (d) By Hit and Trial method,

Let $2^{ax+1}, 2^{bx+1}, 2^{cx+1}$ are in GP.

$\Rightarrow 2^{2(bx+1)} = 2^{ax+1}.2^{cx+1}$

$\Rightarrow 2(bx + 1) = ax + cx + 2$

or $2b = a + c$

$\Rightarrow a, b, c$ are in AP.

Hence, 2^{ax+1}, 2^{bx+1} and 2^{cx+1} are in GP $\forall\ x \neq 0$

38. (d) a, b, c are in H.P $\Rightarrow \frac{2}{b} = \frac{1}{a} + \frac{1}{c}$

Now, $\left(\frac{1}{b} + \frac{1}{c} - \frac{1}{a}\right)\left\{\frac{1}{c} + \frac{1}{a} - \frac{1}{b}\right\}$

$$= \left\{\frac{1}{c} - \left(\frac{1}{a} - \frac{1}{b}\right)\right\}\left\{\frac{1}{c} + \left(\frac{1}{a} - \frac{1}{b}\right)\right\}$$

$$= \frac{1}{c^2} - \left(\frac{1}{a} - \frac{1}{b}\right)^2 = \frac{1}{c^2} - \left(\frac{2}{b} - \frac{1}{c} - \frac{1}{b}\right)$$

$$= \frac{1}{c^2} - \left(\frac{1}{b} - \frac{1}{c}\right)^2 = \frac{2}{bc} - \frac{1}{b^2}$$

Here a is eliminated.

Similarity if we eliminate, b and c we will get (b) and (c) options.

39. (a) Given series 1.2.3 + 2.3.4 + 3.4.5 + ... nth term

$T_n = n(n+1)(n+2) = n^3 + 3n^2 + 2n$

Sum $S_n = \Sigma T_n = \Sigma n^3 + 3\Sigma n^2 + 2\Sigma n$

$$= \frac{n^2(n+1)^2}{4} + \frac{n(n+1)(2n+1)}{2} + n(n+1)$$

$$= \frac{n(n+1)}{4}\{n(n+1) + 2(2n+1) + 4\}$$

$$= \frac{n(n+1)}{4}\{(n+2)(n+3)\}$$

but given $S_n = \dfrac{n(n+1)(n+2)(n+3)}{p}$

On comparing $p = 4$

40. (c) $a_1, a_2, a_3, \ldots\, a_n$ are in A.P.

so $a_2 - a_1 = a_3 - a_2 = \ldots = a_n - a_{n-1} = d$

Now, $\dfrac{1}{a_1a_2} + \dfrac{1}{a_2a_3} + \ldots + \dfrac{1}{a_{n-1}a_n}$

$$= \frac{1}{d}\left\{\left(\frac{1}{a_1} - \frac{1}{a_2}\right) + \left(\frac{1}{a_2} - \frac{1}{a_3}\right) + \ldots + \left(\frac{1}{a_{n-1}} - \frac{1}{a_n}\right)\right\}$$

$$= \frac{1}{d}\left\{\frac{1}{a_1} - \frac{1}{a_n}\right\} = \frac{1}{d}\left\{\frac{a_n - a_1}{a_1 a_n}\right\}$$

$$= \frac{1}{d}\left\{\frac{a_1 + (n-1)d - a_1}{a_1 a_n}\right\} = \frac{n-1}{a_1 a_n}$$

41. (a) In HP, $T_m = n$ and $T_n = m$

$\Rightarrow A + (m-1)D = \dfrac{1}{n}$ and $A + (n-1)D = \dfrac{1}{m}$

On solving, $A = D = \dfrac{1}{mn}$

So, $(m+n)^{th}$ term $= \dfrac{1}{A + (m+n-1)D}$

$$= \frac{1}{\frac{1}{nm} + (m+n-1)\left(\frac{1}{mn}\right)} = \frac{mn}{m+n}$$

42. (b) S = 0.2 + 0.04 + 0.00006 + ...∞

$$= \frac{2}{10} + \frac{4}{1000} + \frac{6}{100000} + \ldots\infty$$

$$S = \frac{2}{10} + \frac{4}{10^3} + \frac{6}{10^5} + \ldots\infty$$

This is AGP, with common ratio $\dfrac{1}{10^2}$.

So, $S = \dfrac{2}{10} + \dfrac{4}{10^3} + \dfrac{6}{10^5} + \ldots\infty$

$$\frac{1}{10^2}S = [\,] + \frac{2}{10^3} + \frac{4}{10^5} + \frac{6}{10^7} + \ldots\infty$$

$$S\left(\frac{99}{100}\right) = \frac{2}{10} + \frac{2}{10^3} + \frac{2}{10^5} + \ldots\infty$$

$$S\left(\frac{99}{100}\right) = \frac{\frac{2}{10}}{1 - \frac{1}{100}} = \frac{20}{99}$$

$$S = \frac{2000}{9801}$$

43. (b) $2^{1/4} . 4^{1/8} . 8^{1/16} . 16^{1/32}$

$= 2^{1/4} . 2^{2(1/8)} . 2^{3(1/16)} . 2^{4(1/32)} \ldots\ldots\infty$

$$= 2^{1/4[1 + 2\left(\frac{1}{2}\right) + 3\left(\frac{1}{2}\right)^2 + 4\left(\frac{1}{2}\right)^3 + \ldots\infty]}$$

$$= 2^{1/4[1-1/2]^{-2}}$$

$\because \{1 + 2x + 3x^2 + 4x^3 + \ldots\, \infty = (1-x)^{-2}\}$

$= 2^{1/4 \times 4} = 2$

44. (c) $\log_x y, \log_z x, \log_y z$ are in GP.

$\Rightarrow (\log_z x)^2 = \log_z y \times \log_y z$

$\Rightarrow (\log_z x)^2 = \log_x z$

$\Rightarrow (\log_z x)^3 = 1$

$\Rightarrow \log_z x = 1$

$\Rightarrow x = z$ ...(i)

Now, x^3, y^3, z^3 are in AP.

$\Rightarrow 2y^3 = x^3 + z^3$

$\Rightarrow y^3 = z^3$ {From eq. (i)}

$\Rightarrow y = z$ ...(ii)

Now, given $xyz = 64$

$\Rightarrow x^3 = 4^3$

$\Rightarrow x = 4 = y = z$

Hence, x, y, z are in both AP and GP.

45. (c) From the solution of Q. 44,

$x = y = z = 4$

So, $xy = yz = zx = 16$

Hence, xy, yz, zx are also in both AP and GP.

46. (c) $1 + \dfrac{1}{2} + \dfrac{1}{4} + \ldots + \dfrac{1}{2^{n-1}} < 2 - \dfrac{1}{1000}$

$$\frac{1\left(1 - \frac{1}{2^n}\right)}{1 - \frac{1}{2}} < 2 - \frac{1}{1000}$$

$$2 - \frac{1}{2^{n-1}} < 2 - \frac{1}{1000}$$

$$\Rightarrow \frac{1}{2^{n-1}} > \frac{1}{1000}$$

$\Rightarrow 2^{n-1} < 1000$ $\{\because 2^9 < 1000\}$

$\Rightarrow n - 1 = 9$

$\Rightarrow n = 10$

47. (d) Let pth, qth and rth terms of GP are 27, 8 and 12.

So, $AR^{p-1} = 27$

$AR^{q-1} = 8$

$AR^{r-1} = 12$

$\therefore 12^3 = 27 \times 8^2$

$\Rightarrow (AR^{r-1})^3 = (AR^{q-1})^2 (AR^{p-1})$

$R^{3r-3} = R^{2q-2+p-1}$

$3r - 3 = 2q + p - 3$

or $2q + p - 3r = 0$

There are infinite solutions for this equation.

Hence, there are infinitely many series.

48. (b) $\because a, x, y, z, b$ are in AP

and $x + y + z = 15$ ...(i)

a, p, q, r, b are in HP

and $\frac{1}{p}+\frac{1}{q}+\frac{1}{r}=\frac{5}{3}$ ...(ii)

From eq. (i),

$x+y+z=3\left(\frac{a+b}{2}\right)$

$\Rightarrow a + b = 10$ ...(iii)

From eq. (ii),

$\frac{1}{p}+\frac{1}{q}+\frac{1}{r}=\frac{3}{2}\left(\frac{1}{a}+\frac{1}{b}\right)$

$\frac{5}{3}=\frac{3}{2}\times\frac{a+b}{ab}$

$\Rightarrow ab = 9$ ...(iv)

From eq. (iii) and (iv),

$a = 9$ and $b = 1$

$\therefore a, x, y, b$ are in AP.

$\Rightarrow a = 9, x = 7, y = 5,$

$z = 3$ and $b = 1$

So, $xyz = 3 \times 5 \times 7 = 105$

Now, a, p, q, r, b are in HP.

So, $\frac{1}{9},\frac{1}{p},\frac{1}{q},\frac{1}{r},\frac{1}{1}$ are in AP.

$d=\frac{1-\frac{1}{9}}{4}=\frac{2}{9}$

$\Rightarrow \frac{1}{p}=\frac{1}{9}+\frac{2}{9}\Rightarrow p=3$

and $\frac{1}{q}=\frac{1}{9}+2\times\frac{2}{9}$

$\Rightarrow q=\frac{9}{5}$ and $\frac{1}{r}=\frac{1}{9}+3\times\left(\frac{2}{9}\right)\Rightarrow r=\frac{9}{7}$

Hence, $pqr=3\times\frac{9}{5}\times\frac{9}{7}=\frac{243}{35}$

49. (b) From solution of Q. 48.

50. (c) From solution of Q. 48.

51. (a) Given in AP,

$T_6 = a + 5d = 2$ and $d > 1$

Product of first, fourth and fifth terms

$P = a(a + 3d)(a + 4d) = (2 - 5d)(2 - 2d)(2 - d)$

$P = 8 - 32d + 34d^2 - 10d^3$

$P' = -32 + 68d - 30d^2$

For greatest value, $P' = 0$

$\Rightarrow 30d^2 - 68d + 32 = 0$

$\Rightarrow (3d - 2)(5d - 8) = 0$

$\Rightarrow d=\frac{2}{3}$ or $\frac{8}{5}$

Now, $P'' = 68 - 60d$

At $d=\frac{2}{3}, P''=28>0$ and at $d=\frac{8}{3}, P''=-28<0$

$\Rightarrow$ P is minimum at $d=\frac{2}{3}$ and maximum at $d=\frac{8}{5}$.

52. (b) From the solution of Q. 51.

For greatest product, $d=\frac{8}{5}$

So, $T_6 = a + 5d = 2$

$\Rightarrow a+5\left(\frac{8}{5}\right)=2$

$\Rightarrow a = -6$

53. (a) $a = 120°, d = 5°$

So, sum of angles $=\frac{n}{2}[2(120°)+(n-1)5°]$

$\Rightarrow \frac{n}{2}[5n+235]=(n-2)\times180°$

$\Rightarrow n^2 - 25n + 144 = 0$

$\Rightarrow (n - 16)(n - 9) = 0$

$\Rightarrow n = 9, 16$

If $n = 16$ then largest angle

$= T_{16} = 120° + (16 - 1)5°$

$= 195° > 180°$

It is not possible.

So, $n = 9$ is the correct answer.

54. (a) Largest angle $= T_9$

$= a + 8d$

$= 120° + 8 \times 5° = 160°$

55. (a) $0.3 + 0.33 + 0.333 + \ldots.\ n$ terms

$=\frac{3}{9}\{0.9 + 0.99 + 0.999 + \ldots.\ n \text{ terms}\}$

$=\frac{3}{9}\{(1-0.1)+(1-0.01)+(1-0.001)\ldots n \text{ terms}\}$

$$= \frac{1}{3}\left\{n - \left[\frac{1}{10} + \frac{1}{10^2} + \frac{1}{10^3} + ... n \text{ terms}\right]\right\}$$

$$= \frac{1}{3}\left\{n - \frac{\frac{1}{10}\left(1 - \frac{1}{10^n}\right)}{1 - \frac{1}{10}}\right\}$$

$$= \frac{1}{3}\left\{n - \frac{1}{9}\left(1 - \frac{1}{10^n}\right)\right\}$$

56. (d) Given, in AP, $S_m = n$ and $S_n = m$

$$\Rightarrow \frac{m}{2}[2a + (m-1)d] = n$$

and $\frac{n}{2}[2a + (n-1)d] = m$

$\Rightarrow 2am + (m^2 - m)d = 2n$ ...(i)

and $2an + (n^2 - n)d = 2m$ ...(ii)

From (i) – (ii)

$2a(m - n) + d\{(m^2 - n^2) - (m - n)\} = 2(n - m)$

$\Rightarrow (m - n)\ \{2a + d(m + n - 1)\} = - 2(m - n)$

$\Rightarrow 2a + (m + n - 1)d = - 2$ ...(iii)

Now, sum of $(m + n)$ terms

$$S_{m+n} = \frac{m+2}{2}[2a + (m+n-1)d]$$

$$= \frac{m+n}{2}(-2) = -(m+n) \quad \text{\{From (iii)\}}$$

57. (b) Given, $S_n = n^2 - 2n$

$T_n = S_n - S_{n-1}$

$= \{n^2 - 2n\} - \{(n-1)^2 - 2(n-1)\}$

$T_n = 2n - 3$

For fifth term, put $n = 5$

$T_n = 2(5) - 3 = 7$

58. (c) $\frac{1}{2} + \frac{3}{4} + \frac{7}{8} + \frac{15}{16} + ...n$ terms

$$= \left(1 - \frac{1}{2}\right) + \left(1 - \frac{1}{4}\right) + \left(1 - \frac{1}{8}\right) + \left(1 - \frac{1}{16}\right) + ... n \text{ terms}$$

$= (1 + 1 + 1 + ... n$ terms$)$

$$-\left(\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + ...n \text{ terms}\right)$$

$$= n - \frac{\frac{1}{2}\left(1 - \frac{1}{2^n}\right)}{1 - \frac{1}{2}} = n - 1 + 2^{-n}$$

59. (c) Let the roots be α and β.

$\alpha + \beta = - b$ and $\alpha\beta = c$

Given, $\alpha + \beta = \frac{1}{\alpha^2} + \frac{1}{\beta^2}$

$$\alpha + \beta = \frac{(\alpha+\beta)^2 - 2\alpha\beta}{\alpha^2\beta^2}$$

$$\Rightarrow -b = \frac{b^2 - 2c}{c^2}$$

$$\Rightarrow 2c = b^2 + bc^2$$

$$\Rightarrow \frac{2}{b} = c + \frac{b}{c}$$

$\Rightarrow c, \frac{1}{b}, \frac{b}{c}$ are in A.P.

$\frac{1}{c}, b, \frac{c}{b}$ are in H.P.

60. (a) Let the roots be α and β.

Then, $\alpha + \beta = -\frac{1}{a}$ and $\alpha\beta = \frac{c}{a}$

Given, $\alpha + \beta = \frac{1}{\alpha^2} + \frac{1}{\beta^2}$

$$\Rightarrow \alpha + \beta = \frac{(\alpha+\beta)^2 - 2\alpha\beta}{(\alpha\beta)^2}$$

$$\Rightarrow -\frac{1}{a} = \frac{\frac{1}{a^2} - \frac{2c}{a}}{\frac{c^2}{a^2}} \Rightarrow -\frac{c^2}{a} = 1 - 2ac$$

or $- c^2 = a - 2a^2c$

$2a^2c = a + c^2$

$\Rightarrow a, a^2c$ and c^2 are in A.P.

61. (d) $S_n = nP + \frac{n}{2}(n-1)Q$

$$\Rightarrow \frac{n}{2}\{2a + (n-1)d\} = \frac{n}{2}\{2P + (n-1)Q\}$$

On comparing $a = P$ and $d = Q$

62. (a) $6^{1/2} \times 6^{1/4} \times 6^{1/8} \times 6^{1/6} \times ... \infty$

$$= 6^{\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + ...\infty}$$

$$= 6^{\frac{1/2}{\left(1 - \frac{1}{2}\right)}} \quad \left\{\text{It is GP of infinite terms } S_\infty = \frac{a}{1-r}\right\}$$

$$= 6^{\frac{1/2}{\frac{1}{2}}} = 6$$

63. (b) Given, $a_1 = a_2 = ... = a_{10} = 150$

So, number of notes counted in first 10 minutes

$= 150 \times 10 = 1500$

But total notes are 4500.

So, remaining notes are 3000.

Given, $a_{10}, a_{11}, a_{12}....$ is an AP.

Let there are n terms in this AP.

Then, sum of AP = 3000

$\Rightarrow \frac{n}{2}[2\times(148)+(n-1)(-2)]=3000$

$\Rightarrow n^2 - 149n + 3000 = 0$

$\Rightarrow (n-24)(n-125) = 0$

$\Rightarrow n = 24$ or $n = 125$

$\because$ $n = 125$ is not possible.

So, $n = 24$

Hence, total time taken = 10 + 24 = 34 minutes

64. (b) $y = x + x^2 + x^2 + ... \infty$

It is GP of infinite terms.

So, $y = \frac{x}{1-x}\left\{S_\infty = \frac{a}{1-r}\right\}$

$\Rightarrow y - xy = x$

$\Rightarrow x(1 + y) = y$

$\Rightarrow x = \frac{y}{1+y}$

65. (c) AM – GM > 1

$\Rightarrow \frac{x_1+x_2}{2} - \sqrt{x_1x_2} > 1$

$\Rightarrow x_1 + x_2 - 2\sqrt{x_1x_2} > 2$

$\Rightarrow (\sqrt{x_1} - \sqrt{x_2})^2 > (\sqrt{2})^2$

$\Rightarrow |\sqrt{x_1} - \sqrt{x_2}| > \sqrt{2}$

66. (d) Let $S = 1.3 + 2.3^2 + 3.3^3 + ... + n.3^n$

$3S = \quad + 1.3^2 + 2.3^3 + ... + n.3^{n+1}$

On subtracting,

$-2S = 3 + 3^2 + 3^3 + ... + 3^n - n.3^{n+1}$

$= \frac{3(3^n-1)}{2} - n\cdot 3^{n+1}$

$\Rightarrow 2S = \frac{3^{n+1} - 3 - 2n.3^{n+1}}{2}$

$\Rightarrow S = \frac{(2n-1).3^{n+1}+3}{4}$

On comparing with $\frac{(2n-1).3^a + b}{4}$

We get $a = n + 1$ and $b = 3$

67. (b) Let $T_n = 121^n - 25^n + 1900^n - (-4)^n$

$\because n \in N$

So, put $n = 1$

$T_1 = 121 - 25 + 1900 + 4 = 2000$

Hence, the given expression is always divisible by 2000.

68. (c) Let the numbers be a and b then

AM = $\frac{a+b}{2}$ and GM = $\sqrt{ab}$

Given, $\frac{AM}{GM} = \frac{5}{3} \Rightarrow \frac{a+b}{2\sqrt{ab}} = \frac{5}{3}$

$\Rightarrow \frac{a+b+2\sqrt{ab}}{a+b-2\sqrt{ab}} = \frac{5+3}{5-3}$

$\Rightarrow \left(\frac{\sqrt{a}+\sqrt{b}}{\sqrt{a}-\sqrt{b}}\right)^2 = \frac{8}{2}$

$\Rightarrow \frac{\sqrt{a}+\sqrt{b}}{\sqrt{a}-\sqrt{b}} = \frac{2}{1}$

$\Rightarrow \frac{\sqrt{a}+\sqrt{b}+\sqrt{a}-\sqrt{b}}{\sqrt{a}+\sqrt{b}-\sqrt{a}+\sqrt{b}} = \frac{2+1}{2-1}$

$\Rightarrow \frac{\sqrt{a}}{\sqrt{b}} = \frac{3}{1} \Rightarrow \frac{a}{b} = \frac{9}{1}$

69. (a) $x = 1 - y + y^2 - y^3 + ... \infty$

This is GP of infinite terms.

So, $x = \frac{1}{1-(-y)} \Rightarrow x = \frac{1}{1+y}\left\{\because S_\infty = \frac{a}{1-r}\right\}$

70. (c) For G.P. $T_n = ar^{n-1}$

Given, $T_3 = 3$

$\Rightarrow ar^2 = 3$ ...(1)

Product of first five terms

$= a(ar)(ar^2)(ar^3)(ar^4)$

$= a^5r^{10} = (ar^2) = 3^5 = 243$